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"Stock PIKs"- Taking a firm by its tails

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Taking a firm by its tails

by

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Abstract

Payment-in-kind bonds (PIKs) make interest payments in the form of an issue of additional bonds rather than cash. This research provides a rationale for the recent PIK issuance by firms with low credit ratings. PIKs offer a financially constrained firm in need of restructuring both an immediate automatic stay and a prepackaged bankruptcy procedure, features that make PIKs better than alternative debt instruments. In many instances PIKs are structured to facilitate a contingent transfer of control to PIK holders, and provide an avenue of obtaining equity in the firm whether the firm value is high or low in the future. The barbell strategy of acquisition that involves a deal with the equity holders (if the firm prospects improve), and a deal with the debt holders (if the firm defaults) dominates the cost of acquisition before the firm defaults, or after the firm goes bankrupt.

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1. Introduction

A payment-in-kind bond (PIK) is a debt contract wherein interest is paid in the form of additional bonds, rather than cash. Table I summarizes data on PIK issues for the five year period 2003-07. PIKs have seen increased issuance in the recent past, with some issuance after the financial crisis of 2007. The data reveals that more than half of the issuing firms were corporations with non-investment grade ratings, that were financially constrained at the time of issue. Also, the stated purpose of the PIK issue is a restructuring of the firm operations, in more than half of these cases. A few issues are associated with Leveraged Buyouts (LBOs) wherein PIKs alleviate the interest charge during a period of restructuring, when there is no room for further debt service. In the case of PIKs associated with LBOs, PIKs can again be likened to companies with financial constraints that need restructuring.

At first glance, a PIK is not very different from a zero coupon bond. Both securities make a single bullet payment at maturity, but beyond this, PIKs and Zeros differ significantly. Zeros are issued by firms with a good credit rating, and are purchased by insurance companies, pension funds, and passive and dispersed investors. PIKs, as seen in Table I, are issued by firms with financial constraints and firms in need of restructuring, and are held by active investors, such as private equity funds and hedge funds.[2] The characteristics of firms that issue PIKs are

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[2] A review of prior year issues and articles in the press (e.g., Financial Times article "Payment in kind giving risky financing a new look", May 25, 2006) reveals that these securities have high coupon rates with equity warrants attached to the bonds, and these bonds are held by hedge funds and active investors. Warren Buffet was amongst the active investors for some of these securities.

crucial in understanding the role of PIKs and its contract design. This paper offers an analysis of the stated rationale for PIK financing. We ask - under what conditions does PIK financing make economic sense for a firm that wants to restructure? How do the features of a PIK contract facilitate a transfer of control of the firm to PIK holders, as is often the case?

PIK financing creates substantial value for existing claimholders over and above the amount raised (existing equity and debt prices would appreciate). Therefore, unlike standard debt offerings, PIK financing typically requires negotiations between the firm's existing claimholders and the new PIK investors on the terms of the deal. PIKs allow the firm to postpone coupon payments at a time of financial fragility when funds are needed to implement a restructuring plan. The automatic stay on coupons implicit in the contract provides a chance for firm prospects to improve. This automatic stay benefits both current equity holders and existing bond holders. Equity holder claims are akin to an out-of-the-money call option that gains from the PIK cash infusion, when otherwise they would be worth little. Similarly, the firm can continue to service its existing debt, and existing bond holders avoid a costly bankruptcy for the foreseeable future. PIK investors understand that a surplus (value) is created by PIK financing and, therefore, bargain for a portion of the benefits generated because of this financing. While negotiations provide a mechanism to split the surplus between the various parties, in order to be feasible PIKs must ensure that at least one party is better off while the others are not worse off.

The differences in interests and the stakes involved make the various parties agree upon the division of the surplus at the time of the PIK financing, thus avoiding a potentially costly conflict later (see Bulow and Shoven, 1978). Because equity holders garner the upside potential in firm value, they agree that, if the firm is solvent at the maturity of the PIK debt, they will pay PIK holders the principal amount plus any accumulated coupons. These payments may include a debt-for-equity swap or equity warrants if the firm is solvent at maturity of the PIK contract. In many instances, the payout gives PIK holders substantial warrants that provide an avenue to become a significant shareholder in the firm.

However, given the financial constraints of the firm, the risk that the firm will not be solvent at the maturity of the PIK bonds is high. In the event of insolvency equity holders receive nothing, and PIK holders and the existing bond holders become joint claimants to the residual assets. The partitioning of the assets of the bankrupt firm is the outcome of a bargaining process between senior bond holders and PIK holders that also occurs when PIKs are issued. In this negotiation, PIK holders attempt to claim a significant fraction of the assets of the firm in case the firm does not survive, and some even bargain for control. We show that the bargaining between PIK holders and the existing firm's bond holders affects and is affected by the bargaining between PIK holders and the firm's equity holders.

One advantage of PIK contracts over alternative debt contracts is that PIKs offer a financially constrained firm in need of restructuring both an immediate automatic stay and a prepackaged bankruptcy procedure. These features of PIK contracts reduce the deadweight costs of bankruptcy both potential costs to the claimants of a constrained firm. [3]

Besides the valuation implications, the size of the payments under PIK financing and the potential for equity swaps makes a change in control likely, and consequently PIKs are structured to facilitate this outcome. We show that PIKs are a more effective mechanism to take control of the firm than the alternative of an outright acquisition of the equity of the firm by the same investors. If PIK holders were instead to buy out the equity of the firm right away, they would still have to

[3] See Tashidian, Lease and McConnell (1996) for an empirical study of prepackaged bankruptcies. Chatterjee, Dhillon and Ramirez (1996) find that prepackaged bankruptcies are used by firms that are economically viable but face severe liquidity constraints.

provide funds for restructuring as well as for interim coupon flows on outstanding debt. In return, they would reap only the upside potential of the firm's value. They would get no payout were the firm to go bankrupt.

Similarly, PIK investors could choose not to provide financing, but let the firm go bankrupt and then purchase its assets. If restructuring costs are the same in either case, PIK financing compared to the purchase of the assets of the bankrupt firm changes a sure cost of acquisition into an expected cost of buying out current debt holders plus interim coupon funding. For reasonable parameter values, PIK financing is preferable.

What makes PIKs interesting is that they provide PIK investors with a way of obtaining equity at both high and low firm values. The barbell strategy involving a deal with the equity holders (if the firm improves) and a deal with the debt holders (if the firm defaults) dominates an acquisition from the equity holders before the firm defaults, and an acquisition from the bond holders after the firm goes bankrupt.

It is the design of a PIK contract which makes it attractive compared to other debt contracts. The drawback of regular debt financing is that it requires the firm to pay coupons at a time of low earnings (liquidity shortage). As a result, more cash is needed to pay the coupons (relative to PIK financing), and the firm is more likely to go bankrupt before the maturity of the debt. Therefore, the ex-ante firm value is higher with PIKs than if straight coupon debt is issued. Furthermore, PIK financing also mitigates the transfer of benefits of additional financing to the firm's other claimholders, thus resolving the debt overhang problem. The reason is that PIK coupons added to the face value of the debt make the claims of PIK holders grow at a faster rate than the claims of current debt holders.

Convertible bonds are not optimal either because they are bets on the upside potential of a firm and require coupon payments, even though these are lower than the coupons of regular debt.

Interestingly, the role of PIK financing is similar in many respects to that of preferred stock in later stage investments by venture capital funds. Venture capital firms provide financing in the form of preferred stock with no definite maturity that allows investors to make intermediate payments but can roll over coupons after consultation with debt holders. The roll over feature reduces the cash burning rate of the firm and the likelihood of forced stoppage. In general, the interest rates on these loans are low and these early stage firms have little or no regular debt on their books, as opposed to a setting with existing debt considered in this paper. Also, preferred stock holders can convert their loans into regular equity at a later date if the firm goes public or is bought out. At the same time, preferred securities have priority over regular equity were the firm to go bankrupt (see Sahlman (1990), Bascha and Walz (2001)).

Our model uses a continuous time approach that allows us to nest the results in a broad literature on structural models of capital structure and credit risk that have used this approach in the past. This also permits us to make the computations and numerical results comparable to the extant literature. We wish to provide a practical answer on a security design that is welfare improving relative to more standard financial contracts, and thus offer an explanation as to why some financial contracts exist in particular settings. Agency costs are implicit in this set up in that equity holders, who control the assets, maximize the value of the equity, and they are the ones that decide whether to declare bankruptcy or not.

The article is organized as follows: Section 2 describes the model of a firm with PIKs. Section 3 discusses why PIKs are an efficient contract for restructuring a financially distressed firm compared to regular debt, convertible bonds or equity.

Section 4 discusses PIKs as a mechanism for controlling the firm. Section 5 characterizes the bargaining game between the stakeholders that sets the terms of a PIK contract, Section 6 concludes the paper.

2. A model with equity, debt and PIKs

We outline a model of a firm that needs funds for restructuring. Consider a firm with productive assets that generate operating cash flows or earnings before interest and taxes of $\delta(t)$. The earnings follow a continuous time process with constant proportional volatility under the risk neutral measure:

$$\frac{d\delta(t)}{\delta(t)} = \mu dt + \sigma dz(t) \tag{1}$$

where μ is the total expected rate of increase of $\delta(t)$, σ is the instantaneous volatility of the earnings, and dz is the increment of a standard Brownian motion. If the firm were financed entirely with equity, equity holders would receive after tax a flow of $(1-\tau)\delta(t)$ at all times. The unlevered value of the firm is equal to the present value of these payments: $V(t) = \frac{(1-\tau)\delta(t)}{r-\mu}$. Our assumption about the earnings process implies that the unlevered value of the assets is lognormally distributed i.e., $Ln\left(\frac{V(T)}{V(0)}\right) \sim N\left(\left(\mu - 0.5\sigma^2\right)T, \sigma\sqrt{T}\right)$.

We assume that the firm is partly financed with debt, and that the debt is a consol bond with coupon flow *CH* per period, where *C* is the coupon rate and *H* is the face value of bonds sold originally. The cash flow requirements for coupon payments are normally met through internal cash flow generation $\left(\delta(t) > CH\right)$. If the cash flows generated by the firm $\delta(t)$ cannot cover coupon payments $\left(\delta(t) < CH\right)$, the firm faces financial constraints. In a full information setting, debt financing is motivated by tax savings (equal to $\tau\, CH$ each period), but can drive the firm into costly bankruptcy when the equity holders are no longer willing to fund these coupon payments. Let α equal the fraction of the asset value that current debt holders receive in the event that the firm goes bankrupt, leaving equity holders with nothing, where $0 < \alpha < 1$.

Assume that the firm needs additional funds to pay for restructuring costs and to keep servicing the coupon payments on outstanding bonds. This is the starting point in our analysis. At this time firm value equals V_L. This value is higher than the endogenous liquidation barrier, V_B,[4] at which the equity value would be driven to zero if there were no restructuring costs. Equity holders are unwilling to provide new funds for restructuring because the cash infusion results in a transfer of wealth to the debt holders.[5] The restructuring costs can be seen as a one-time investment made at time 0 to realign the business, without which the firm will need to suspend operations and declare bankruptcy. These costs are distinct from the bankruptcy costs $(1-\alpha)$ that are borne by debt holders were the firm to default.

Suppose that when firm value equals V_L the financially constrained firm issues a new class of claims: payment–in-kind debt (PIK).[6] A PIK is a debt contract wherein interest is paid in the form of additional bonds, rather than cash. In the next section we show why the financially distressed firm resorts to PIKs to solve its funding needs. Our objective is to clarify that PIKs are particularly useful contracts when the firm is unable to pay restructuring costs while simultaneously servicing debt obligations.

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[4] The endogenous bankruptcy barrier in the absence of restructuring costs is the point at which equity holders are no longer willing to fund losses in the firm.

[5] This is the underinvestment effect first analyzed by Myers (1977).

[6] A financially distressed firm with cash flow problems would issue additional debt because upon returning to profitability, and if the restructuring plan succeeds, the tax shields are valuable.

The amount raised from the sale of PIKs is denoted Z, and the maturity of the PIKs is set at the expected period necessary to reorganize the firm, denoted *T*. Since the cash raised from selling PIKs is used to service the existing debt and to restructure the firm, both existing debt holders and equity holders benefit from the refinancing with PIKs. Therefore, PIKs necessarily involve negotiations between the firm's existing claimholders and PIK investors (discussed in Section 5). This explains why PIKs are concentrated in the hands of active investors, normally hedge funds.

The feature that PIKs do not require intermediate interest payments and are used to pay coupon on existing debt provides the debtor with an automatic stay and avoidance of bankruptcy. This is equivalent to giving the firm liquidity when liquidity is most valuable. The payment received by a PIK holder at maturity is equal to the initial amount lent to the firm plus accrued coupons. If the firm is solvent at *T*, PIK holders receive:

$$PK(T) = Z + \frac{C_{PIK} Z}{r}\left(e^{rT} - 1\right) \tag{2}$$

where $PK(T)$ is the total payout at maturity to PIK holders, and the coupon rate on PIK debt is denoted C_{PIK}.

If the firm does not restructure immediately, it would need to file for bankruptcy. If the firm raises cash, the business would continue and improve, once the restructuring is completed at time *T*. To incorporate the effects of the restructuring, we assume that in *T* periods the value of the firms' assets (or earnings) will increase by a factor of $\lambda \geq 1$ if the firm is solvent. Any operating cash flows generated by the firm during the restructuring period are also directed toward restructuring costs, and equity holders do not receive any intermediate dividends.

Since PIKs in general have a shorter maturity than the current debt, PIKs could effectively obtain a senior priority. We assume, however, that current bonds are protected and that PIKs have equal priority with current debt holders.[7] We assume that, upon the issuance of PIKs, both existing debt and PIKs are repaid at *T*, and after this date the firm reverts to an all equity firm. This assumption is not critical to the analysis but helps disentangle the role of PIKs from the choice of the terminal capital structure and priority considerations.

If the firm were to go bankrupt at time *T*, the possible bankruptcy payouts are split: a proportion p $(0 \leq p \leq 1)$ is received by current debt holders and (1-*p*) is paid to PIK holders. In this section, we consider C_{PIK} and *p* as given parameters. Later, we relax this assumption in a model of bargaining among PIK holders, equity holders and original bond holders.

Given that the firm receives an inflow of cash (*Z*) from the sale of PIKs, we want to analyze its impact on the value of the various claims. Suppose V_{BT} is the value of firm assets at the maturity (T) of the PIK contract below which the equity holders are unwilling to pay back PIK holders and declare bankruptcy. Evidently V_{BT} is equal to the face value of existing debt (*H*) plus the terminal value of PIKs $(PK(T))$. Then, using the earnings value process in equation (1) and risk-neutral valuation, and given that $Ln\left(\frac{V(T)}{V(0)}\right) \sim N\left(\left(\mu - 0.5\sigma^2\right)T, \sigma\sqrt{T}\right)$, the price of the outstanding debt at time 0 is the value of two components – the interim coupons and the payment at time *T*:

$$D(0) = \underbrace{\frac{CH}{r}\left(1 - e^{-rT}\right)}_{Interim\ \ Coupons} + e^{-rT}\left[\underbrace{H\left(1 - N(d)\right)}_{Face\ \ Value\ \ Repaid} + \underbrace{p\alpha V_L e^{\mu T} N(d - \sigma\sqrt{T})}_{Bankruptcy\ \ Payoff}\right] \tag{3}$$

[7] The analysis with subordinated PIK debt is more complicated and does not add relevant insights to the more important issues of PIKs' role and design.

where $d = \frac{Ln(V_{BT} / \lambda V_L) - (\mu - 0.5\sigma^2)T}{\sigma\sqrt{T}}$, $N(d)$ is the cumulative normal density function and corresponds to the probability that $\lambda V(T) \le V_{BT}$, $D(0)$ is the value of existing debt at time 0, V_{BT} is the bankruptcy level at the maturity of the PIK contract, and H is the face value of debt repaid. The first term on the right hand side in equation (3) is the value of the intermediate coupon payments to existing debt holders, made possible by the sale of PIK debt. The second term has two terms in the brackets: the first is the value of existing debt if the firm becomes solvent and PIK debt is repaid (if $\lambda V(T) > V_{BT}$), and the last term is the bankruptcy payout if the firm is not solvent at maturity of the PIK contract (if $\lambda V(T) \le V_{BT}$).

The payout to PIK holders equals the sum of the payout if the firm is solvent at T plus the payout in bankruptcy. Its present value gives the value of the PIK debt:

$$PK(0) = e^{-rT}\left[\underbrace{PK(T)(1 - N(d))}_{PIK \quad repayment} + \underbrace{(1-p)\alpha V_L e^{\mu T} N(d - \sigma\sqrt{T})}_{Bankruptcy \quad Payoff}\right] \tag{4}$$

The overall firm value accrues to the three claimants: original bond holders, equity holders and PIK holders. The firm value after the issuance of PIKs is given by:

$$\begin{aligned} F(0) = & \underbrace{e^{-rT} V_L e^{\mu T}\left[\lambda\left(1 - N(d - \sigma\sqrt{T})\right) + \alpha N(d - \sigma\sqrt{T})\right]}_{Asset \quad payoffs \quad at \quad time \quad T} \\ & + \underbrace{e^{-rT}\tau\left(CHT + C_{PIK} ZT\right)(1 - N(d))}_{deferred \quad tax \quad benefits} + \underbrace{\frac{CH}{r}\left(1 - e^{-rT}\right)}_{Interim \quad Coupons} \end{aligned} \tag{5}$$

Where $F(0)$ is the firm value at time 0. The first line of the right hand side of equation (5) is the value of the firm if the firm is restructured and solvent at T, ($\lambda V(T) > V_{BT}$), and the bankruptcy payout if the firm is not solvent at the maturity of the PIK contract ($\lambda V(T) \le V_{BT}$). The second line is the tax benefit of debt that can be availed if the firm were to become solvent.[8] Equity holders, denoted by $S(t)$, are the residual claimants:

$$S(t=0) = F(0) - D(0) - PK(0) \qquad (6)$$

In summary, the firm needs a cash infusion for restructuring when its value reaches V_L. The firm raises an amount Z via an issuance of PIK debt with maturity T and a payment-in-kind coupon rate C_{PIK}, which in turn give the promised payment to PIK holders $PK(T)$ if the firm survives at T. Given the amount raised via the PIK sale and the coupons on existing debt (C), we can compute the value of the remaining claims – existing debt, equity and the firm value (equations (3) through (6)).

3. The efficiency of PIKs

In this section we show that PIKs are Pareto improving debt contracts for three reasons. First, PIKs offer a temporary workout that allows the firm to avoid the immediate costs of bankruptcy. Second, PIKs exchange zero debt payments when firm earnings are low for repayment when the firm's earnings (and value) are high. This makes the firm's capital structure approach the optimal capital structure: when the firm has low earnings it reduces interest payments, and when the firm has high earnings, the PIK debt is repaid. Third, PIKs solve the underinvestment problem that would pertain if the firm were to finance its needs with additional equity or the issuance of other forms of debt, such as straight coupon debt or convertible bonds.

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[8] We assume that PIK coupons enjoy tax benefits. Bali (2005) provides an overview of the legal issues relating to deductibility of PIK coupon payments. Consistent with our formulation, the Revenue Reconciliation Act of 1989 removed any tax timing incentives for the issuance of PIKs.

Let us first characterize the magnitude of the total gain in firm value as a result of the PIK financing, and assess the gains that accrue to debt holders, equity holders and PIK holders.

Proposition 1:

(a) *The surplus created by the introduction of a PIK contract* $\left(C_{PIK}, p, T\right)$ *is equal to the firm value after PIK funding minus the firm value in the absence of any funding, and is given by:*

$$\Pi = \underbrace{e^{-rT} V_L e^{\mu T}\left[\lambda\left(1 - N(d - \sigma\sqrt{T})\right) + \alpha N(d - \sigma\sqrt{T})\right]}_{Firm\ \ Value\ \ because\ \ of\ \ survival} + \underbrace{e^{-rT}\tau\left(CHT + C_{PIK}ZT\right)\left(1 - N(d)\right)}_{deferred\ \ tax\ \ benefits} \underbrace{- Z - \alpha V_L}_{Outside\ \ Options} \tag{7}$$

(b) *The distribution of the total surplus* Π *amongst current debt holders, equity holders and PIK holders from the introduction of a PIK contract is equal to:*

$$\Pi_D = \frac{CH}{r}\left(1 - e^{-rT}\right) + e^{-rT}\left[H\left(1 - N(d)\right) + p\alpha V_L e^{\mu T} N(d - \sigma\sqrt{T})\right] - \alpha V_L \tag{8}$$

$$\Pi_E = \lambda V_L e^{(\mu - r)T}(1 - N(d - \sigma\sqrt{T}) - e^{-rT}\left[H + PK(T) - \tau\left(CHT + C_{PIK}ZT\right)\right]\left(1 - N(d)\right) \tag{9}$$

$$\Pi_{PIK} = e^{-rT}\left[PK(T)\left(1 - N(d)\right) + (1 - p)\alpha V_L e^{\mu T} N(d - \sigma\sqrt{T})\right] - Z \tag{10}$$

Proof: See Appendix.

Feasibility of the PIK contract requires that equations (7) to (10) are individually and simultaneously positive. In equation (7), the first term in brackets in the first line is the expected value of the firm after completing the restructuring and if the firm is solvent by the time the PIKs mature. The second term in brackets in the first line is the expected value of the firm at the maturity of the PIK contract if the firm is not solvent when the PIKs mature. The first term on the second line is the gain from the interim tax deductibility of revenues, that applies because the company continues in business but that would be lost if the firm were to declare bankruptcy right away. The last two terms are the outside options of the various claimholders. These terms capture the payoffs to claim holders that are deducted from the overall gain to give the net gain from the PIK deal.

Note that the payment profile of the PIK contract is deferred and state contingent. At the outset, when the firm value is low, PIKs provide a cash infusion Z and require no coupon payments. The cash injected via the PIK sale results in enhanced earnings once the restructuring is complete (increase in firm value by a factor λ), and a part of the cash is set aside to pay coupons to the current debt holders. Equity holders repay the PIK coupon and principal at maturity only if the firm value recovers to a level V_{BT}. Thus, initially the leverage ratio -measured by the market value of the outstanding debt relative to firm value- is approximately 1, because the firm is likely to file for bankruptcy. As we show later, financing with PIKs results in a surplus to all claim holders, and the claim of equity holders is positive. Hence, the ratio of the market value of current debt plus PIKs relative to the value of the firm is less than 1. The firm value refinanced with PIKs is then closer to the optimal leverage than it was before the PIK issue.

The net surplus that accrues to debt holders (equation (8)) is composed of the same components as in equation (3): the interim coupon payments, the value of debt if the firm survives, and the share of payouts in bankruptcy at T (the latter being a prepackaged bankruptcy agreement that depends on the bargaining power of the parties at the time of the PIK financing). The last term is the payoff received were the firm to go bankrupt right away.

Similarly, equation (9) gives the surplus that accrues to equity holders. This is obtained by giving equity holders a second chance, from the PIK automatic workout. The terms in equation (10) represent the payments to PIK holders net of their cash injection, Z.

PIK holders receive compensation if the firm is either solvent or bankrupt at *T*. C_{PIK} determines the upside compensation and defines the barrier below which the firm defaults at the maturity of the PIK contract. If the firm is not solvent, PIK holders get (1-*p*) of the bankrupt firm assets. Alternatively, the PIK contract can include upside compensation in the form of cashless warrants, which would transfer wealth from equity holders to PIK holders.

From the expression for Π, the appropriateness of PIK financing depends on: (1) the *type of the firm*, namely on the deadweight costs of bankruptcy (1-α), the expected rate of earnings increase that captures in part the economic health of the firm (μ), and the volatility of earnings (σ), and (2) the *requirements and risks imposed by the restructuring*, which directly determine the terms of the contract: the amount raised via the PIK sale (*Z*), the PIK coupon rate (C_{PIK}) and the maturity (*T*).

Remark 1: *The sensitivity of the overall surplus* Π *in Proposition 1 to the type of firm and the contract type is given by (using proposition 1):*

$$\frac{\partial \Pi}{\partial \alpha} = -V_L\left(1 - e^{(\mu - r)T} N(d - \sigma\sqrt{T})\right) \tag{11}$$

$$\begin{aligned} \frac{\partial \Pi}{\partial \sigma} &= (\lambda - \alpha) V_L e^{(\mu - r)T} T\left[\frac{n(d - \sigma\sqrt{T})}{\sigma\sqrt{T}}\right]\left(\frac{d}{\sigma} + \frac{1}{\sqrt{T}} - \sqrt{T}\right) \\ &\quad + \frac{n(d)\left(\tau(CHT + C_{PIK} ZT)\right)}{\sigma\sqrt{T}}\left(\frac{d}{\sigma} + \frac{1}{\sqrt{T}}\right) \end{aligned} \tag{12}$$

$$\frac{\partial \Pi}{\partial \mu} = TF(0) + V_L e^{(\mu - r)T} T\left[\frac{n(d - \sigma\sqrt{T})(\lambda - \alpha)}{\sigma\sqrt{T}} + \frac{n(d)\left(\tau(CHT + C_{PIK} ZT)\right)}{\sigma\sqrt{T}}\right] \tag{13}$$

$$\begin{aligned} \frac{\partial \Pi}{\partial Z} &= -V_L e^{(\mu - r)T}\left[\frac{n(d - \sigma\sqrt{T})(\lambda - \alpha) + n(d)\tau(CHT + C_{PIK} ZT)}{\sigma\sqrt{T}}\right] \\ &\quad \times \left(\frac{\lambda V_L}{V_{BT}}\right)\left(\frac{e^{C_{PIK} T}}{r}\left(e^{rT} - 1\right)\right) \end{aligned} \tag{14}$$

$$\frac{\partial \Pi}{\partial C_{PIK}} = -V_L e^{(\mu - r)T} T \left[\frac{n(d - \sigma\sqrt{T})(\lambda - \alpha) + n(d)\tau(CHT + C_{PIK} ZT)}{\sigma\sqrt{T}} \right] \times \left(\frac{\lambda V_L}{V_{BT}} \right) PK(T) + \tau ZT(1 - N(d)) \tag{15}$$

where $n(d)$ and $N(d)$ denote the standard normal density function and the cumulative normal density function, respectively.

From equation (11), firms with a higher residual value after bankruptcy have a lower surplus from PIK financing, primarily because such firms have a more valuable outside option for current debt holders. Thus firms with assets that can be easily sold and with lower bankruptcy costs find PIKs a less viable avenue of financing. Equation (12) shows that PIK financing benefits firms whose gains from restructuring (λ plus tax deferral), net of the recovery amount were the firm to go bankrupt right away, are high. Equation (13) shows that an expected higher growth in earnings, μ, increases the surplus. That is, an economically sound firm has higher chances of survival from the restructuring effort that PIKs afford.

With respect to the contract features, equation (14) shows that the greater the amount Z needed for restructuring and for servicing existing debt, the lower the surplus. This is evidently so because the higher is the amount borrowed, the higher is the repayment threshold, V_{BT}, and consequently the higher must be the earnings of the firm at the point at which equity holders are willing to repay PIKs. Equation (14) shows that this higher barrier reduces the option value of the firm by the factor $\left(\frac{e^{C_{PIK}T}}{r} \left(e^{rT} - 1 \right) \right)$. Thus, the firm value and tax benefits decrease when the amount of PIK refinancing, Z, increases. Therefore, there is a level of restructuring costs above which PIKs are a less interesting form of financing. The PIK coupon rate impacts the surplus in much the same way as Z.

3.1 Regular bonds versus PIKs

Other securities can also provide financially constrained firm additional funds for restructuring. PIKs, however, allow for financing when the issue of additional regular debt, convertible debt or an immediate equity infusion is not viable. PIKs add value by reducing the probability of financial distress and by solving the underinvestment problem created by a debt overhang when other securities are not able to do so.

Suppose that a firm instead finances its needs with the sale of additional *regular debt* with maturity T and face value Z and of equal priority with existing debt. The primary difference in the interim cash flows between PIKs and regular debt is that when new regular debt is sold, the firm has to pay coupons denoted C_{reg} to the new debt holders throughout the restructuring period. In contrast, PIKs require payments when the restructuring is complete and the firm has an enhanced level of earnings. In the case of new regular debt issuance, the new debt holders would share the bankruptcy payoffs with the former debt holders in proportion to the face values of debt.

In this setting with regular debt there is a possibility that the firm may go bankrupt before the restructuring is complete if the firm is not able to service the newly issued debt. Suppose $I_{\{\tau<T\}}$ is the indicator function whose value equals 1 if the firm goes bankrupt (at time τ) and $E(I_{\{\tau<T\}})$ is the associated probability of bankruptcy before time T. Now the value of the newly issued regular debt can be written as:

$$D_{reg}(t=0,C_{reg}) = \underbrace{E\left(\int_0^T e^{-rt} C_{reg} Z I_{\{\tau>t\}}\right)}_{Interim\quad Coupons}$$

$$+e^{-rT} E\left[I_{\{\tau>T\}} \underbrace{Z\left(1-N(d(V_{BT}(reg))\right)}_{Face\quad Value\quad Repaid} + I_{\{\tau>T\}} \underbrace{\alpha \frac{Z}{Z+H} V_L e^{\mu T} N(d(V_{BT}(reg)-\sigma\sqrt{T})}_{Bankruptcy\quad Payoff} \right]$$

(16)

where the bankruptcy barrier $V_{BT}(reg) = H + Z$ and E is the expectation operator. The tradeoff between financing with regular debt relative to PIKs is that even though regular debt has a positive probability of bankruptcy before time T, the bankruptcy barrier at maturity of the regular debt contract, $V_{BT}(reg) = H + Z$, is lower in comparison with PIK debt- $V_{BT}(PIK) = H + Z + \frac{C_{PIK}}{r}(e^{rT} - 1)$. However, regular debt holders do not have the attendant clauses to ensure the upside and downside payoffs that accrue to PIK holders because of sharing in bankruptcy or via warrants. Thus, regular debt changes the chance of financial distress and may not be viable if most of the gains to the additional financing accrue to the existing claimants.

Remark 2*: a) The firm has a lower chance of survival with regular debt financing relative to PIKs when:*

$$E\left(1 - I_{\{\tau<T\}}\right) N\left(d(V_{BT}(reg)\right) < N\left(d(V_{BT}(PIK))\right)$$

where N is the cumulative normal density function and $d = \frac{Ln\left(V_{BT} / \lambda V_L\right) - (\mu - 0.5\sigma^2)T}{\sigma\sqrt{T}}$.

b) Regular debt financing does not address the underinvestment problem and is not feasible when $D_{reg}(t=0, C_{reg}) < Z$ *.*

Proof: *Follows directly from the preceding discussion.*

Remark 2 provides a simple tradeoff facing the stakeholders of a firm - when V_L is low the probability of running out of cash to pay regular debt holders is high and regular debt financing is not feasible because $I_{\{\tau<T\}}$ is high, but PIK financing is a possibility so long as the restructured firm will generate enough cash flows so that PIK holders do not face an immediate loss.

3.2 Convertible bonds versus PIKs

PIKs provide an equity stake in the company whether the restructuring goes well and the firm recovers or whether the firm does poorly and loses its residual assets to debt holders. Convertible bonds also provide equity in these states. Why then shouldn't such a firm use convertible debt to finance its restructuring costs? Alternatively, how are PIKs justified, given the possibility of convertible debt?

To answer this question, suppose the cash for restructuring *Z* is raised by attaching an option to a regular bond that allows the bond holders to convert the bond into a certain number of shares. That is, the convertible bond holders receive a coupon payment C_{conv} unless the newly issued bonds are converted to equity by time *T*. These new bonds can be converted into equity if the value of assets reaches or exceeds V_U, a value that is higher than the current value of the firm's assets and the bankruptcy boundary. For simplicity, suppose the conversion is possible at maturity of the bonds only. On conversion, suppose that the additional payout over and above the face value of debt is equivalent to γ times the value of the firm assets V_U. If, on the other hand there is not enough cash to pay the bond holders at a time before *T*, the firm files for bankruptcy and convertible bond holders split the assets with existing bond holders under equal priority. Then, the value of a convertible bond is simply the value of a regular bond plus the upside equity option:

$$D_{conv}(t=0, C_{conv}) = D_{reg}(t=0, C_{conv}) + E\left(e^{-rT}\gamma V_U I_{\{\tau>T\}}\right) \quad (17)$$

When the value of the firm assets is high relative to the interim bankruptcy barrier, the value of coupon flows to a convertible bond is lower than flows to a regular fixed coupon bond. Convertible bond holders trade off the reduced coupon flows with the potential payments on conversion. Typically convertible bonds are such bets on the upside (see Hoffmeister (1977)), but do not contain special clauses for downside payments. When firm value is very low, the conversion option has little value, and the coupon rate rises. The payment of intermediate coupons increases the likelihood of bankruptcy before the restructuring is complete. As shown before in the case of regular coupon bonds, the coupon may be too high to be feasible when the firm has low earnings. The following remark summarizes this result:

Remark 3: *Convertible debt reduces the chance of survival and does not address the underinvestment problem when:*

$E\left(1-I_{\{\tau<T\}}\right)N\left(d(V_{BT}(conv)\right) < N\left(d(V_{BT}(PIK))\right)$ *and* $D_{conv}(t=0, C_{conv}) < Z$

where $V_{BT}(conv) = H + Z$

Proof: *This follows directly from the preceding discussion.*

One way to reduce the coupon on newly issued debt is to ask the current claimholders of the firm to share with the new debtors the outcome of the restructuring, whether the outcome is positive or negative. This is precisely what PIK financing achieves. If the firm goes bankrupt, PIK holders negotiate to receive a proportion $\alpha(1-p)$ of the residual value of the firm that is higher than the ratio received by convertible or regular debt holders. Thus, the sensitivity of the value of PIKs to sharing one dollar in the downside is, in general, much smaller than the sensitivity of the value of the convertible security in this scenario. On the other hand, if the firm recovers, PIKs receive a larger share than convertibles and regular bonds. Therefore, the payment PIKs receive in the extremes is higher than the payments received by convertibles – i.e., the butterfly payoff in the case of PIKs is higher than that for convertibles. For current bondholders to continue to be paid with the funds raised from PIKs, they must give up a sizeable portion of their claim in the event that the firm does not recover from the restructuring effort. This is *precisely* why PIKs always involve a negotiation between PIK holders and current bondholders, something that does not occur when convertible bonds are issued. Convertibles are held mostly by widely dispersed investors, while holdings of PIKs are highly concentrated. Furthermore, convertible bond holders do not have expertise in managing the assets of a firm as PIK holders do. These salient features characterize the difference between PIKs and convertibles, even when both securities have equity payments on the upside as well on the downside. [9]

3.3 Equity versus PIKs

Next we show that PIKs solve the underinvestment problem created by the refusal of equity holders to provide additional funds at time 0, although equity holders are willing to retire PIKs at *T*. Equity financing is not incentive compatible at time 0 because all the gains accrue to existing bond holders while equity holders bear an immediate cost. But why would equity holders be willing to refinance the PIK debt at maturity *T*, using equity, when they would not finance the firm at time 0? Because the repayment of PIK debt by equity holders occurs *only if* the firm's

[9] The type of coupon- floating or fixed- is not central to this argument. Because floating rate convertibles require coupon payments, they would generally be subject to the same limitations as regular convertibles as long as the floating rate is tied to some interest rate index that is independent of the firm behavior.

asset value has recovered to some level higher than what it was at the outset, and the firm does not need to incur any additional restructuring costs beyond that date.

For a contract to be feasible, PIK holders must account for the ex-post behavior of equity holders, and that behavior should be consistent with ex-ante expectations. A PIK contract requires the PIK holders to impose the condition that equity holders do not face a loss when they inject cash. If the amount of equity infusion required at the maturity of the PIK contract is more than the value of the equity in the firm after this infusion, equity holders would allow the firm to go bankrupt. Thus, the level of assets below which equity holders will be unwilling to pay back PIK holders, V_{BT}, must imply that equity holders do not bear an immediate loss. Therefore, the contingent financing (payment only if the firm value is above V_{BT}) does not penalize equity holders with an immediate loss.

Remark 4: *Cash infusion by equity holders to retire PIK debt at maturity does not result in a transfer of wealth to senior bond holders, and a corresponding loss to equity holders.*

Proof: See appendix

The features of the PIK contract play an important role in mitigating wealth transfers from PIK holders to the other claim holders, especially when existing bond holders continue to be serviced with the money raised from selling PIKs. PIK financing avoids a wealth transfer to equity and debt holders because PIK coupons are cumulated on the face value of the bond- the claim of PIK holders grows at a faster rate than the claims of current debt holders or equity holders. The higher PIK coupon requires more of a cash infusion from equity holders later if the firm is solvent, and also a higher level of sharing if the firm goes bankrupt. Thus, PIKs address underinvestment by mitigating wealth transfers amongst claimants, and at the same time increase firm value and, consequently, the firm's debt capacity.

Gertner and Scharfstein (1991) explain how the issuance of bank debt in the presence of outstanding public debt can affect incentives to invest in a financially constrained firm. The possibility of running into problems in the future and the need to renegotiate makes concentrated bank debt more easy to roll over if it is needed. Bank debt plays a similar role for Gertner and Scharfstein to the role of PIKs herein, which alleviate the debt overhang problem and foster additional investment. The simplicity of rolling over bank debt is in a way equivalent to the automatic delay of the payment of coupons in PIK debt. While in Gertner and Scharfstein concentrated bank debt solves the underinvestment problem associated with debt overhang by institutional design and ex-post renegotiation, PIKs solve the problem by contract design and ex-ante bargaining.

4. Controlling a financially distressed firm with PIKs

Next we show that PIKs provide an effective avenue for controlling the firm. In a particularly interesting case, New Look (an apparel company) structured a cash infusion that was financed by the management via a PIK offering instead of a management buyout. A PIK contract achieved that more inexpensively than other more obvious alternatives. Consider two obvious ways that investors may try to gain control of a distressed firm in need of restructuring: (1) by purchasing the outstanding equity, or (2) by purchasing the assets of the firm after the firm has gone bankrupt.[10]

[10] We do not consider an exchange offer to replace existing debt, since an exchange offer requires buying all the debt of the firm, as well as funding all the restructuring costs. It is cheaper to buy the assets of the bankrupt firm.

The first alternative, an outright purchase of the equity of the firm, requires investors to fund restructuring costs as well as interim coupon flows. For the same time horizon (*T*), control via the acquisition of equity requires the same amounts of funds as PIK financing, but offers control only if the firm is solvent. PIK financing, on the other hand, offers the possibility of controlling the firm both on the upside (in case of a swap of the final payout on PIKs for equity and any warrants), and on the downside if the firm goes bankrupt (when 1-*p* is negotiated to be higher than 0.5). Thus, for the same investment, there is a higher probability of gaining control of the firm through PIKs even though the probability of survival may increase with equity financing.

Allowing the firm to go bankrupt and purchasing firm assets immediately after that, will incur costs of αV_L, but save the interim coupons $\frac{CH}{r}\left(1-e^{-rT}\right)$ to the current bond holders. If restructuring costs are the same in either case, PIK financing relative to the purchase of the assets of the bankrupt firm changes a sure cost of αV_L - $\frac{CH}{r}\left(1-e^{-rT}\right)$ for an expected cost of $p\alpha V_L e^{(\mu-r)T}N(d)$, for purchase of the residual assets of the bankrupt firm. For reasonable parameter values (α, p), PIK financing is preferred. These results are summarized in the following proposition:

Proposition 2: *PIK financing is a more efficient way to gain control of the distressed firm than:*

(a) *An outright purchase of the firm's equity.*

(b) *The purchase of the assets of the firm after it goes bankrupt when* αV_L - $\frac{CH}{r}\left(1-e^{-rT}\right) > p\alpha V_L e^{(\mu-r)T}N(d-\sigma\sqrt{T})$.

Proof: *See Appendix*

The primary advantage of a PIK contract, from the perspective of PIK investors is that PIKs get a fraction of the firm both on the upside and on the downside. This barbell strategy allows PIK holders to get a portion of the surplus on both sides of the return spectrum, and that proves to be better than other ways of getting control, which controls the firm only on one side – equity on the upside, and straight debt on the downside. The likelihood of obtaining control with PIK debt depends on the features of the contract, (C_{PIK},p,T), which depend on the bargaining power of the PIK investors compared to the firm's current claim holders.

Remark 5*: PIK financing results in PIK holders gaining control from a barbell like payoff to the PIK holders –*

(a) *On the upside, if the firm does well, PIK holders can swap their terminal payoff for equity or obtain warrants. The probability of control conditional on solvency is defined as:*

$$\text{Prob}\left\{S\left(T^{+}\right) < 2PK(T) \mid V_{T^{+}} > V_{BT}\right\}.$$

(b) *On the downside they gain control via the sharing of bankruptcy payoffs when (1-p) > 0.5.*

Proof: *See Appendix*

Figure 1 graphs the probability of control for different coupon rates and different maturities of PIKs conditional on firm solvency. By control we mean that PIK holders are allowed to swap their debt repayment for an equivalent number of shares. An increase in the coupon rate increases the face value of the PIK bond and the number of outstanding shares that are transferred to PIK holders. Note, however, that an increase in the coupon rate increases the payment to PIK holders while at the same time it reduces the chance that the equity value will exceed the payment to PIK holders by enough for equity holders to retain control.

5. Surplus sharing with bargaining

An important aspect of any refinancing is its implication for the distribution of the overall gain among the parties. The extent of the surplus (value creation beyond the amount infused) depends on the terms of the PIK contract, (C_{PIK}, p), for a given contract maturity *T*, which is set by the estimated interval needed to restructure the firm. The very different interests and the need to avoid potential conflicts motivate the parties to agree on the partition of future profits at the moment of the PIK refinancing. We assume that the outcomes involving the PIK refinancing are contractible at time 0.

Here, we first consider the sharing of the gains when there is a single take-it or leave-it offer. Next, we consider a more detailed setting, where the PIK holders negotiate with the debt holders and the equity holders separately to set the sharing and the terms of the PIK contract.

5.1 Distribution of gains with a take it or leave it offer

Suppose that the various claimholders have one opportunity to say yes or no to the proposed refinancing of the firm with PIK debt, with given terms. Each of the players has a veto in that any player can negate any proposed agreement by refusing to agree. In the case of disagreement, the PIK refinancing deal collapses, and equity holders receive 0, the firm defaults and current debt holders receive αV_L. Then, PIK holders keep their cash (*Z*) that would otherwise be injected into the firm. If a deal is approved, the surplus Π characterized in equation (7) is created. One strategy for player *i* where $i \in (D, E, PIK)$ is to ask for a share a_i of the surplus, net of the outside options:

$$a_E + (a_D - \alpha V_L) + (a_{PIK} - Z) = \Pi \tag{18}$$

Given that the total of the asked surplus from the claimants is equal to the total surplus, no money is left on the table. If a player asks for more than the total surplus Π, it would imply a negative payoff to at least one other player. The Nash bargaining solution is the outcome that maximizes the product of the utilities of the various players (assuming risk neutrality):

$$\begin{aligned} &Max(a_D - \alpha V)a_E(a_{PIK} - Z) \\ &s.t. \quad a_D + a_E + a_{PIK} = \Pi + \alpha V_L + Z \end{aligned} \tag{19}$$

Appendix C shows that the solution to this maximization problem is $a_D^* - \alpha V_L = a_{PIK}^* - Z = a_E^*$. This solution is efficient and invariant to the disagreement point. It shows that the total surplus is split among the three claim holders, taking into account their different outside options, such that equations (8) to (10) are all equal, $\Pi_D^* = \Pi_E^* = \Pi_{PIK}^*$. Claim holders with the better outside options come out of the bargaining game in a better position. Equity holders will get less, followed by PIK holders (because in general $Z < \alpha V_L$) and debt holders will receive the biggest share of all.

5.2 Separate bargaining with claimholders

The Nash equilibrium solution in the previous section assumes that the various claim holders will get together to decide on the split of the surplus when they have equal bargaining power. It is more realistic to think that PIK holders conduct separate discussions with equity holders and with debt holders, and the outcome of these negotiations set an optimal pair (C_{PIK}^*, p^*) for a given *T*.

Consider first how C_{PIK} is linked to the surplus obtained by equity holders. Equity holders earn a surplus only if the firm recovers and is solvent at time *T*. C_{PIK} determines the bankruptcy barrier at time *T*, since coupons are paid in kind and accrue to the face value of PIK debt. Equity holders are willing to pay back

PIK holders the amount $PK(T) = Z + \frac{C_{PIK} Z}{r}\left(e^{rT} - 1\right)$ at the maturity of the PIK debt contract only if the residual value of the equity, after this payment, is higher than $PK(T) + H$

If the firm value is below V_{BT}, equity holders refuse to pay PIK holders and the firm is declared bankrupt. Therefore, the outcome of the bargaining game between equity holders and PIK holders has a decisive impact on the likelihood of bankruptcy when PIKs mature. C_{PIK} determines not only the split of the surplus between equity holders and PIK holders, but also the expected deadweight losses from bankruptcy.

The fraction p determines the sharing of payouts at T if the firm is bankrupt. Note that the partitioning of assets involves PIK holders and current debt holders only. Even if equity holders do not garner any part of the residual assets, however, changes in p are reflected in changes in C_{PIK}, and vice-versa. Therefore, C_{PIK} and p affect one another. PIKs, besides providing an immediate automatic stay in exchange for a higher payoff afterward, are structured as a prepackaged bankruptcy procedure. The terms of the automatic stay and the prepackaged bankruptcy involve all the claimholders because of their interdependence. These features of PIKs – automatic stay and prepackaged bankruptcy- can reduce the deadweight costs of bankruptcy, which are significant for the types of firms we are discussing.

The lower bound on the extent of sharing of bankruptcy payouts by the current debt holders in bankruptcy is given by setting the value of debt in equation (3) equal to αV_L. This is the amount that current debt holders receive if the firm defaults right away. A solution to this equation provides the minimum p that current debt holders are willing to accept.

Remark 6*: The minimum proportion of payoffs in bankruptcy that current debt holders are willing to accept is given by:*

$$p_{debt}^{min} = \frac{\alpha V_L - \frac{CH}{r}\left[1 - e^{-rT}\right] - e^{-rT} H(1 - N(d))}{\alpha e^{-rT} N(d - \sigma\sqrt{T})} \tag{20}$$

Proof: Set the value of debt in equation (3) equal to αV_L and solve for *p*.

Comparative statics allow us to analyze how this minimum level of payout to debt holders in bankruptcy depends on α and on C_{PIK} (see results in Appendix C). A higher α increases the minimum level of payouts acceptable to debt holders in direct proportion to V_L. This occurs because current debt holders are trading off bankruptcy right now versus accepting a PIK financing and sharing the proceeds with PIK holders if later the firm goes bankrupt. With a higher α, a higher amount is received by debt holders if the firm goes bankrupt right away, thus p_{debt}^{min} increases. Also, a higher α increases the present value of the debt, conditional on firm solvency at maturity, and therefore reduces p_{debt}^{min}, although the magnitude is smaller than the first term. Overall, an increase in α increases p_{debt}^{min}. Similarly, an increase in C_{PIK} reduces p_{debt}^{min} because a decrease in the probability that the firm will survive and receive the face value of debt decreases p_{debt}^{min}.

There is also a minimum level of bankruptcy payouts that must be made to PIK holders for them to accept the deal. Recall that the amount contributed by PIKs depends on the amount of cash flows, *Z*, needed to finance the operations of the firm for a given period. PIK holders receive an amount $PK(T)$ at maturity. Then, for PIK financing to be feasible, an appropriate payout is needed if the firm were to go bankrupt. Using equation (4) and solving for *p* we get:

Remark 7*: The maximum proportion of payoffs* $p_{debt}^{\max}$ *in bankruptcy that PIK holders are willing to pay debt holders is given by:*

$$p_{debt}^{max} = 1 - \frac{Z - \left[PK(T)(1 - N(d)\right]}{\alpha\left[e^{(\mu - r)T} V_L N(d - \sigma\sqrt{T})\right]} \tag{21}$$

Proof: Set the value of PIK in equation (4) equal to Z and solve for *p*.

Again, we analyze how this maximum level of sharing depends on the residual proportion of assets that debt holders receive in bankruptcy, α, and the coupon rate on the PIK debt C_{PIK}. As α increases, p_{debt}^{max} increases because it increases the assets left in bankruptcy. Similarly, an increase in C_{PIK} decreases p_{debt}^{max}, because of an increase in the chance of bankruptcy, but on the other hand it increases p_{debt}^{max}, because the value of PIK debt ($PK(T)_{solv}$) increases.

The outcome of the bargaining game between PIK holders and equity holders, on the one hand, and between PIK holders and debt holders, on the other hand, is characterized by the domain Ω for parameter pairs (C_{PIK}, p), and a given *T*, that make the contract feasible and efficient:

$$\Omega \equiv \underset{\substack{0 \le p \le 1 \\ C_{PIK} \ge r}}{(C_{PIK}, p)} \;\; s.\;\; t. \;\; \{\Pi_{PIK} + \Pi_E, \Pi_D \ge 0\}. \tag{22}$$

Figure 2 graphs the feasible pair (C_{PIK}, p) for a PIK contract of maturity *T*, given other parameters that determine the value of the levered firm and the losses in case of bankruptcy. The upper boundary of this domain corresponds to the payout restrictions for debt holders set in equation (20). The lower boundary corresponds to the payout restriction set in equation (21). The domain clearly shows that the coupon rate C_{PIK} restricts the choices for the bankruptcy payout *p* and vice versa. For example, the horizontal line in Figure 2 depicts that the maximum feasible values of the fraction of the asset values in bankruptcy, p, that current debt holders receive 0.22 when the PIK coupon rate is 14%. In the example, we have assumed that the maturity of the PIK contract is T=2 years and α =0.5

The total surplus is maximized when C_{PIK} is set at the lowest possible level. This coincides with the maximum surplus to equity holders. This is because the only source of market imperfection is related to bankruptcy and missing the opportunity of a recovery, and a low PIK coupon level will minimize the probability of going bankrupt and the resulting deadweight costs of bankruptcy. Hence, the incentives of equity holders and PIK holders are aligned in the sense that each wants the minimum possible coupon in order to increase the total surplus and their own portion of the gains. The fact that debt holders and PIK holders incentives are not aligned, because each wants to negotiate for a higher fraction of the payout in bankruptcy, means there are incentives for bargaining between PIK holders and debt holders. This bargaining, in turn, makes equity holders and PIK holders bargain too.

Note that PIK holder surplus is maximized when the coupon rate on PIKs is minimized (because it increases the chance of survival) and is coupled with a minimum possible payout to debt holders (lowest possible p). Indeed, it is possible to show that when PIK holders have all the bargaining power and are free to set the pair (C_{PIK}, p), the optimal parameter values correspond to the lowest possible coupon and the payout given by the solution to equation (26). Debt holder surplus is maximized when the payout in bankruptcy to debt holders is maximized (maximum p), given C_{PIK}. Also, given the parameter p, debt holders benefit with a lower C_{PIK}, because there is a larger probability that the debt holders will be able to enjoy continued coupon payments if the firm is healthy at the maturity of the PIK contract.

Note that in many cases, $p_{debt}^{\min} \leq p \leq p_{debt}^{\max}$, leads to PIK holders taking control of the firm in bankruptcy. When PIK holders have much of the bargaining power, and they want to get control, they cannot just constrain their offer to $p<0.5$ in the event of bankruptcy, but also must choose the payoffs that maximize their chance of being in control on the upside. On the upside, gains can be transferred via cashless warrants.

6. Conclusions

We examine why financially constrained firms issue payment-in-kind bonds (PIKs) for restructuring their operations. The introduction of PIK debt increases the value of a financially constrained firm in need of cash for restructuring over and above other forms of financing, such as regular debt, convertibles and equity.

The fact that the introduction of a PIK contract increases firm value opens up the possibility of bargaining among claimants. In the bargaining game, the terms of a PIK contract involve negotiations between PIK holders and equity holders, and between PIK investors and original bond holders. In this sense, PIKs contracts simultaneously provides an immediate automatic stay and a prepackaged bankruptcy procedure, features that reduce the deadweight costs of bankruptcy that is of significant importance for the types of firms involved in this form of financing.

PIKs, held mostly by activist investors, are a more effective mechanism to take control of a firm than the alternatives of an outright acquisition of its equity, or letting the firm go bankrupt and then buying its assets. What makes PIKs effective for control purposes is that they provide investors with a barbell strategy that obtains equity at extremes of the distribution of firm values- at both high and low firm values. PIKs can thus be seen as bets by creditors on the volatility of a firms' prospects.

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Appendix A

Proof of Proposition 1

The surplus is equal to the value of the firm with PIK financing less the value of the outside options of the claim holders: $\Pi = F(0) - Z - \alpha V_L$. Substituting the value of the firm using equation (5) gives the desired result. The surplus to debt, equity and PIK holders follows directly from the fact that the surplus to each claimant is equal to the market value after PIK financing minus their outside options, using equations (3), (4) and (6). The surplus division is efficient in the sense that the total surplus is equal to the sum of the three parts.

Proof of Remark 4

Equity holders are willing to retire and pay back PIK holders the amount $PK(T) = Z + \frac{C_{PIK} Z_{PIK}}{r}\left(e^{rT} - 1\right)$ at the maturity of the PIK debt contract only if the residual value of the equity, after this payment, is higher than $PK(T) + H$. Hence, at time T, after the PIK principal and its accrued coupons are paid by equity holders, it must be that $S\left(V\left(T^{+}\right)\right) \geq PK(T) + H$. Thus equity holders gain from the second chance provided by the PIK sale and the contingent financing does not result in an immediate loss to equity holders. Regular bond holders approve the deal only if their payoff is more than their outside option.

Proof or Proposition 2

An outright purchase of the equity of the firm, costs the investors the restructuring costs as well as interim coupon flows: $Z + \frac{CH}{r}\left(1 - e^{-rT}\right)$. Payoffs to equity holders are 0 if the firm goes bankrupt with no control and they get control if the firm is solvent. PIK financing, on the other hand, offers control of the firm both on the upside (in case of a swap of the final payout on PIKs for equity and any warrants), as well as on the downside if the firm goes bankrupt (1-p>0.5). Thus, for the same investment, there is a higher probability of gaining control of the firm through PIKs.

Purchase of firm assets right after bankruptcy costs $\alpha V_L + Z$ but saves $\frac{CH}{r}\left(1 - e^{-rT}\right)$. Because costs of restructuring Z are incurred in either case, purchase of the assets of the bankrupt firm relative to PIK financing changes a sure cost of $\alpha V_L - \frac{CH}{r}\left(1 - e^{-rT}\right)$ incurred now to an expected cost of $p\alpha V_L e^{(\mu - r)T} N(d)$ incurred if the firm is in bankruptcy at maturity.

Proof of Remark 5
If the PIK covenant allows for a debt for equity swap, on the upside the PIK holders gain control if their equity stake is greater than 0.5. On the downside, were the firm to go bankrupt, PIK holders gain control if 1-*p*>0.5. Cost of control with an outright purchase of the firm 's equity costs Z (or more) because the firm will have to fund restructuring costs as well as fund interim coupon flows until the firm is solvent. For the same time horizon (*T*), control costs the same but provides control only on the upside (i.e., conditional on survival of the firm).

Appendix B

To solve the bargaining game we use the Lagrange multiplier function:
$L = (a_D - \alpha V_L) a_E (a_{PIK} - Z) - \lambda(\sum_i a_i - \Pi - \alpha V_L - Z)$, which depends on the vector of asks, a_i, as well λ, requiring that: $\frac{\partial L}{\partial a_D} = a_E (a_{PIK} - Z) + \lambda = 0$, $\frac{\partial L}{\partial a_E} = (a_D - \alpha V)(a_{PIK} - Z) + \lambda = 0$, $\frac{\partial L}{\partial a_{PIK}} = a_E (a_D - \alpha V_L) + \lambda = 0$ and $\sum_i a_i = \Pi + \alpha V + Z$. Solving gives $a_D^* - \alpha V_L = a_{PIK}^* - Z = a_E^*$. This solution is efficient and invariant to the disagreement point. It shows that the total surplus is split among the three claim holders, taking into account their different outside options, such that $\Pi_D^* = \Pi_E^* = \Pi_{PIK}^*$.

Appendix C

Comparative static results for $p_{debt}^{\min}$ and for $p_{debt}^{\max}$:

$$\frac{\partial p_{debt}^{min}}{\partial \alpha} = \frac{\frac{CH}{r}\left[1 - e^{-rT}\right] + e^{-rT} H (1 - N(d))}{\alpha^2 e^{-rT} N(d - \sigma\sqrt{T})} > 0 \qquad \text{(C1)}$$

$$\frac{\partial p_{debt}^{min}}{\partial C_{PIK}} = -\frac{e^{-rT} H}{\alpha e^{-rT}\left(N(d - \sigma\sqrt{T})\right)} \left(n(d)T \frac{\lambda V_L}{V_{BT}} PK(T) \right)$$

$$- \frac{\alpha V_L - \frac{CH}{r}\left[1 - e^{-rT}\right] - e^{-rT} H (1 - N(d))}{\alpha e^{-rT}\left(N(d - \sigma\sqrt{T})\right)^2} \left(n(d - \sigma\sqrt{T})T \frac{\lambda V_L}{V_{BT}} PK(T) \right) < 0 \qquad \text{(C2)}$$

$$\frac{\partial p_{debt}^{max}}{\partial \alpha} = \frac{Z - PK(T)(1 - N(d))}{e^{(\mu - r)T} V_L N(d - \sigma\sqrt{T})(\alpha)^2} > 0 \tag{C3}$$

$$\frac{\partial p_{debt}^{max}}{\partial C_{PIK}} = -\frac{Z - PK(T)(1 - N(d))}{\alpha V_L e^{(\mu - r)T} \left(N(d)\right)^2} \left(n(d - \sigma\sqrt{T})T \frac{\lambda V_L}{V_{BT}} PK(T) \right) + \frac{PK(T)\left(T + n(d)T \frac{\lambda V_L}{V_{BT}} PK(T) \right)}{\alpha V_L e^{(\mu - r)T} N(d - \sigma\sqrt{T})} \tag{C4}$$

Table 1
Recent issues of Payment-In-Kind bonds

This table provides a list of firms that have issued payment-in-kind bonds over the years 2006 and 2007. Data is obtained from Standard and Poors, Bloomberg Information Services and from reports on company web sites. The table reports the approximate proceeds in millions of dollars as well as the long term credit rating assigned to the company by S&P at the time of the issue.

Year	Issuer	Purpose	Industry	($ mill)	S&P	Spread
2007	BMS	Dividend	Info Tech	149	CCC+	700
2007	TerreStar Networks Inc.	Restructuring	Telecom	500	NA	1027
2007	Beverages & More	Expansion	Retail	29	NA	905
2007	Marsico	Dividend	Investment	400	CCC+	800
2007	Marsico	Dividend	Investment	275	NA	800
2006	Hard Rock Park	Restructuring/Exp	Entertainment	50	NA	973
2006	Tim Hellas	LBO	Telecom	610	NA	825
2006	Houghton Mifflin	Restructuring	Publishing	297	CCC+	675
2006	Pipe Holdings	Restructuring	Manufacturing	121	NA	825
2006	UGS	Restructuring	Software	297	B-	500
2006	American Achievement	Restructuring	Manufacturing	147	CCC+	863
2006	Libbey Glass	Restructuring	Manufacturing	100	NA	1152
2006	GNC	Restructuring	Lifestyle prod.	421	CCC+	675
2006	Eircom	Restructuring	Telecom	545	B-	700
2006	United Components	Restructuring	Manufacturing	227	CCC+	700
2006	Hellas Finance	No Information	Investment	262	NA	800
2006	Panrico	LBO	Food Products	290	NA	NA
2005	Aero Invest	Dividend	Aircraft leasing	478	NA	850
2005	Ardagh Glass	Inves/Restructuring	Manufacturing	535	CCC	800
2005	Innophos	Restructuring	Chemicals	120	NA	975
2005	K&F aircraft brakes	LBO	Manufacturing	55	NA	850
2005	Malcolm Glazer	LBO	Sports	515	NA	1150
2005	New Look	Restructuring	Apparel	645	NA	1050
2005	Viasystems	Restructuring	Computer	100	NA	750
2005	Warner Music Group	Restructuring	Entertainment	200	B+	700
2005	Wornick	LBO	Food Products	26	B+	1050
2004	Cognis Holding	Div/Bridge to IPO	Chemicals	675	B+	900
2004	Eco-bat	Dividend	Recycling	318	NA	700
2004	ISS A/S (PurusCo)	Investment	Investment	170	B+	800
2004	Jefferson Smurffit	Dividend	Packaging	414	CCC	850
2004	Kabel Deutschland	Div/Bridge to IPO	Telecom	510	BB+	850
2004	Sealy Corporation	LBO	Manufacturing	75	NA	NA
2004	VNU World	LBO	Investment	127	NA	NA
2003	Norcross Safety Products	Acquisition/Exapnsion	Manufacturing	100	B-	900
2003	Northrop Grumman	Restructuring	Aviation	600	BB+	900
2003	TDC A/S	LBO	Telecom	382	NA	NA

Figure 1

Probability of control in solvency

This figure provides a graphical depiction of the probability that PIK holders will control the firm conditional on the firm being solvent. The Y-axis is the probability that PIK holders will gain control and the X-axis is the ratio of the firm value at which the firm faces restructuring costs (VL) to the bankruptcy barrier in the absence of such one-time costs (VB). We assume that current debt has a constant coupon flow $CH = 20$, the risk free rate $r = 0.05$, tax rate $\tau = 0.35$, recovery rate $\alpha = 0.5$ and asset volatility $\sigma = 0.45$, $\lambda = 1.5$ and $p = 0.5$.

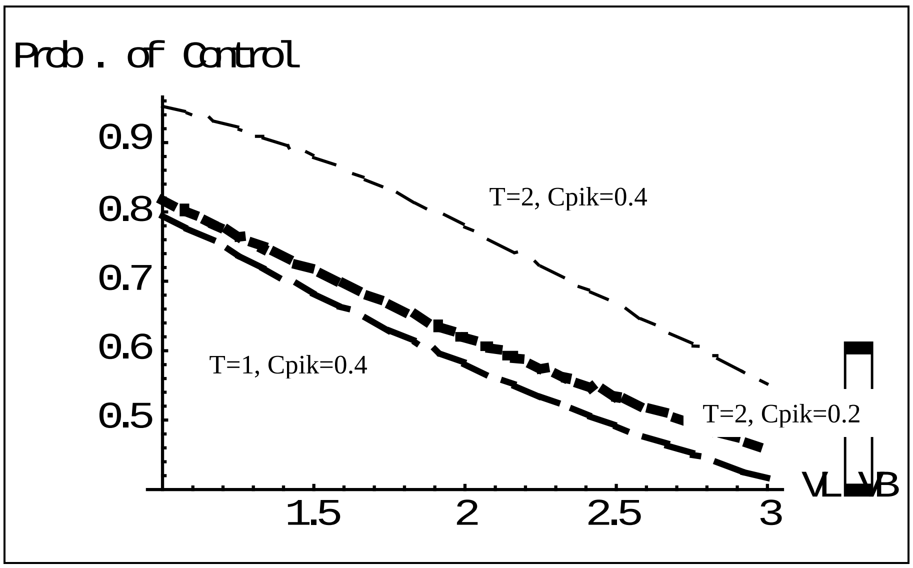

Figure 2
Domain of Feasible Parameter Values and the Surplus

This figure provides a graphical depiction of the domain of feasible values (Cpik, p) for the PIK contract. We assume that current debt has a constant coupon flow $CH = 20$, the risk free rate $r = 0.05$, tax rate $\tau = 0.35$, $Z = 25$, $\lambda = 1.5$.