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The Cost of Capital, Asset Prices, and the Effects of Monetary Policy*

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Abstract

The primary objective of this paper is to study the interaction between monetary policy, asset prices, and the sources of technological progress. We develop a two sector model in which financial institutions promote risk sharing and fiat money alleviates trade frictions. Since the price of capital goods depends on inflation, the Friedman Rule may be sub-optimal. In addition, different sources of productivity can affect the degree of risk sharing. Although the optimal money growth rate falls in response to an increase in productivity in either sector of the economy, monetary policy should react more aggressively to investment-specific productivity. Our results are broadly consistent with U.S. monetary policy during the postwar period.

JEL Classification: E44, E52, E31, O16

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1 Introduction

One of the primary goals of central banks is to determine the degree of policy intervention to regulate the amount of investment activity in the economy. Obviously, productivity growth is an important driving force for investment. However, recent work identifies multiple sources of growth. Notably, Greenwood, Hercowitz, and Krusell (1997) attribute nearly 60% of economic growth in the United States to productivity growth in the capital sector.¹ The remaining 40% stems from neutral technological progress. Moreover, Greenwood and Jovanovic (1999) and Hobijn and Jovanovic (2001) demonstrate that capital embodied productivity results in lower equity prices. This implies that the source of productivity growth in the economy bears significant implications for the direction of asset prices and investment activity. While technological change in the capital sector can cause equity prices to fall, neutral growth raises the demand for capital. Consequently, these arguments suggest that monetary policy should be chosen according to the sources of technological progress.

This paper seeks to develop a framework to study the relationships between monetary policy, asset prices, and technological change. In particular, our approach features two primary elements. First, following Abel (2003), we construct an adjustment costs model of investment in which the relative price of equipment is endogenous. Second, individuals also encounter idiosyncratic risk.^{2,3} While financial institutions help insure against such risk, monetary policy affects the efficiency of risk sharing. Interestingly, as the economy includes two sectors of production, the level of productivity in each sector not only has an impact on the relative price of capital – it also has important implications for the degree of risk sharing that occurs. In this manner, there are interesting connections between monetary policy and the sources of productivity in the economy.

We proceed by providing specific details about our modeling framework. As in Schreft and Smith (1997), we construct a two-period overlapping generations model in which individuals encounter liquidity risk and information frictions generate a transactions role for money. In the model, individuals are born in two different geographically separated locations. Within each location, agents have complete information regarding others' asset holdings. In contrast, across locations, there is incomplete information such that individuals do not have the ability to establish and trade claims to assets. In this manner, private information leads to a transactions role for money. Therefore, if an individual is forced to trade outside of his location of residence, he must liquidate asset holdings and acquire money balances. Moreover, monetary policy affects the degree to which individuals are insured against liquidity risk.

¹In addition, they determine that the relative price of capital goods has fallen by around 3% per year in the United States during the postwar period. See also the discussion in Jones (1994).

²Abel (2003) studies the effects of aggregate uncertainty from random population growth on asset prices. In his work, demographic shocks affect the price of capital goods.

³In Greenwood, Hercowitz, and Krusell, both neutral and capital-embodied productivity shocks occur. However, they study a representative agent, infinite-horizon economy. Consequently, idiosyncratic shocks do not take place and there is no role for risk sharing between groups of individuals.

In addition to two separate locations, we consider a production economy in which there are two different production sectors: a consumer goods sector and a capital sector. In the capital sector, firms incur adjustment costs and the equilibrium price reflects decisions by two different groups of participants in the market. On the supply side, firms produce capital in order to maximize profits. On the demand side, financial institutions (which we refer to as banks) seek to acquire capital goods based upon the degree of liquidity risk and anticipated earnings from capital. In particular, banks choose diversified portfolios of assets in order to provide risk pooling services. In this manner, the attitudes of individuals regarding their tolerance for risk will affect portfolio choices.

As a benchmark, we study an economy in which individuals' demand for money is inelastic and production in each sector is of the Cobb-Douglas form. In this setting, the Fisher equation holds and inflation only affects the value of real balances. While monetary policy does not affect the demand for money, it does affect the efficiency of risk-sharing in the economy. Moreover, the resulting degree of tractability allows us to easily evaluate the determinants of the price of capital and its rate of return. Although individuals in our framework engage in risk-sharing, the predictions from the benchmark model are consistent with the insights provided in the literature on investment-specific technological change. That is, the direction of productivity in each sector of the economy affects the direction of asset prices and the amount of investment.

To begin, neutral productivity raises the productivity of capital. Following the logic of Tobin's Q (Tobin 1969), neutral productivity increases the value of the existing capital stock and generates more demand for capital goods. In turn, the increase in capital accumulation expands the productive capacity of the capital sector. Due to the availability of capital, neutral progress does not affect the price of capital or the value of Q (the rate of return to capital) – the net impact comes from additional capital accumulation. While expected income is higher when neutral productivity is higher, the degree of risk sharing remains the same. In contrast, in response to an increase in productivity in the capital sector, the supply of capital will be higher. In turn, the price of capital goods falls. However, as in Greenwood, Hercowitz, and Krusell, Q does not react to capital-embodied productivity.

Interestingly, these results indicate that the extent of risk sharing is *independent* of stock prices and productivity in either sector of the economy. Consequently, optimal monetary policy does not depend on equity prices. However, optimal policy does provide full insurance against liquidity risk – this policy is the Friedman Rule.⁴ Nevertheless, capital accumulation responds more to technological change in the capital sector than neutral progress. Through higher levels of savings, expected income will also grow more.

While the benchmark setup provides a tractable framework to study the relation-

⁴Reed and Waller (2006) study an endowment economy in which information frictions prevent individuals from establishing insurance contracts. In this setting, fiat money allows risk sharing to occur. In their model, the Friedman Rule is also the optimal monetary policy since it leads to efficient risk sharing.

ships between stock prices, productivity, and risk sharing in the economy, it fails to generate two important predictions concerning the impact of monetary policy. First, since money demand is inelastic, the Fisher equation holds. Yet, a wide array of evidence appears to reject the Fisher hypothesis. For example, using aggregate values of the New York Stock Exchange, Fama and Schwert (1977) conclude that inflation leads to lower nominal equity prices. This finding receives support in a number of cross-country studies.⁵ Second, monetary policy has an important impact on investment. Based upon annual data for the United States, Ahmed and Rogers (2000) point out that inflation is associated with higher levels of investment activity. This *also* indicates that inflation is associated with lower (real) equity returns.

In order to address the relationships between inflation and investment that appear in the data for the United States, we proceed by examining economies in which money demand is elastic. If households are not too risk averse, higher rates of inflation lower the demand for money since it has a higher opportunity cost. As inflation leads to more investment and capital formation, it also lowers the returns to equity. In this setting, the model yields novel results regarding optimal monetary policy. Notably, the Friedman Rule may not be the optimal monetary policy if the price of capital goods responds to inflation. At the Friedman Rule, money is costless to hold and individuals are fully insured against liquidity risk. However, as the capital stock is relatively low, *the Friedman Rule may lead to an excessively high cost of investment*. As a result, it may be welfare-improving to increase the rate of money growth so that more capital formation occurs (thereby expanding the productive capacity of the capital sector and lowering the relative price of equipment) at the cost of incomplete risk sharing.

In an economy with a general CES production function, interesting connections between monetary policy and the sources of productivity emerge. For example, neutral productivity raises the demand for capital goods and generates higher rates of return to equity. If the central bank pursues the same money growth rate, general productivity growth leads to higher expected consumption, but less risk sharing. In return, the optimal monetary policy provides more liquidity insurance in response to neutral technological change. By comparison, investment-specific productivity generates lower equity prices, but much higher equity returns (higher values of Q). Consequently, capital embodied productivity further distorts risk-sharing. Therefore, optimal policy should react more aggressively to investment-specific productivity than neutral growth. This implies that monetary policy should be designed according to the sources of productivity in the economy. Moreover, our results are broadly consistent with U.S. monetary policy during the postwar period.

The paper is organized as follows. In Section 2, we describe the benchmark model. Section 3 extends the analysis to an economy with elastic demand for money. Finally, we offer concluding remarks in Section 4. Most of the technical details are presented in the Appendix.

⁵In economies with relatively low inflation rates, Gultekin (1983) and Boyd, Levine, and Smith (2001) also observe that inflation adversely affects equity returns.

2 The Benchmark Model

The objective of this paper is to examine the relationships between monetary policy, asset prices, and the sources of productivity in the economy. As a benchmark, we begin by studying an economy in which individuals' demand for money is inelastic. The resulting degree of tractability allows us to easily evaluate the determinants of the price of capital along with its rate of return. In this setting, we find that the direction of productivity in each sector of the economy affects the direction of stock prices and the amount of investment. We also examine the effects of productivity in each sector for risk sharing and optimal monetary policy.

2.1 The Environment in the Benchmark Economy

The economy consists of two distinct geographic locations. For example, the locations could be viewed as separate islands. Within each location, there is an infinite sequence of two-period lived overlapping generations, plus an initial group of old individuals. At the beginning of each date, a continuum of ex-ante identical young workers are born with unit mass. Furthermore, agents only derive utility from consumption (c_t) in old-age:

$$u(c_t) = \frac{c_t^{1-\theta}}{1-\theta} \text{ for } \theta \neq 1 \quad (1)$$
$$u(c_t) = \ln c_t \text{ otherwise}$$

where θ is the coefficient of relative risk aversion. As a benchmark, we focus on studying economies in which individuals' demand for money is inelastic. This occurs if the substitution effect from an increase in the return to money is exactly offset by the income effect. In our framework, this takes place when $\theta = 1$.

Each young agent is endowed with one unit of labor. Since there is no disutility of labor effort, an individual's labor supply is independent of wages. In contrast, agents are retired when old. As a result, the total labor supply at each date is equal to the total population mass of young individuals.

On each island, there are two types of firms. The first type uses labor (L_t) and capital (K_t) to produce the economy's consumption good. We refer to these firms as consumer goods producers. Total output per worker produced in period t is given by a standard Cobb-Douglas production function of the form, $y_t = Ak_{y,t}^\alpha$, where A is an exogenous technology parameter and $k_{y,t}$ is capital per worker employed in the consumer goods sector. In addition, α is the capital share of total output.

In contrast, a capital firm uses the consumption good and capital to produce next period's capital stock. Following Basu (1987) and Abel (2003), we adopt a log linear specification of the classical capital production function. In particular, capital is produced using a Cobb-Douglas production technology:

$$k_{t+1} = ai_t^\rho k_{k,t}^{1-\rho} \quad (2)$$

where i_t is the amount of investment per worker in period t . Equivalently, i_t is the amount of the economy's consumption good used as an input in the production process. As in Kydland and Prescott (1982), it takes time to build productive capital. In addition, $k_{k,t}$ is the capital stock per capita used by a capital producer. Moreover, the parameter a measures the level of productivity in the production of new capital, with $a > 0$.

On the other hand, $\rho \in [0, 1]$, indicates the importance of new capital investment relative to the existing stock of capital in producing new capital. That is, higher values of ρ indicate that new capital investment is more important in capital production relative to the existing stock of capital. Equivalently, it is the investment share of new capital production. If $\rho = a = 1$, the production of capital goods becomes identical to the one sector model with complete depreciation. Specifically, one unit of foregone consumption generates one unit of new capital.

There are two types of assets in this economy: money (fiat currency) and capital. The monetary base per worker is given by M_t . Assuming that the price level is common across locations, we refer to P_t as the number of units of currency per unit of consumer goods at time t . Thus, in real terms, the supply of money per worker is $m_t = M_t/P_t$. At the initial date 0, the generation of old depositors at each location is endowed with the aggregate stock K_0 and the initial aggregate money stock $M_0 > 0$.

Due to private information, depositors face a trading friction. Each island is characterized by complete information about agents' asset holdings, but communication across islands is not possible. As a result, individuals do not have the ability to issue private liabilities. Moreover, they are also subject to relocation shocks. The probability of relocation, π , is exogenous, publicly known and is the same across locations.

As in standard random relocation models, fiat money is the only asset that can be carried across locations. Furthermore, currency is universally recognized and cannot be counterfeited – therefore, it is accepted in both locations. In this manner, money facilitates transactions made difficult by spatial separation and limited communication. Thus, money has an advantage over holdings of capital in terms of liquidity. Consequently, although it is dominated in rate of return, it is accepted as a medium of exchange on each island.

Since money is the only asset that can cross locations, depositors who learn they will be relocated will liquidate all their asset holdings into currency. Random relocation thus plays the same role that liquidity preference shocks perform in Diamond and Dybvig (1983). As banks provide insurance against the shocks, each young depositor will put all of her income in the bank rather than holding assets directly.

In addition to depositors, there is a monetary authority that follows a constant money growth rule. In particular, the money stock evolves according to:

$$M_{t+1} = \sigma M_t \tag{3}$$

where $\sigma > 1$ is the gross growth rate in the money supply. Equivalently, in real terms:

$$m_{t+1} = \sigma m_t \frac{P_t}{P_{t+1}} \quad (4)$$

At this juncture, we describe the timing of events at each date. For simplicity, we consider that there are two different stages of activity. During Stage *I*, trade in factor markets occurs. In Stage *II*, agents born at date t deposit their funds at banks. In turn, banks acquire portfolios of assets on behalf of their depositors. We elaborate immediately below.

At the beginning of period t , young depositors are born. Banks receive the capital they ordered in the previous period (which they paid $P_{k,t-1}$ per unit). Subsequently, banks announce the schedule of deposit rates $r_{m,t}$ and $r_{n,t}$ for deposits received in period t . They also place orders for capital to be delivered next period, k_{t+1} . This pins down the amount of cash reserves to obtain, m_t .

Next, consumer goods producers choose how much capital to rent from banks, $k_{y,t} \equiv \frac{K_{y,t}}{L_t}$, and how much labor to hire, L_t , to produce the economy's output, y_t . Capital producers decide to rent $k_{k,t}$ and select an amount of investment (i_t) to produce next period's capital stock, k_{t+1} .⁶ Investment and rental payments ($r_{k,t}$) are funded by retained earnings from the previous period. Afterwards, workers provide labor services and production takes place at consumer goods producers. In turn, labor and capital receive their marginal products, w_t and $r_{y,t}$. At the end of Stage *I*, banks pay returns to non-movers with the income they earned from the rental services of capital. A summary of events occurring in Stage *I* of period t is illustrated in Figure 1 below.

⁶The assumption that capital producers simultaneously sell and rent capital is motivated by the expression for Tobin's Q . In particular, our framework explicitly separates the market value of capital from the replacement costs. While the market value of capital is given by the rental rate, the replacement costs of capital are determined by the price of new capital goods.

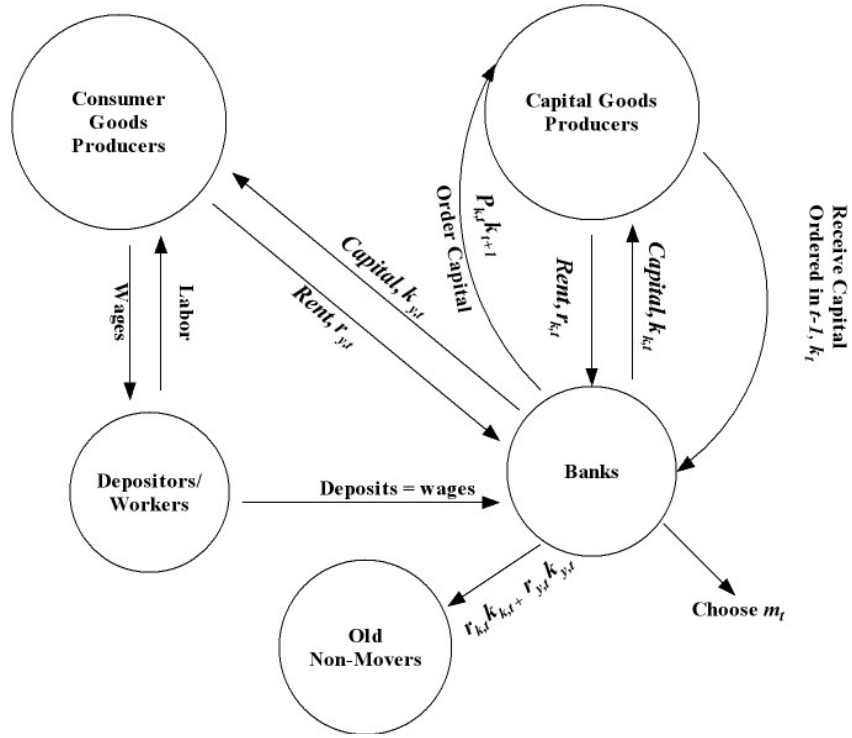


Figure 1: Stage I of Period t : Trade in Factor Markets

At the beginning of Stage II, workers deposit their earnings in bank accounts. Afterwards, old movers show up at banks to exchange their money holdings for $m_{t-1} \frac{P_{t-1}}{P_t}$ units of goods. The payments for money acquired in period t are financed by deposits received from the young workers. Subsequently, banks receive monetary injections from the central bank that are consistent with the money growth rule, $M_{t+1} = \sigma M_t$. With the remaining amount of deposits, banks pay for capital purchases. Then, old agents consume and die. Finally, young individuals learn their location status for the next period. Individuals who must trade in the other island liquidate their deposits and acquire money balances from banks. Relocation occurs and period t ends. A sketch of the events in Stage II is summarized in Figure 2 below.

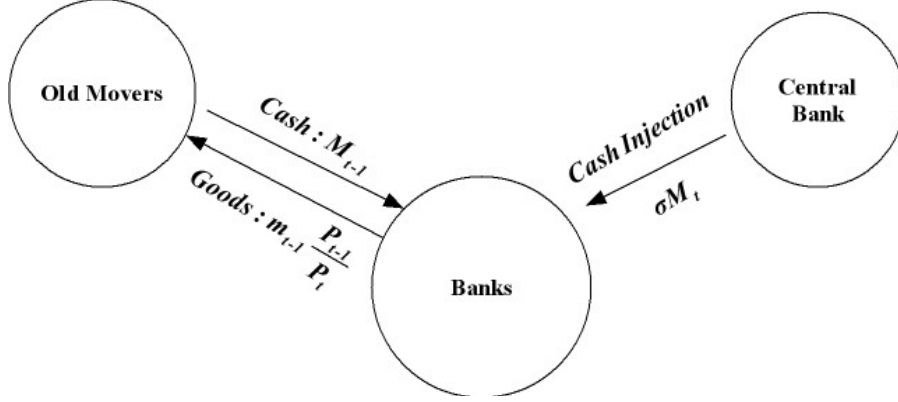


Figure 2: Stage *II* of period t : Acquisition of Money Balances

2.2 Trade

2.2.1 The Capital Sector

A typical capital producer chooses the amount of inputs to maximize its profits. The problem is given by:

$$\underset{k_{k,t}, i_t}{Max} P_{k,t} k_{t+1} - r_{k,t} k_{k,t} - i_t \quad (5)$$

subject to

$$k_{t+1} = a i_t^\rho k_{k,t}^{1-\rho}$$

A capital producer rents capital up to the point where the marginal revenue from one additional unit of capital is equal to its marginal cost. The marginal benefit from using one additional unit of capital, $k_{k,t}$ is equal to the increase in revenue, $P_{k,t} \frac{\partial k_{t+1}}{\partial k_{k,t}}$. In addition, the marginal cost is simply the rental rate paid to banks. Algebraically:

$$r_{k,t} = P_{k,t} \frac{\partial k_{t+1}}{\partial k_{k,t}} = (1 - \rho) a P_{k,t} \left(\frac{i_t}{k_{k,t}} \right)^\rho \quad (6)$$

As the rental cost of capital rises, the demand for capital by a capital producer falls. Furthermore, as a capital firm becomes more productive (higher a), the marginal benefit from using capital rises. In turn, the ratio of capital to investment increases. Equivalently, the rental cost of capital can be deflated by its purchase price to obtain the effective rental cost, $\frac{r_{k,t}}{P_{k,t}}$.

Similarly, each capital firm chooses the amount of investment such that the marginal revenue from one additional unit of investment is equal to its marginal cost.

The marginal revenue is equal to the value of additional units of capital produced with one additional unit of the consumption good, $P_{k,t} \frac{\partial k_{t+1}}{\partial i_t}$. However, the marginal cost of a unit of investment is equal to one unit of goods (the price of one unit of investment). Therefore,

$$P_{k,t} \frac{\partial k_{t+1}}{\partial i_t} = 1$$

Moreover, the profit maximizing choice of investment requires the relative cost of one unit of investment, $\frac{1}{P_{k,t}}$, to be equal to its marginal product, $\frac{\partial k_{t+1}}{\partial i_t}$:

$$\frac{1}{P_{k,t}} = \frac{\partial k_{t+1}}{\partial i_t} = a\rho \left(\frac{k_{k,t}}{i_t} \right)^{1-\rho} \quad (7)$$

Clearly, the marginal revenue from one additional unit of investment rises with the productivity parameter a . Consequently, the investment to capital ratio increases.

Furthermore, combining both first order conditions, (6) and (7), generates the marginal rate of technical substitution, $MRTS$, between investment and capital:

$$MRTS_{i,k} = \frac{\frac{1}{P_{k,t}}}{\frac{r_{k,t}}{P_{k,t}}} = \frac{1}{r_{k,t}} = \frac{\rho k_{k,t}}{(1-\rho) i_t} \quad (8)$$

As in standard Cobb Douglas production functions exhibiting constant returns to scale, the $MRTS$ depends only on the ratio of factor inputs and not on the scale of production.

Upon substituting the expression for the $MRTS$ into the profit-maximizing factor choices by a capital producer, (6) and (7), we get the following condition on the relative costs of inputs:

$$\frac{(1-\rho)^{(1-\rho)} a\rho^\rho}{\left(\frac{1}{P_{k,t}}\right)^\rho} = \left(\frac{r_{k,t}}{P_{k,t}}\right)^{1-\rho} \quad (9)$$

Equivalently:

$$P_{k,t} = \frac{r_{k,t}^{1-\rho}}{(1-\rho)^{(1-\rho)} a\rho^\rho} \quad (10)$$

When this condition is satisfied, a capital producer is indifferent between renting an additional unit of capital and using an additional unit of investment to produce new equipment. Notably, since the price of investment is equal to one, this condition reduces to a relationship between the purchase price of capital and its rental cost. As a result, the *rental* cost of capital is positively related to the *replacement* cost of capital.

Finally, for a given amount of capital rental, we get an expression for the supply of a capital producer by combining capital output (2) with the investment choice (7) :

$$k_{t+1}^S = a^{\frac{1}{1-\rho}} (\rho)_t^{\frac{\rho}{1-\rho}} P_{k,t}^{\frac{\rho}{1-\rho}} k_{k,t} \quad (11)$$

The quantity of capital produced by a capital firm depends on its price and the amount of capital rented. Clearly, the supply of capital is positively related to its price ($P_{k,t}$) and the quantity of capital ($k_{k,t}$) it seeks to rent. In addition, for a given price of equipment, a firm's output rises as the sector becomes more productive.

2.2.2 The Consumer Goods Sector

Analogous to the capital sector, consumer goods producers operate in a perfectly competitive market. By constant returns to scale in the consumption good technology, the wage rate per worker is expressed as:

$$w_t = Af(k_{y,t}) - k_{y,t}Af_k(k_{y,t}) = w_t(k_{y,t}) = (1 - \alpha)Ak_{y,t}^\alpha \quad (12)$$

Moreover, the rental rate paid by a consumer good producer in period t is:

$$r_{y,t} = A\alpha k_{y,t}^{\alpha-1} \quad (13)$$

2.2.3 A representative bank's problem

Due to perfect competition in the deposit market, banks choose portfolios to maximize the expected utility of each depositor. Since financial intermediaries reduce depositors' consumption variability, each of them chooses to deposit all of their income. The bank promises a gross real return r_t^m if the young individual will be relocated and a gross real return r_t^n if not.

As of period t , a bank determines the amount of real money balances to hold, m_t . In addition, it chooses how much capital to purchase from firms at the market price, $P_{k,t}$. This is equivalent to choosing how much capital to rent out to capital firms, $k_{k,t+1}$ and to the consumer goods sector, $k_{y,t+1}$, in $t+1$. Moreover, given that banks are the sole suppliers of capital, $k_{t+1} = k_{k,t+1} + k_{y,t+1}$. The bank's balance sheet is expressed by:

$$m_t + P_{k,t}k_{t+1} \leq w_t ; t \geq 0 \quad (14)$$

Announced deposit returns must satisfy the following constraints. First, since currency is the only asset that can be transported across locations, relocated agents will choose to liquidate their asset holdings into currency. Depending on the bank's money holdings and the inflation rate, the return to movers satisfies:

$$\pi r_t^m w_t \leq m_t \frac{P_t}{P_{t+1}} \quad (15)$$

In addition, we choose to study equilibria in which money is dominated in rate of return. Therefore, banks will not carry money balances between periods t and $t+1$. The bank's total payments to non-movers are therefore paid out of its return to capital in $t+1$:

$$(1 - \pi)r_t^n w_t \leq r_{k,t+1}k_{k,t+1} + r_{y,t+1}k_{y,t+1} \quad (16)$$

Thus, each bank chooses values of $r_t^m, r_t^n, m_t, k_{k,t+1}$, and $k_{y,t+1}$ in order to solve the problem:

$$\underset{r_t^m, r_t^n, m_t, k_{k,t+1}, k_{y,t+1}}{Max} \quad \pi \frac{(r_t^m w_t)^{1-\theta}}{1-\theta} + (1-\pi) \frac{(r_t^n w_t)^{1-\theta}}{1-\theta} \quad (17)$$

subject to (14), (15), and (16), with $\theta = 1$.

A typical bank rents capital to both sectors as long as it yields the same rate of return. That is,

$$r_{k,t+1} = r_{y,t+1} = r_{t+1} \quad (18)$$

In turn, the bank's demand for capital is expressed by:

$$k_{t+1}^D = \frac{(1-\pi)}{P_{k,t}} w(k_{y,t}) = \frac{(1-\pi)}{P_{k,t}} (1-\alpha) A k_{y,t}^\alpha \quad (19)$$

Alternatively,

$$P_{k,t} k_{t+1}^D = (1-\pi) (1-\alpha) A k_{y,t}^\alpha \quad (20)$$

As agents have log preferences, the income and substitution effects from a change in the rate of return to capital exactly offset each other. Consequently, as observed in (20), total expenditures on capital are independent of its rate of return. Since total expenditures do not depend on the price of capital, the demand for capital will be lower if its purchase price ($P_{k,t}$) is higher. By comparison, the demand for capital will be higher if $k_{y,t}$ is higher. This results from the complementarity between capital and labor in the consumer goods sector – if workers have access to more equipment, they earn higher wages. Due to higher earnings, individuals deposit more funds in the bank and investment will be higher. In addition, if agents are less likely to be relocated, there is less need for banks to insure agents against liquidity shocks in the economy. As a result, expenditures on capital will increase.

By the marginal rate of technical substitution in the consumer goods sector, we can eliminate $k_{y,t}$ from the bank's demand for capital. This is achieved by using the return from the consumer goods sector, (13). In particular, the demand for capital by a typical bank becomes:

$$k_{t+1}^D = \frac{(1-\pi)}{P_{k,t}} (1-\alpha) A^{\frac{1}{1-\alpha}} \alpha^{\frac{\alpha}{1-\alpha}} \left(\frac{1}{r_t} \right)^{\frac{\alpha}{1-\alpha}} \quad (21)$$

where $w_t = (1-\alpha) A^{\frac{1}{1-\alpha}} \alpha^{\frac{\alpha}{1-\alpha}} \left(\frac{1}{r_t} \right)^{\frac{\alpha}{1-\alpha}}$.

The intuition behind equation (21) is analogous to that of (19). In particular, as the rental cost of capital rises, the demand for capital and labor declines. This occurs because capital and labor are complementary inputs. As a result, wages (deposits) fall, thereby lowering banks' demand for capital.

With remaining deposits after payments to purchase capital, the bank acquires real money balances. In particular, using the bank's balance sheet, (14), and the demand for capital, (19), the demand for real money balances is:

$$m_t = \pi w(k_{y,t}) = \pi (1 - \alpha) A^{\frac{1}{1-\alpha}} \alpha^{\frac{\alpha}{1-\alpha}} \left(\frac{1}{r_t} \right)^{\frac{\alpha}{1-\alpha}} \quad (22)$$

As the income and substitution effects of a change in the return to money exactly offset each other, the demand for real money balances is perfectly inelastic with respect to inflation. That is, the bank allocates a constant fraction of its deposits into cash reserves. In contrast, a higher rental rate lowers the demand for money since it is associated with lower wages and deposits.

Finally, the returns on deposits to movers and non-movers are:

$$r_t^m = \frac{P_t}{P_{t+1}} \quad (23)$$

$$r_t^n = Q_{t+1} \equiv \frac{r_{t+1}}{P_{k,t}} \quad (24)$$

where $\frac{P_t}{P_{t+1}}$ and $Q_{t+1} \equiv \frac{r_{t+1}}{P_{k,t}}$ are the gross real rates of return to money and capital respectively. From equation (24), the return to non-movers is equal to Q_{t+1} .

As denoted, we define Q_t to be the return to capital in the model. This follows Tobin's theory of investment in which Q is essentially a measure of the returns to capital. According to Tobin, one important aspect of overall investment is the market valuation of firms' existing capital stock. In our model, the market value of capital is given by the rental rate.⁷ The second component of investment behavior is the replacement cost of capital. This is given by the purchase price of new capital goods, $P_{k,t}$. In contrast to Tobin's model, we pursue a general equilibrium theory of investment behavior. In particular, both components of investment behavior are jointly determined.

2.3 General Equilibrium

We now combine the results of the preceding section and characterize the equilibrium for the benchmark economy. In equilibrium, labor effort receives its marginal product, (12), and the labor market clears:

$$L_t = 1 \quad (25)$$

Furthermore, the price of capital is expressed by (10). Moreover, capital must earn the same return in both sectors, (18), and the capital market clears. In particular, the supply of capital by capital firms, (11), must be equal to the demand for capital by banks, (21), with $k_t = k_{k,t} + k_{y,t}$.

As mentioned in the previous section, the expression for the supply of capital, (11), does not take into account the profit maximizing choice of how much capital to

⁷In standard Q -Theory models of investment, the market value of a firm's existing capital stock reflects its discounted flow of revenues. However, in our overlapping generations model, capital depreciates completely each period. Hence, total income from capital is equal to rental income.

rent. Since capital is a predetermined variable, we can use the demand for capital in the consumer goods sector, from (13), to eliminate $k_{k,t}$ from the capital supply equation:

$$k_{t+1}^S = a^{\frac{1}{1-\rho}} (\rho)^{\frac{\rho}{1-\rho}} P_{k,t}^{\frac{\rho}{1-\rho}} \left(k_t - \left(\frac{\alpha A}{r_t} \right)^{\frac{1}{1-\alpha}} \right) \quad (26)$$

Intuitively, for a given stock of capital input and a given price of capital, we observe the following. As the rental cost of capital to a consumer goods producer rises, its demand falls. Since capital is a state variable, the remaining quantity available to a capital firm increases. Consequently, a capital firm's output rises. In contrast, the demand for capital is expressed by equation (21).

Finally, the money market must clear in equilibrium. In particular, we can combine the evolution of real money balances, (4), with the demand for cash by banks to obtain the evolution of the rental cost of capital:

$$r_{t+1} = \left(\frac{P_{t+1}}{P_t} \frac{1}{\sigma} \right)^{\frac{1-\alpha}{\alpha}} r_t \quad (27)$$

Equivalently, we can deflate the rental cost of capital by the purchase price of equipment to generate the evolution of the real return to capital:

$$Q_{t+1} = \left(\frac{P_{t+1}}{P_t} \frac{1}{\sigma} \right)^{\frac{1-\alpha}{\alpha}} \left(\frac{P_{k,t}}{P_{k,t+1}} \right) Q_t \quad (28)$$

2.3.1 Steady-State Analysis

The steady-state behavior of the economy is characterized by the capital market clearing conditions. To begin, note that there are essentially two distinct markets for capital. We refer to the first market as the market for new capital goods. Please refer to the upper-left quadrant of Figure 3 for a graph representing activity in this market.

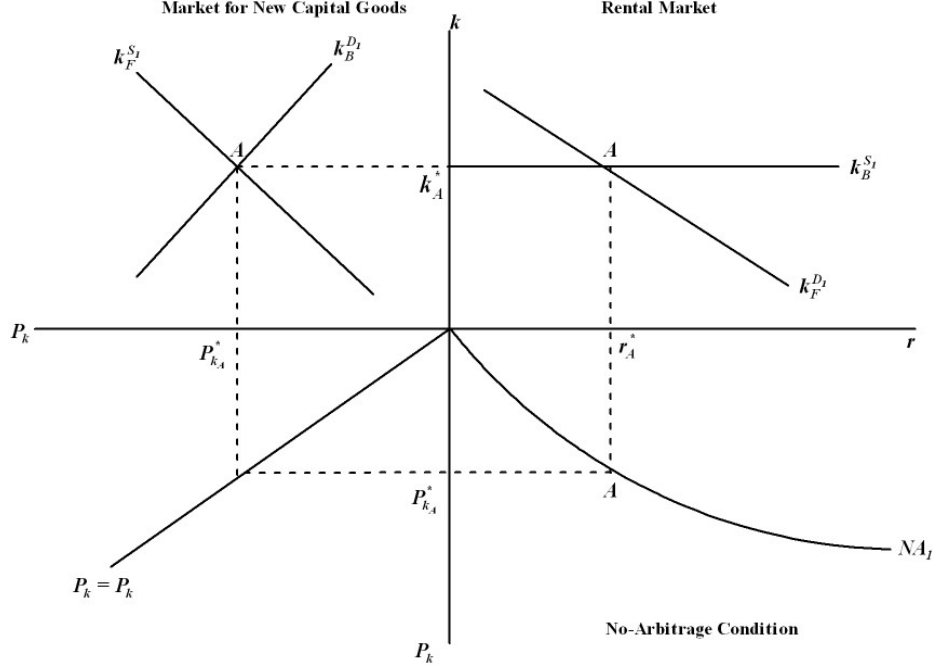


Figure 3: A Partial Equilibrium Perspective of the Benchmark Economy

In the market for equipment, capital-producing firms choose how much to produce, depending upon the price (replacement cost), P_k :

$$k_F^S(P_k; k_k) = a^{\frac{1}{1-\rho}} (\rho)^{\frac{\rho}{1-\rho}} P_k^{\frac{\rho}{1-\rho}} k_k$$

In Figure 3, the capital supply curve is upwards-sloping. Since higher values of P_k raise the marginal revenue from producing another unit of capital, capital producers generate more output at higher prices. In contrast, the demand for capital by banks is given by:

$$k_B^D(P_k; w) = \frac{(1 - \pi)w}{P_k}$$

As previously mentioned, banks allocate a particular amount of deposits to expenditures on capital. That is, *total* expenditures on capital do not depend on P_k . This leads to the downward-sloping demand curve for capital in the economy – since total expenditures on capital are fixed, higher prices translate into a lower quantity of capital demanded. The demand curve for banks is also depicted in Figure 3.

An equilibrium in the output market is a pair (P_k^*, k^*) in which $k_F^S(P_k^*; k_k) = k_B^D(P_k^*; w)$. Consequently, given activity in other sectors of the economy, k^* and P_k^* are pinned down in the market for new capital goods.

We refer to the second market for capital as the rental market. Please refer to the upper-right quadrant of Figure 3 for a graph representing activity in this market. In contrast to the market for new capital goods, banks rent capital to consumer and capital goods producers. The supply curve of capital by banks is given by:

$$k_B^S(w, P_k) = \frac{(1 - \pi)w}{P_k}$$

The price of new equipment and wages determine the position of banks' supply in the rental market. In particular, the supply curve is represented by a horizontal line in the rental market – this occurs since banks' portfolio choices are independent of the return from capital.

We now turn to the demand for capital by capital and consumer goods producers. In particular, the demand by consumer goods producers is given by:

$$\left(\frac{A\alpha}{r}\right)^{\frac{1}{1-\alpha}} = k_y$$

In contrast, the demand curve by capital producers depends upon their anticipated amount of production, k_0 :

$$k_k = \left(\frac{(1 - \rho)P_k}{r}\right) k_0$$

As a result, it is clear that the rental demand curve is downward-sloping – as the rental rate rises, both types of firms choose to rent less capital inputs.

Finally, note there is another equilibrium condition to take into account. This is the no-arbitrage condition on the part of capital producers:

$$P_k = \frac{r^{1-\rho}}{(1 - \rho)^{(1-\rho)} a \rho^\rho}$$

The graphical representation of the no-arbitrage curve is shown in the bottom, right-hand side of Figure 3. There are two important components of its graph. First, in order for capital producers to remain indifferent between using another unit of investment or renting another unit of capital, P_k must be increasing in r . Second, the relationship is strictly concave – if P_k falls from a relatively high initial price level, the adjustment in the rental rate must be stronger. Finally, the bottom left-hand quadrant in Figure 3 connects the equilibrium price of capital in the output market to the no-arbitrage curve.

As we will discuss below, our representation of activity in the steady-state using both the rental price of capital and the price of new equipment allows us to draw a number of insights into the behavior of the economy. However, since it involves four endogenous variables, the four variable system renders it difficult to prove existence of steady-state equilibrium. Consequently, we choose to reduce the system to two variables: $Q \equiv \frac{r}{P_k}$ (the return to capital) and k (the steady-state stock of capital).

To begin, the supply and demand for capital in the steady-state are:

$$k^S = a^{\frac{1}{1-\rho}} (\rho)^{\frac{\rho}{1-\rho}} P_k^{1-\rho} \left(k - \left(\frac{\alpha A}{r} \right)^{\frac{1}{1-\alpha}} \right) \quad (29)$$

$$k^D = \frac{(1-\pi)}{P_k} (1-\alpha) A^{\frac{1}{1-\alpha}} \alpha^{\frac{\alpha}{1-\alpha}} \left(\frac{1}{r} \right)^{\frac{\alpha}{1-\alpha}} \quad (30)$$

Moreover, the price and the rate of return on equipment in the steady-state are given by:

$$P_k = \frac{r^{1-\rho}}{(1-\rho)^{(1-\rho)} a \rho^\rho} \quad (31)$$

$$Q = \frac{r}{P_k} = (1-\rho)^{(1-\rho)} a \rho^\rho r^\rho \quad (32)$$

We start by describing the supply of capital in the steady-state.

The Supply of Capital As a first step, we substitute the expression for P_k and Q into the supply equation:

$$k^S = \frac{1}{Q^{\frac{1-\rho(1-\alpha)}{(1-\alpha)\rho}} [Q - (1-\rho)]} \frac{\alpha}{(1-\alpha)} \lambda \quad (33)$$

where $\lambda \equiv (1-\alpha) \alpha^{\frac{\alpha}{1-\alpha}} (1-\rho)^{\frac{(1-\rho)}{(1-\alpha)\rho}} a^{\frac{1}{\rho(1-\alpha)}} \rho^{\frac{1}{1-\alpha}} A^{\frac{1}{1-\alpha}}$. In order to guarantee a positive supply of capital, we shall consider cases in which $Q > (1-\rho)$. It follows that the supply of capital is strictly decreasing in its effective rental cost. As $Q \equiv \frac{r}{P_k}$, higher values of the rental cost imply that capital producing firms will choose to rent less capital from banks. In turn, the supply of capital will be lower as the effective rental cost of capital is higher. Moreover, without loss of generality we assume that $(1-\rho) < \frac{1}{\sigma}$. The supply of capital is illustrated in Figure 4.

The Demand for Capital From the no-arbitrage and rate of return conditions, we obtain the demand for capital:

$$k^D = (1-\pi) \frac{\lambda}{Q^{\frac{1-(1-\alpha)\rho}{(1-\alpha)\rho}}} \quad (34)$$

As shown in (34), the demand for capital is strictly decreasing in its rate of return. The reasoning is as follows. By the expression for the rate of return to equity, (32), the real return to capital is increasing in its rental rate. If the rental rate is higher, consumer goods producers will choose to rent less capital. Due to the complementarity between labor and capital in the consumer goods sector, wages will be lower. This causes deposits to fall and leads to less demand for capital by banks. In addition, by the no-arbitrage condition in the capital goods sector, (31), the price of new equipment increases if the rental rate is higher. In turn, the demand for capital by banks declines further. Consequently, the demand for capital is decreasing

in its rate of return. Please refer to Figure 4 for an illustration of the demand curve.

Proposition 1. *Suppose that $(1 - \rho) < \frac{1}{\sigma}$. Under this condition, a steady-state exists and is unique if $\frac{1}{\sigma} < \frac{1}{(1-\pi)} \frac{\alpha}{1-\alpha} + (1 - \rho)$.*

Following arguments in the appendix, the supply and demand for capital always intersect once for all $Q > (1 - \rho)$. Moreover, we also seek to study steady-states in which money is dominated in rate of return. As a benchmark, suppose that both assets yield the same rate of return. That is, $Q = \frac{1}{\sigma}$. Upon substitution into the expressions for the supply and demand for capital, the curves intersect if $\frac{1}{\sigma} = \frac{1}{(1-\pi)} \frac{\alpha}{1-\alpha} + (1 - \rho)$. Alternatively, if $\frac{1}{\sigma} < \frac{1}{(1-\pi)} \frac{\alpha}{1-\alpha} + (1 - \rho)$, the rate of return to capital is not high enough to clear the market. While the relatively high price of new capital (P_k) encourages capital producers to supply large amounts of equipment, it leads to less demand by banks. As a result, if $Q = \frac{1}{\sigma} < \frac{1}{(1-\pi)} \frac{\alpha}{1-\alpha} + (1 - \rho)$, the supply of capital exceeds its demand. Consequently, the rate of return (Q) must rise to clear the market. This leads to a higher rate of return to capital than money. Moreover, at $Q > Q^*$, there is excess demand for capital since capital is relatively less expensive to acquire. However, since capital rental is quite costly, capital firms choose to produce relatively less equipment. Thus, the steady-state is unique. These properties are illustrated in Figure 4 below.

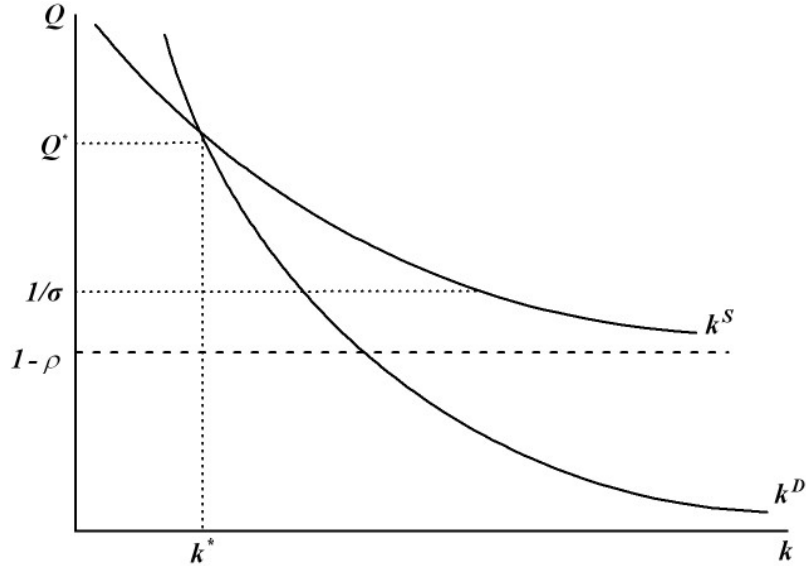


Figure 4. Steady State Equilibrium Under Inelastic Demand for Money

The explicit solutions for Q , k , r , and P_k are:

$$Q^* = \frac{r^*}{P_k^*} = \frac{\alpha + (1 - \pi)(1 - \alpha)(1 - \rho)}{(1 - \pi)(1 - \alpha)} \quad (35)$$

$$k^* = \frac{[(1 - \pi)(1 - \alpha)a]^{\frac{1}{(1-\alpha)\rho}} A^{\frac{1}{1-\alpha}} \alpha^{\frac{\alpha}{1-\alpha}} (1 - \rho)^{\frac{(1-\rho)}{(1-\alpha)\rho}} \rho^{\frac{1}{(1-\alpha)}}}{[\alpha + (1 - \pi)(1 - \alpha)(1 - \rho)]^{\frac{1-(1-\alpha)\rho}{(1-\alpha)\rho}}} \quad (36)$$

$$r^* = \left[\frac{\alpha + (1 - \pi)(1 - \alpha)(1 - \rho)}{(1 - \pi)(1 - \alpha)(1 - \rho)^{(1-\rho)} a \rho^\rho} \right]^{\frac{1}{\rho}} \quad (37)$$

$$P_k^* = \frac{1}{(1 - \rho)^{\frac{(1-\rho)}{\rho}} a^{\frac{1}{\rho}} \rho} \left[\frac{\alpha + (1 - \pi)(1 - \alpha)(1 - \rho)}{(1 - \pi)(1 - \alpha)} \right]^{\frac{1-\rho}{\rho}} \quad (38)$$

In standard random relocation models such as Schreft and Smith (1997, 1998), the price of capital is exogenously set to unity. This is equivalent in our model to the case where $a = \rho = 1$. Our framework generalizes these models to incorporate two sectors of production along with adjustment costs in the capital sector. This enables us to examine the interaction between asset prices and different sources of productivity in the economy. In the discussion below, we draw interesting connections between technological change, stock prices, and the degree of risk sharing.

We begin with the following observation:

Corollary 1. *Monetary policy is superneutral. That is, with the exception of the rate of return to money, inflation does not have any real effects.*

When agents have log preferences, the demand for real money balances is perfectly inelastic with respect to inflation. From a typical bank's balance sheet, (14), this also implies that inflation does not have any impact on the demand for capital. Therefore, the Fisher equation holds. While monetary policy does not have any impact on the amount of money balances, it does affect the degree of risk sharing. At higher inflation rates, the return to money is lower. Consequently, individuals receive less insurance against liquidity risk.

Since monetary policy does not affect portfolio choices under logarithmic preferences, it provides a simple means to evaluate the determinants of the price of capital and its rate of return. Although individuals engage in risk-sharing, the predictions from our framework are consistent with the literature on investment-specific technological change. Notably, Greenwood, Hercowitz, and Krussel (1997) point out that the relative price of capital goods has fallen around 3% per year in the United States. In their work, they attribute the price decline to technological advances in the production of new equipment. The following Corollary characterizes the impact of increasing productivity in the capital sector:

Corollary 2. *Investment-specific productivity lowers the (replacement cost) price of capital goods. Furthermore, the steady-state capital stock is higher in economies*

with a more productive capital sector. However, the rate of return to capital is unaffected.

To gain deep insights into the effects of investment-specific productivity, we begin by reviewing the framework introduced in Figure 3. We initiate our discussion by describing activity in the market for new capital goods. As a result of the higher level of productivity among capital producers, the supply curve of these firms shifts up. (Please refer to Figure 5 below)

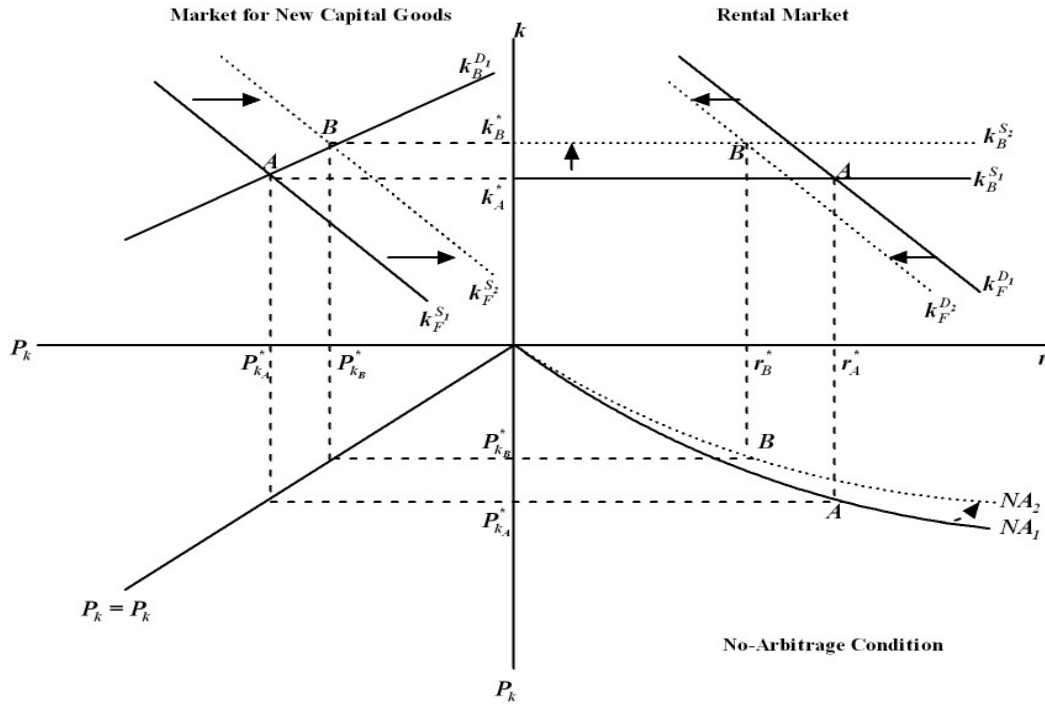


Figure 5. Partial Equilibrium Effects of Investment-Specific Technological Advance

This causes the price of new capital to fall. In turn, the supply of capital by banks in the rental market is higher. In particular, it shifts up to point B in which the total amount of capital produced and rented is the same in both markets. Due to the higher supply of capital, the rental rate falls as well. The decline in both the purchase and rental costs of capital maintains the no-arbitrage relationship among capital producers. All of this activity takes place assuming that deposits are unchanged.

However, because the capital stock is higher, workers in the consumer goods sector have additional capital to work with. Since capital raises workers' productivity, all individuals earn higher wages. Therefore, bank deposits will increase due to higher savings. This further spurs investment and capital accumulation in the economy.

Consequently, investment-specific productivity will have an impact on both the capital demand curve (34) and the supply curve (33). The impact through the supply curve is demonstrated in the graph of the output market in Figure 5.

The presence of a in the capital demand curve comes from two sources. The first effect is a partial equilibrium effect – since higher productivity leads to a lower price, it generates an increase in the quantity of capital demanded by banks. The second effect is a general equilibrium effect – increased capital accumulation leads to an increase in deposits. Figure 6 below shows the net impact on the rental rate and purchase costs. Although the capital supply curve shifts out in response to the technological change in the capital sector, the demand curve shifts out by the same magnitude. In this manner, the return to capital remains the same. This implies that the rental rate falls by the same amount as the purchase costs.

Thus, the return to capital does not respond to investment-specific productivity. From the no-arbitrage condition, equation (31), banks will continue to purchase additional capital until firms are indifferent between renting an additional unit of capital or allocating an additional unit of revenues to investment. As banks purchase additional capital, the rental rate falls. Consequently, capital accumulation increases from k_1 to k_2 in the Figure.

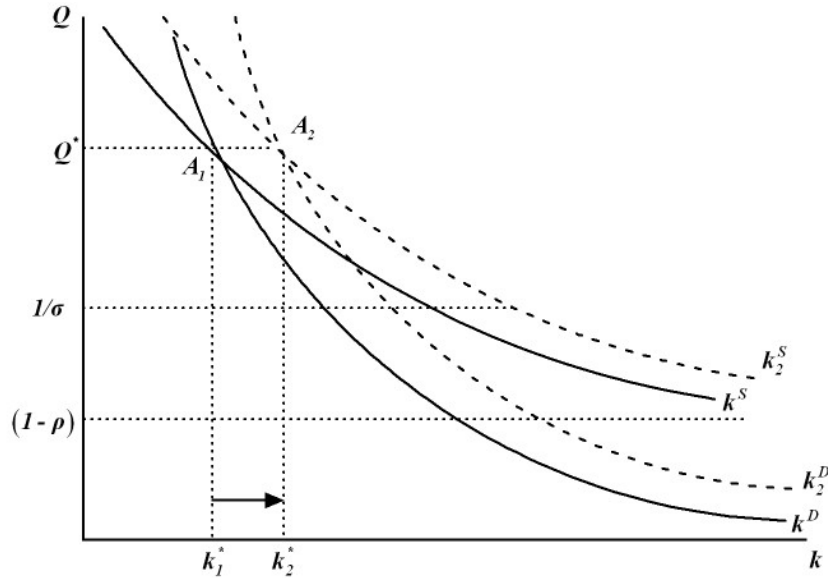


Figure 6: Effects of Higher Productivity in the Capital Sector

While the relative price of capital falls in response to investment-specific technological

change, the decline in the rental rate leads to lower earnings from capital investment. As a result, the return to capital remains the same.

Therefore, investment-specific technological change fuels investment and capital accumulation. Notably, Greenwood et. al. attribute nearly 60% of economic growth in the United States to productivity growth in the capital sector. The remaining 40% stems from neutral technological progress. Although financial institutions allow individuals to engage in risk-sharing, our model is consistent with predictions from the literature on investment-specific technological change. That is, the direction of productivity in each sector of the economy affects the direction of stock prices and the amount of investment. From equation (36), it is clear that both the level of neutral productivity (A) and investment-specific productivity (a) lead to an increase in capital accumulation and higher expected income. However, as Q does not react to the level of productivity in either sector, the results from the benchmark setup indicate that the extent of risk sharing is *independent* of stock prices and investment. Nevertheless, capital accumulation responds more to technological change in the capital sector than neutral progress. Through higher levels of savings, expected income will grow more.

As demonstrated, the degree of tractability in the benchmark economy generates closed-form solutions for Q and other endogenous variables. However, since the Fisher equation holds, the monetary side of the economy is separate from real activity. In this setting, monetary policy only influences the cost of liquidity risk. The optimal policy is associated with full insurance – this policy is the Friedman Rule.

In order to determine the effects of monetary policy on Q and investment behavior, we must allow for the portfolio choices of financial institutions to respond to rates of return. That is, we need to consider the effects of monetary policy on asset prices in economies with elastic demand for money. Following standard random relocation models, we retain the assumption that capital completely depreciates after production occurs.

3 Economies with Elastic Demand for Money

3.1 The Model with a High Degree of Risk Aversion

A key departure from the benchmark model occurs when agents' coefficient of relative risk aversion, θ , is not equal to one. In this case, the demand for real money balances is:

$$m = \frac{w(k_y)}{\left[1 + \frac{(1-\pi)}{\pi} (\sigma Q)^{\frac{1-\theta}{\theta}}\right]} \quad (39)$$

To make our results comparable to previous work, we follow Schreft and Smith (1998) by assuming that agents have a relatively high degree of risk aversion. That is, $\theta > 1$. In this setting, the demand for real money balances is increasing in the rate of return to capital. This reflects depositors' attitudes regarding risk. Notably, the relocation shock is responsible for agents to experience two different location states – a 'good'

state in which they earn a relatively high rate of return and a ‘bad’ state in which they earn a low rate of return. If $\theta > 1$ as in Schreft and Smith (1998), depositors are particularly sensitive to low levels of consumption in the bad state. If the return to capital is high, depositors obtain a high level of consumption in the good state. This leads to a higher variability in their income and therefore less risk sharing. As agents are highly risk averse, banks will seek to provide more liquidity insurance to their depositors. This is possible if banks acquire more money balances and less capital. In turn, the demand for capital is decreasing in Q :

$$k^D = \frac{1}{1 + \frac{\pi}{1-\pi} (\sigma Q)^{\frac{\theta-1}{\theta}}} \frac{\lambda}{Q^{\frac{1-(1-\alpha)\rho}{(1-\alpha)\rho}}} \quad (40)$$

The second term in the right hand side of (40) is identical to that under log preferences (from equation (34)). However, from the first term in (40), the fraction of deposits allocated to purchases of new capital varies with its rate of return. Consequently, the demand curve is flatter if agents have elastic demand for money balances and $\theta > 1$.

Proposition 2. *Suppose that $(1 - \rho) < \frac{1}{\sigma}$. Under this condition, a steady-state where money is dominated in rate of return exists and is unique if $\frac{1}{\sigma} < \frac{1}{(1-\pi)} \frac{\alpha}{1-\alpha} + (1 - \rho)$. Furthermore, if agents are more risk averse (as indicated by higher values of θ), the rate of return to capital is higher. In addition, the rental cost and the price of new equipment are higher. This leads to less capital accumulation.*

As in the previous section, the supply and demand for capital always intersect once in an economy with highly risk averse individuals. Moreover, the conditions for existence and uniqueness from the case where $\theta = 1$ are sufficient for existence and uniqueness if $\theta > 1$. To better understand the reasoning, please refer to Figure 7 below:

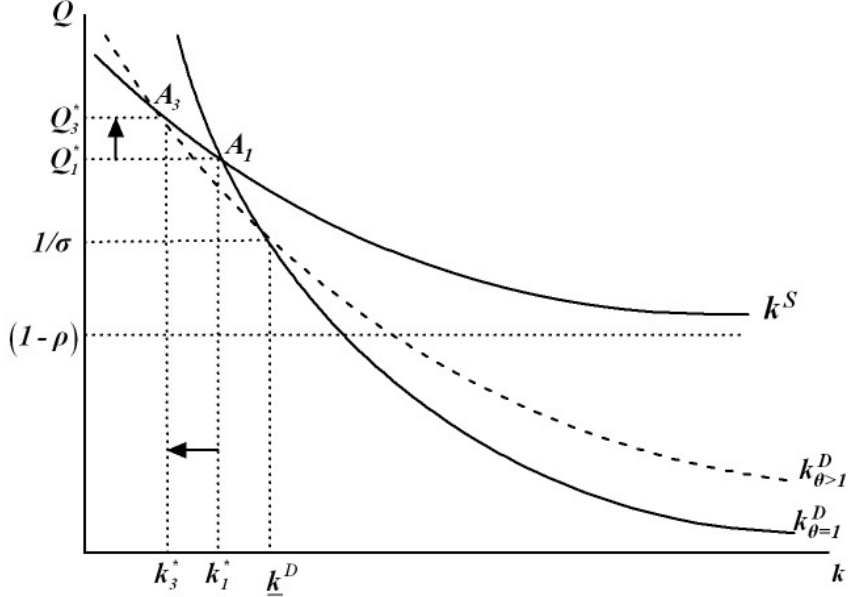


Figure 7: Steady-State Equilibrium ($\theta > 1$)

Note that the steady-state rate of return in the case of log preferences ($\theta = 1$) is equal to Q_1^* . In an economy in which individuals are more risk averse ($\theta > 1$), banks will acquire greater money balances in order to provide more liquidity insurance. Therefore, at $Q = Q_1^*$, the demand for capital goods will be lower if $\theta > 1$ compared to the log case. This implies that there will be an excess supply of capital if $Q = Q_1^*$. In order for the economy to be in equilibrium, the effective price of capital goods (P_k/r) must fall so that capital firms will lower their amount of production. Consequently, the steady-state rate of return must be higher than Q_1^* if $\theta > 1$. Thus, if a steady-state exists in an economy where $\theta = 1$, then it also exists if $\theta > 1$.

At this juncture, we discuss the impact of monetary policy on asset prices and real equity returns. While inflation did not have any real effects in the benchmark economy, the following Proposition demonstrates that the Fisher hypothesis fails to hold under higher degrees of risk aversion:

Proposition 3. *Higher rates of money growth lead to higher prices of capital goods. However, the marginal product of capital and the return to equity are also higher. Nevertheless, inflation adversely affects capital formation.*

In this economy, monetary policy affects the need for banks to provide insurance against liquidity risk. In particular, under a higher inflation rate, the return to money is lower. As a result, banks demand more cash reserves to compensate their highly risk averse depositors for the loss in purchasing power in the bad state. Therefore,

they purchase less capital for a given rate of return. This causes the demand curve in Figure 8 to shift back.

At the rate of return Q_A^* , the capital market is in excess supply. Consequently, the relative cost of capital (P_k/r) must fall in order to clear the market. As illustrated in the Figure, the steady-state of the economy moves from A to B . In the economy with higher inflation, B , the capital stock is lower and its marginal product is higher. Although the relative cost (P_k/r) is lower, the price of new capital will be higher. This is observed by the no-arbitrage condition in the rental market, (31). Furthermore, since the return to capital is higher in economy B , this implies that inflation also leads to less risk sharing.

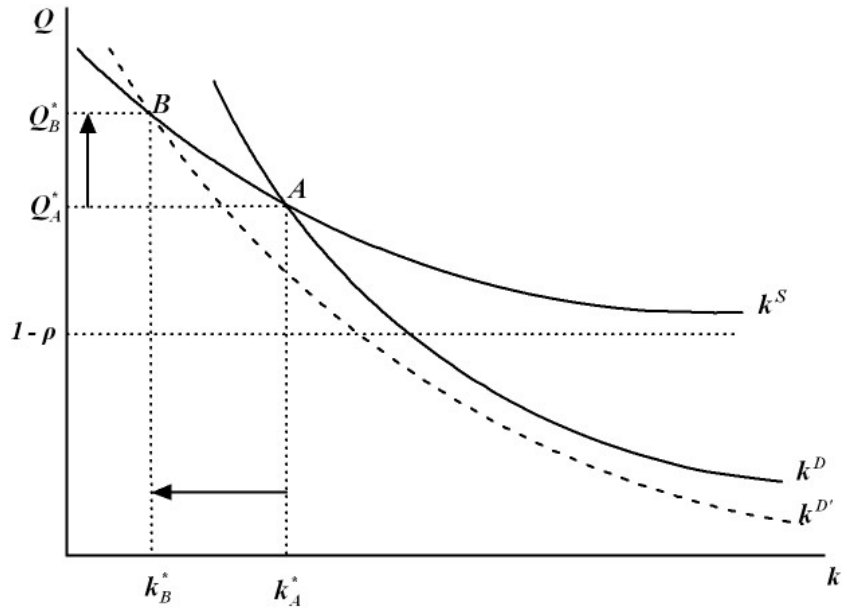


Figure 8: The Impact of Higher Inflation ($\theta > 1$)

Notably, many monetary models predict there should be a positive relationship between real returns to equity and inflation. In a cash-in-advance economy, Stockman (1981) argues that inflation restricts the amount of investment since money has less purchasing power. By diminishing returns, the marginal product of capital must be higher. In this manner, the return to capital is increasing in the inflation rate. Moreover, Schreft and Smith (1998) obtain the same result. Interestingly, when the price of equipment is endogenously determined (as in our model), the adverse effects of inflation are magnified. The intuition is as follows. If the price of capital is exogenous and equals one as in Schreft and Smith (1998), inflation operates through two major channels. First, higher inflation reduces the demand for capital since banks seek to provide greater liquidity insurance. Second, as the capital stock falls,

its rental rate rises. This also discourages capital accumulation. Moreover, in our economy, inflation affects capital accumulation through an additional channel – the endogenous price of new capital goods. In particular, as explained in Proposition 2, the price of equipment rises with inflation. This provokes banks to lower their amount of investment. Consequently, the adverse effects of inflation on expected income are stronger if the price of capital is endogenously determined. In addition, inflation interferes with the ability of the financial sector to promote risk sharing. This occurs because inflation has a stronger effect on the return to capital in our setting compared to the one sector economy in Schreft and Smith (1998).

Furthermore, it is important to note that the result in Corollary 2 still holds under more general preferences. That is, the steady state capital stock increases when the capital sector is more productive. However, as in the previous section, the return to capital does not respond to investment-specific technological change.

While the preceding discussion describes a role for monetary policy in influencing equity returns, evidence for low inflation countries such as the United States indicates that inflation simultaneously lowers equity returns *and* promotes capital accumulation. In order to provide a framework in which money growth stimulates investment activity, we choose to study an economy in which money demand is decreasing in the inflation rate. Such a relationship is plausible for a number of reasons. For example, in a standard model of money demand, higher inflation raises the costs of holding money. In the discussion below, incorporating lower levels of risk aversion produces results that are consistent with the evidence on inflation, investment, and returns to equity.

3.2 An Economy in which Inflation Lowers Money Demand

In the previous section, introducing relatively high levels of risk aversion yields money demand functions which respond to monetary policy. In particular, banks provide more liquidity insurance at higher inflation rates. However, standard models of money demand imply that inflation should lower the demand for money in the economy. Furthermore, the predictions regarding monetary policy are not in line with available evidence for the United States. For these reasons, we turn to economies in which agents are less risk averse. Although banks continue to provide risk pooling services, there is less emphasis on protecting agents from low levels of consumption in the event of relocation. At higher inflation rates, liquidity insurance comes at the cost of devoting less funds to capital. As banks choose to hold less of the dominated asset, our results mirror standard views of money demand such as Baumol (1952) and Tobin (1956).

Similar arguments explain the effects of the return to capital on banks' portfolio choices. First, as shown in the benchmark model, higher returns to capital generate higher revenues for banks, but also imply that capital is more costly to rent. In this manner, higher capital returns are associated with less investment since there will be less demand for the rental services of capital. This explains the downward-sloping demand curve for capital in the benchmark model. In contrast, in this economy,

agents are less risk averse. Higher rates of return raise the costs of acquiring money balances and lead to more demand for capital by banks. This provides a rationale for an upward-sloping demand curve for capital. Therefore, as we show in the Appendix, the demand curve for capital is backward bending. At relatively low rates of return, the effective rental costs of capital will be low. As a result, initial increases in Q generate an increase in the demand for capital. At higher values of Q , capital becomes much more expensive to rent. Consequently, banks acquire less capital. This is illustrated in Figure 9 below.

Proposition 4. *Suppose that $(1 - \rho) < \frac{1}{\sigma}$. Under this condition, a steady state exists and is unique if $\frac{1}{\sigma} < \frac{1}{(1-\pi)} \frac{\alpha}{1-\alpha} + (1 - \rho)$.*

Avoiding unnecessary repetition, the condition for existence is identical to that in previous sections. A graphical illustration of the steady state is presented in Figure 9 below.

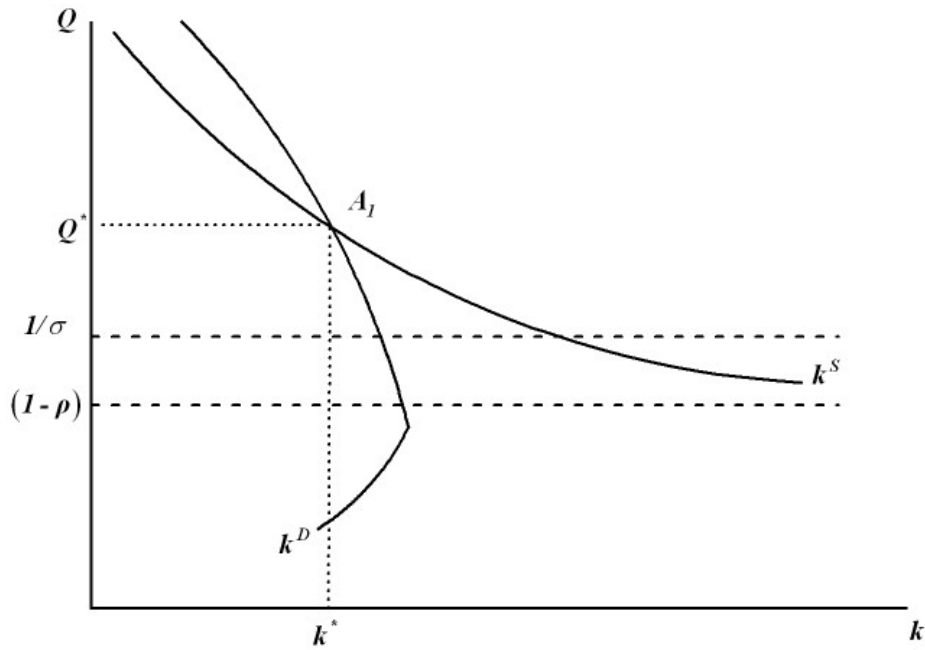


Figure 9: Steady-State Equilibrium when $\theta < 1$

While the Fisher hypothesis fails to hold under high degrees of risk aversion, the effects of inflation on equity returns are inconsistent with the data. In light of this discrepancy, we consider the impact of money growth if agents are less risk averse:

Proposition 5. *Higher rates of money growth lead to lower prices (replacement costs) of new equipment. Moreover, the marginal product of capital and its rate of return also fall. In contrast, inflation leads to higher capital accumulation.*

For a given real return to capital, higher inflation rates raise the opportunity cost of holding money. In turn, banks allocate a larger fraction of their deposits to capital. This causes the demand curve for capital to shift out in Figure 10 below. At the rate of return Q^* , the capital market is in excess demand. Consequently, the rate of return to capital must fall in order for the market to clear. This is illustrated as a movement from A_1 to A_2 . In the economy with higher inflation, A_2 , the capital stock is higher and its marginal product is lower. Moreover, by the no-arbitrage condition (31), the price of capital is also lower.

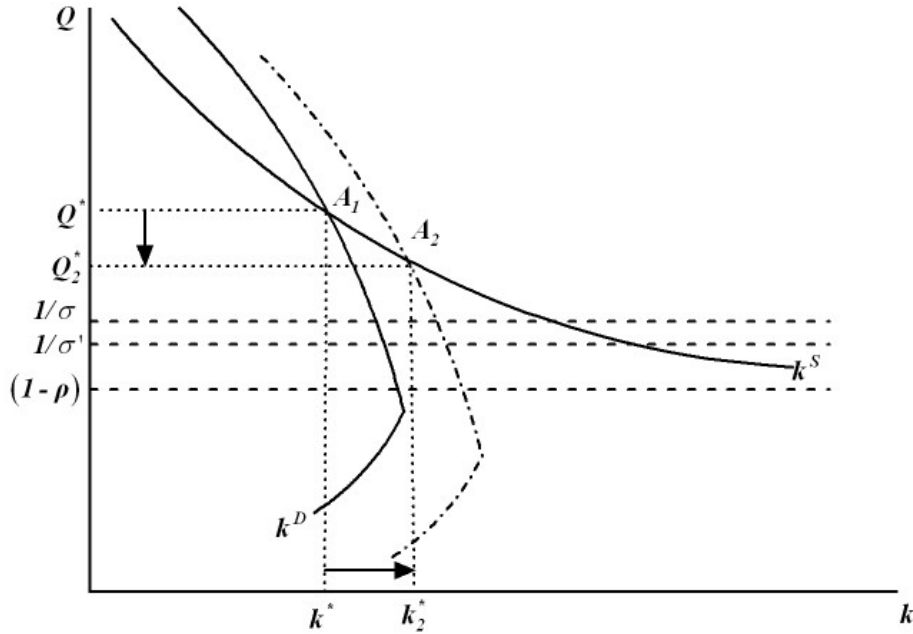


Figure 10: The Impact of Higher Inflation if $\theta < 1$

Intuitively, for a given stock of capital, higher rates of money growth raise the market value of capital relative to its replacement cost (higher value of Q). The higher value of Q stimulates investment and capital accumulation. By diminishing returns, income generated from capital declines. As the economy's productive capacity expands, the supply of new equipment rises leading to a decline in its price. Therefore, inflation causes stock prices and real equity returns to fall.

While inflation leads to higher expected income from higher wages, it is easy to verify that it leads to less risk sharing. That is, the return to movers declines relative to non-movers. This occurs for two reasons. First, a higher inflation rate directly lowers the return to money and thereby, that to movers. Moreover, as banks hold a less liquid portfolio under a higher inflation rate, the return in the bad state declines further. In contrast, since banks allocate a larger fraction of their deposits into capital, the return in the good state does not fall as much when the return to capital declines.

Ignoring the impact of monetary policy on the replacement cost of capital would overstate the effects of monetary policy in most models. For example, if P_k is exogenously set to unity, the bulk of a monetary expansion on equity returns is reflected through the rental rate. Since the price of new equipment is not allowed to fall with income from capital, the effects of monetary policy are likely to be miscalculated. This is important to consider in studying the determination of optimal monetary policy and calculating the welfare costs of inflation.

3.2.1 Optimal Monetary Policy

As noted in the introduction, much empirical research points out that inflation lowers returns to equity. Moreover, Ahmed and Rogers find evidence of a Tobin effect (Tobin 1965) for the United States. If agents are not too risk averse, our model generates the same outcomes. As the model's predictions are consistent with empirical observations, we proceed to study the determination of optimal monetary policy. In the presence of a Tobin effect, there are obvious trade-offs – inflation promotes capital accumulation which increases earnings and bank deposits. However, it leads to lower rates of return and less risk sharing – both the return to money and capital fall as inflation rises.

We assume that the monetary authority chooses the rate of money growth to maximize the expected utility of a representative generation of depositors:

$$\omega = \left[\pi (r^m)^{1-\theta} + (1 - \pi) (r^n)^{1-\theta} \right] \frac{w (k_y)^{1-\theta}}{1 - \theta} \quad (41)$$

Noting that $\gamma(Q, \sigma) = \frac{m}{w}$ reflects the fraction of deposits allocated to cash reserves, the gross rates of return to movers and non-movers are given by:

$$r^m = \frac{\gamma(Q(\sigma), \sigma)}{\pi} \frac{1}{\sigma} \quad (42)$$

$$r^n = \frac{(1 - \gamma(Q(\sigma), \sigma))}{1 - \pi} Q(\sigma) \quad (43)$$

Using (42), (43) and the steady-state demand for cash reserves, (39), the welfare function can be written as:

$$\omega(\sigma) = \frac{\pi^\theta}{1 - \theta} \frac{1}{\sigma^{1-\theta}} \Psi(\sigma) \quad (44)$$

where $\Psi(\sigma) \equiv \frac{w(k_y(\sigma))^{1-\theta}}{\gamma^\theta(Q(\sigma), \sigma)}$. The optimal rate of money growth depends on the net effects of inflation:

$$\omega'(\sigma) = \frac{\pi^\theta}{1-\theta} \left[-\frac{(1-\theta)}{\sigma^{2-\theta}} \Psi(\sigma) + \frac{1}{\sigma^{1-\theta}} \Psi'(\sigma) \right] \quad (45)$$

where:

$$\Psi'(\sigma) = \frac{1}{\gamma^\theta(Q(\sigma), \sigma)} \left\{ (1-\theta) w(k_y(\sigma))^{-\theta} \frac{\partial w(k_y)}{\partial k_y} \frac{\partial k_y}{\partial \sigma} - \frac{w(k_y(\sigma))^{1-\theta}}{\gamma(Q(\sigma), \sigma)} \theta \frac{d\gamma(Q(\sigma), \sigma)}{d\sigma} \right\} \quad (46)$$

The term $-\frac{(1-\theta)}{\sigma^{2-\theta}} \Psi(\sigma)$ represents the social marginal cost of inflation. Since inflation lowers the return to money, it directly leads to less consumption by movers. The second term, $\frac{1}{\sigma^{1-\theta}} \Psi'(\sigma)$, reflects the net social marginal benefit of inflation. Its first component, $(1-\theta) w(k_y(\sigma))^{-\theta} \frac{\partial w(k_y)}{\partial k_y} \frac{\partial k_y}{\partial \sigma}$, shows that the Tobin effect generates higher bank deposits. Finally, $-\frac{w(k_y(\sigma))^{1-\theta}}{\gamma(Q(\sigma), \sigma)} \theta \frac{d\gamma(\sigma)}{d\sigma}$ shows that inflation affects the relative rates of return between movers and non-movers. Although inflation lowers the returns of agents in both states, the relative return to non-movers increases. In this manner, at higher inflation rates, non-movers earn higher returns at the expense of movers.

Substituting functional forms for the production technologies yields:

$$\omega'(\sigma) = \frac{[(1-\alpha) A k_y^\alpha]^{1-\theta}}{\sigma^{2-\theta}} \frac{\pi^\theta}{\gamma^\theta} \left[[\alpha - (1-\gamma(\sigma))(1-\alpha)\rho] \frac{\sigma}{k_y} \frac{\partial k_y}{\partial \sigma} - \gamma(\sigma) \right] \quad (47)$$

We obtain the following Proposition:

Proposition 6. *Suppose $\theta = \frac{1}{2}$. If $\rho > \frac{\alpha}{1-\alpha}$, welfare is decreasing in inflation. Consequently, the Friedman Rule is the optimal policy. However, if $\rho < \frac{\alpha}{1-\alpha}$, the optimal money growth rate exceeds the Friedman Rule.*

Interestingly, Proposition 6 highlights the role of our two-sector model for developing insights into optimal monetary policy. In a standard one-sector model with complete depreciation of physical capital, the capital stock is simply given by foregone consumption in the previous period. In this sense, ρ would be equal to one. In addition, α represents the capital intensity of the consumer goods sector. Based upon the growth-accounting literature, it is common to set α somewhere around one-third. Therefore, the Friedman Rule is likely to be the optimal policy in standard monetary, neoclassical growth models. However, in our framework, this is not necessarily true – if the capital sector is relatively capital-intensive (ρ less than one), the optimal money growth rate exceeds the Friedman Rule.

Intuitively, at the Friedman Rule, money is costless to hold and individuals are fully insured against liquidity risk. However, as the capital stock is relatively low,

the Friedman Rule may lead to an excessively high cost of investment. As a result, it may be welfare-improving to increase the rate of money growth so that more capital formation occurs (and therefore, higher expected utility) at the cost of incomplete risk sharing.

We proceed to study optimal monetary policy under different degrees of technical change. We present the following observation:

Proposition 7. *The optimal monetary policy is independent of technology parameters.*

As discussed in the previous sections, technological advance in either sector promotes capital accumulation. In addition, as shown in the appendix, the elasticity of capital with respect to inflation is also independent. In this manner, the source of technological change does not affect the degree of risk sharing in the economy and thereby optimal monetary policy – this occurs regardless of the optimality of the Friedman Rule.

3.3 The Model Under General Degrees of Substitution in the Consumer Goods Sector

The preceding analysis generates a number of important observations regarding investment, stock prices, and technological change. Although individuals in the model engage in risk-sharing, the predictions from the benchmark model are consistent with the insights provided in the literature on investment-specific technological change. Notably, capital-embodied productivity leads to lower equity prices. Incorporating elastic demand for money produces results in line with the available evidence on inflation, equity prices, and investment activity in the United States.

Interestingly, as the model explicitly determines both the price of new equipment along with the rental costs of capital, our framework may be interpreted as a monetary, general equilibrium model of Tobin’s Q theory of investment. The return to capital is equal to the income earned from capital divided by its replacement cost – Tobin’s Q .

Due to the increased attention to the role of the stock market for macroeconomic activity in recent years, there have been a number of studies which derive estimates of Q for the United States. In particular, Hall (2001) provides a time-series of calculations of Q from 1946 through 1999. From the mid 1950s to about 1970, Q generally increased. In contrast, during the next decade, Q consistently fell. Accompanying the surging rise of equities throughout the 1990s, measurements of Q rose substantially.

In the economy with a Cobb-Douglas production technology, the elasticity of substitution in the consumer goods sector is equal to one. Under this specification, the return to capital (Q) only reflects the extent of liquidity risk and the willingness of financial market participants to tolerate such risk. The resulting degree of tractability allows us to pin down parameters to determine the economy’s optimal monetary

policy. Extending the model to more general elasticities of substitution introduces a role for productivity in each sector to drive the behavior of Q . In this manner, we aim to illustrate that it is important to consider the sources of productivity growth in the design of optimal monetary policy. Moreover, the predictions from our analysis are broadly consistent with U.S. monetary policy during the postwar period.

To begin, let $y = A [\alpha k_y^\epsilon + (1 - \alpha)]^{\frac{1}{\epsilon}}$, with $\epsilon \in (0, 1)$. The production technology in the capital sector remains the same. In this case, factor prices are given by:

$$r(k_y) = A \alpha k_y^{\epsilon-1} [\alpha k_y^\epsilon + (1 - \alpha)]^{\frac{1-\epsilon}{\epsilon}} \quad (48)$$

$$w(k_y) = A (1 - \alpha) [\alpha k_y^\epsilon + (1 - \alpha)]^{\frac{1-\epsilon}{\epsilon}} \quad (49)$$

Furthermore, define $\psi = (1 - \rho)^{\frac{(1-\rho)}{\rho}} a^{\frac{1}{\rho}} \rho A$. After some algebra, the steady state behavior of the economy is characterized by the following two loci:⁸

$$Q^{\frac{1}{\rho}} = \alpha \psi \left(1 - (1 - \rho) \frac{1}{Q}\right)^{\epsilon-1} k^{\epsilon-1} \left[\alpha \left(1 - (1 - \rho) \frac{1}{Q}\right)^\epsilon k^\epsilon + (1 - \alpha)\right]^{\frac{1-\epsilon}{\epsilon}} \quad (50)$$

and

$$k = (1 - \gamma(Q, \sigma)) \frac{(1 - \alpha) \psi}{Q^{\frac{(1-\rho)}{\rho}}} \left[\alpha \left(1 - (1 - \rho) \frac{1}{Q}\right)^\epsilon k^\epsilon + (1 - \alpha)\right]^{\frac{1-\epsilon}{\epsilon}} \quad (51)$$

Proposition 8. *Suppose that the level of productivity in either the consumer goods sector or the capital sector is sufficiently high. If this holds, a steady-state exists and is unique.*

We next examine the effects of productivity in each sector:

Corollary 3. *The effects of technological change on stock prices and equity returns (Q) depend upon the source of change. At a higher level of investment-specific productivity, both the price of new capital goods and the rental costs of capital will be lower. In contrast, under neutral progress, the price of new equipment and the rental costs will be higher. While both sources of productivity lead to higher capital accumulation and values of Q , investment-specific productivity has a larger impact.*

The results in the Corollary are intuitive. An increase in neutral productivity stimulates deposits and investment in the economy. Therefore, it raises the market value of capital. Since the demand for capital is higher, the rental costs of capital and the price of new capital are also higher. In addition, rental costs increase more than the price of new equipment. In turn, the return to capital will be higher under

⁸Please refer to the Appendix for details.

higher levels of productivity in the consumer goods sector. These predictions differ from the analysis under a Cobb-Douglas production technology. That is, the demand-induced impact of neutral productivity dominates any indirect productivity effects from increased capital accumulation. We obtain analogous predictions for investment-specific technological change.

Interestingly, the model now produces important transmission channels for monetary policy through Tobin's Q . While increasing productivity in the capital sector lowers the cost of new capital goods, neutral productivity raises their cost. Both sources of change raise the rate of return to capital, but Q responds more to investment-specific productivity. Therefore, capital-embodied productivity raises expected income more, but further distorts risk-sharing.⁹

3.3.1 Optimal Monetary Policy

Analogous to the previous section, the central bank chooses the rate of money growth to maximize the welfare of a representative generation of depositors, (41). Using some algebra, the condition for optimal welfare becomes:¹⁰

$$\gamma(\sigma) = \left[(1 - \gamma)(\sigma) - \frac{1}{(1 - \alpha)\rho} k_y^\epsilon(\sigma) \right] \frac{\sigma}{Q} \frac{\partial Q}{\partial \sigma} \quad (52)$$

As in the case of the Cobb-Douglas production technology in the consumer goods sector, the Friedman Rule may be the optimal monetary policy or it may not be. If the production technology in the consumer goods sector exhibits the more general CES form, we are unable to find parameters which pin down the optimal money growth rate. However, the following Proposition explains that we can characterize how the optimal money growth rate depends on the source of productivity in the economy:

Proposition 9. *The optimal monetary policy depends upon the source of productivity growth in the economy. In particular, the central bank's response to higher levels of productivity depends on the elasticity of equity returns with respect to technology parameters. That is, if $\frac{\alpha}{Q^*} \frac{\partial Q^*}{\partial a} < 1$ and $\frac{A}{Q^*} \frac{\partial Q^*}{\partial A} < 2\rho$, the optimal money growth rate falls in response to either source of productivity. However, optimal policy should react more aggressively to investment-specific productivity.*

As previously noted, advances in neutral productivity and investment-specific productivity cause the return to capital to rise. For a given money growth rate, this

⁹Reed and Waller (2006) study an endowment economy which includes both idiosyncratic and aggregate production risk. In some periods, individuals receive income while in others they do not. In the good aggregate state of the economy, individuals who obtain income receive high endowments. In the bad aggregate state, endowments are relatively low. At moderate rates of money growth, there is efficient risk sharing in the low aggregate state, but not the high state.

¹⁰If $\epsilon = 0$, the production technology is of the Cobb-Douglas form. As shown earlier, the optimal inflation rate is independent of technology parameters.

leads to less risk sharing. Since agents are risk averse, the optimal monetary policy seeks to smooth agents' consumption across income states. Therefore, in response to either source of productivity growth, monetary policy should provide more insurance against liquidity risk. This occurs by pursuing lower rates of money growth in order to raise the rate of return to money. By Corollary 3, investment-specific productivity generates much higher equity returns. Consequently, capital-embodied productivity further distorts risk-sharing. In this manner, optimal policy should react more aggressively to investment-specific productivity than neutral growth. This implies that monetary policy should be designed according to the sources of productivity in the economy.

The predictions from the extended model line up well with the available evidence on equity prices, Tobin's Q , the effects of productivity growth, and monetary policy in the United States during the postwar period. To draw such inferences, we rely upon three available sources of information. First, Greenwood and Jovanovic (1999) provide data on the value of equities from 1930 until 1990. Second, Hall's calculations for Q show trends in the return to capital over time. Finally, Greenwood, Hercowitz, and Krussel (1997) compute measures of neutral and capital-embodied productivity from the mid 1950s to about 1990.

We begin with the period from the mid 1950s to 1969. Greenwood and Jovanovic's measures of stock market capitalization show strong growth during this time period. In the mid 1950s, the value of the stock market relative to GDP was about 40%. In contrast, in the late 1960s, market capitalization was on average equal to GDP. In addition, Hall's calculations for Q indicate that the return to capital increased substantially, rising from a trough near 0.5 to over 1.5 by 1969. According to Corollary 3, this correlation reflects strong neutral productivity growth. From the results in Greenwood, Hercowitz, and Krussel, neutral productivity increased around 25% during this time period. Given this information, Proposition 9 implies that the central bank should pursue relatively low rates of money growth since expected income should be strong. Consequently, there is greater need to provide insurance against liquidity risk than to promote investment. From 1954 to 1969, average annual inflation rates were approximately equal to 2.0%.¹¹

As widely observed, the value of equities plummeted during the next decade. Market capitalization fell to 90% of GDP in 1970 and proceeded downward in the early seventies. Equity prices did not rebound until around 1984. Greenwood and Jovanovic contend that the declining value of equities could be attributed to strong gains in capital-embodied productivity that became available. They could also be due to strong declines in neutral productivity – measurements of neutral productivity fell about 7.5%. Therefore, nearly one-third of the gains in neutral productivity that occurred prior to 1970 were erased. Based upon our model, each source of productivity generates conflicting predictions regarding monetary policy. On the one hand, the central bank should pursue low rates of money growth in response to investment-specific technical change. On the other hand, declining neutral productivity should

¹¹ Authors' calculations based upon average annual inflation rates for the CPI-U from the *Economic Report of the President, 2003*.

induce higher rates of money growth in order to promote investment. Estimates of Q imply that the return to capital fell substantially – from this perspective, expected income would be falling while individuals experience greater risk sharing. In these circumstances, inflation would be expected to be higher. From 1970 to 1984, prices grew rapidly – average annual inflation rates exceeded 7%.

Finally, from 1984 through the late 90s, equity prices increased substantially. The return to capital grew at a strong pace. This could be due to strong growth in neutral productivity along with modest increases in capital-embodied technology. Either way, Proposition 9 indicates that there should be less inflation – expected income is likely to be strong so the central bank should promote risk-sharing. From 1985 - 1999, average annual inflation rates fell to almost 3%. Again, the model's predictions regarding equity prices, technical change, and monetary policy are in line with economic activity in the United States.

4 Conclusions

Investment activity plays a significant role in the formulation of monetary policy. Moreover, recent evidence identifies different sources of productivity which drive investment behavior. For instance, Greenwood, Hercowitz, and Krusell (1997), attribute nearly 60% of economic growth in the United States to productivity growth in the capital sector. The remaining 40% stems from neutral technological progress. Since each source of productivity bears different implications for asset prices and investment, this suggests that monetary policy should be directed according to the sources of productivity in the economy.

In order to study the relationships between monetary policy, stock prices, and productivity, we develop a two-sector model in which individuals encounter idiosyncratic liquidity risk and engage in risk-sharing. Consistent with the findings of Ahmed and Rogers (2000), we study an economy in which inflation leads to more investment, but lower equity returns. Interestingly, if the price of capital goods significantly responds to capital accumulation, the optimal monetary policy deviates from the Friedman Rule. Furthermore, we demonstrate that different sources of productivity can affect the degree of risk-sharing. In turn, optimal money growth depends on the level of productivity in each sector. In particular, monetary policy should react more aggressively to investment-specific productivity than neutral change. Interestingly, the predictions from our framework are in line with available evidence on economic activity and monetary policy during the postwar period.

Our framework may be used to study a number of important issues regarding monetary policy, asset prices, and investment. As an example, the model could be extended to consider the implications of technological change for equity prices and inflation-targeting. If inflation tax revenues fund government expenditures, the government's need to raise revenues will depend on the level of productivity in each sector and investment. In this manner, asset prices will have a significant impact on inflation expectations.

In addition, aggregate uncertainty may also be introduced. In the current frame-

work, risk averse individuals encounter financial uncertainty due to idiosyncratic liquidity risk. Given the perfectly anticipated returns to capital, financial institutions choose a diversified portfolio of assets to provide risk sharing opportunities. Nevertheless, asset price volatility is also an important component of macroeconomic behavior. If productivity in each sector is stochastic, financial market participants must deal with financial risk in two forms – liquidity risk and uncertain stock prices. While higher rates of money growth stimulate investment, central banks must also construct monetary policy to help mitigate the costs of volatile equity prices. Therefore, optimal policy should balance the desire to promote expected income while providing insurance against asset price volatility *and* liquidity risk.

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5 Technical Appendix

1. Proving existence and uniqueness for all $\theta > 0$. A steady-state solution for Q is obtained by setting (33) equal to (40) :

$$\eta(Q) = Q^{\frac{1}{\theta}} \left(1 - \left[(1 - \rho) + \frac{\alpha}{(1 - \alpha)} \right] Q^{-1} \right) = \frac{1}{\sigma^{\frac{1-\theta}{\theta}}} \frac{\alpha}{(1 - \alpha)} \frac{\pi}{1 - \pi} \quad (53)$$

where $\left(1 - \left[(1 - \rho) + \frac{\alpha}{(1 - \alpha)} \right] Q^{-1} \right) > 0$ when $Q > (1 - \rho)$. Clearly, $\eta'(Q) > 0$ for all $\theta > 0$. Consequently, capital dominates money in rate of return if $\eta\left(\frac{1}{\sigma}\right) < \frac{1}{\sigma^{\frac{1-\theta}{\theta}}} \frac{\alpha}{(1 - \alpha)} \frac{\pi}{1 - \pi}$. Upon substitution, we get the condition for existence in Propositions 1, 2, and 4. Furthermore, $\eta(Q)$ is strictly increasing in Q . In this manner, $\eta(Q)$ intersects the $\frac{1}{\sigma^{\frac{1-\theta}{\theta}}} \frac{\alpha}{(1 - \alpha)} \frac{\pi}{1 - \pi}$ line only once and the steady state is unique for all $\theta > 0$. This completes the proof that a steady-state exists and is unique for all $\theta > 0$ when $\frac{1}{\sigma} < (1 - \rho) + \frac{\alpha}{(1 - \alpha)} \left(\frac{1}{1 - \pi} \right)$.

2. Proving that Q^* is higher when $\theta > 1$ relative to $\theta = 1$. As $\eta'(Q) > 0$ for $\theta > 0$, it is sufficient to show that $\eta\left(\frac{1}{\sigma}\right)_{\theta=1} > \eta\left(\frac{1}{\sigma}\right)_{\theta>1}$. This is equivalent to showing that $\eta(Q)_{\theta=1}$ with a slope of 1 is steeper than $\eta(Q)_{\theta>1}$. Algebraically:

$$\sigma^{\frac{1-\theta}{\theta}} > \left(1 - \left[(1 - \rho) + \frac{\alpha}{(1 - \alpha)} \right] \frac{1}{\sigma} \right)$$

which clearly holds since $\sigma > 1$. Graphically, the demand for capital rotates counter clockwise around $\left(\underline{k}^D, \frac{1}{\sigma} \right)$, where $\underline{k}^D = (1 - \pi) \lambda \sigma^{\frac{1-(1-\alpha)\rho}{(1-\alpha)\rho}}$ as observed in Figure 7. This completes the proof.

3. Effects of monetary policy: The effects of money growth on equity returns are obtained by taking the derivative of the capital market clearing condition derived above, (53), with respect to σ :

$$\frac{\partial Q}{\partial \sigma} = \frac{-\frac{1-\theta}{\theta} \frac{\alpha}{(1-\alpha)} \frac{\pi}{1-\pi} \frac{1}{\sigma^{\frac{1}{\theta}}}}{\eta'(Q)} \quad (54)$$

As explained previously, $\eta'(Q) > 0$ for all $\theta > 0$. As a result, monetary policy is super-neutral when $\theta = 1$. In addition, $\frac{\partial Q}{\partial \sigma} > 0$ when $\theta > 1$ and $\frac{\partial Q}{\partial \sigma} < 0$ when $\theta < 1$. Equivalently, the effects of σ on k^* , r^* , and P_k^* follow from the effects of Q on (31), (32), and (33). This completes the proof of Corollary 1 and Propositions 3 and 5.

4. Characterizing the demand for capital when $\theta < 1$. Define the fraction of deposits allocated into cash reserves by $\gamma(Q, \sigma)$, with $\gamma(Q, \sigma) = \frac{1}{\left[1 + \frac{1-\pi}{\pi} (Q\sigma)^{\frac{1-\theta}{\theta}} \right]}$.

In this manner, the demand for capital can be written as:

$$k^D = (1 - \gamma(Q, \sigma)) \frac{\lambda}{Q^{\frac{1-(1-\alpha)\rho}{(1-\alpha)\rho}}} \quad (55)$$

It is easily verified that $\lim_{Q \rightarrow \infty} k^D \rightarrow 0$ and for $Q = \frac{1}{\sigma}$, $\gamma(Q, \sigma) = \pi$ and $k^D = (1 - \pi) \sigma^{\frac{1 - (1 - \alpha)\rho}{(1 - \alpha)\rho}}$. Differentiating (55) with respect to Q :

$$\frac{\partial k^D}{\partial Q} = \frac{\lambda(1 - \gamma(Q, \sigma))}{Q^{\frac{1}{(1 - \alpha)\rho}}} \left[\frac{1 - \theta}{\theta} \gamma - \frac{(1 - (1 - \alpha)\rho)}{(1 - \alpha)\rho} \right] \leq 0$$

Clearly, the sign of $\frac{\partial k^D}{\partial Q}$ depends on the sign of the term in bracket. In particular $\frac{\partial k^D}{\partial Q} = 0$ when:

$$\gamma = \frac{\theta}{1 - \theta} \frac{(1 - (1 - \alpha)\rho)}{(1 - \alpha)\rho}$$

Substituting for the expression of γ , we find that $\frac{\partial k^D}{\partial Q} = 0$ when:

$$\hat{Q} = \frac{1}{\sigma} \left(\frac{(1 - \alpha)\rho - \theta}{(1 - (1 - \alpha)\rho)\theta} \frac{\pi}{1 - \pi} \right)^{\frac{\theta}{1 - \theta}}$$

Consequently, $\frac{\partial k^D}{\partial Q} > 0$ for $Q < \hat{Q}$ and $\frac{\partial k^D}{\partial Q} < 0$ for $Q > \hat{Q}$. As a result, the demand for capital is backward bending as illustrated in Figure 9. Moreover, we demonstrated above that the return to capital is falling in σ . This occurs regardless of where the inflection point occurs. Without any loss of generality, Figure 9 illustrates the case where $\hat{Q} < \frac{1}{\sigma}$. This completes the proof that the demand for capital is backward bending when $\theta < 1$.

5. Deriving $\omega'(\sigma)$ in equation (47): As a first step, we need to find an expression for $\Psi'(\sigma)$. In particular, we start by deriving steady-state expressions for $\frac{d\gamma(Q(\sigma), \sigma)}{d\sigma}$ and $\frac{\partial w(k_y)}{\partial k_y} \frac{\partial k_y}{\partial \sigma}$.

Using (13) and (32), the rate of return to capital can be written as:

$$Q = (1 - \rho)^{(1 - \rho)} a \rho^\rho (\alpha A)^\rho k_y^{(\alpha - 1)\rho} \quad (56)$$

Substituting this expression into γ and taking the derivative with respect to σ :

$$\frac{d\gamma(Q(\sigma), \sigma)}{d\sigma} = -\xi \left[-(1 - \alpha) \rho k_y^{-1} \frac{\partial k_y}{\partial \sigma} + \sigma^{-1} \right] \gamma^2 k_y^{(\alpha - 1)\rho \frac{1 - \theta}{\theta}} \quad (57)$$

where $\xi = \frac{1 - \pi}{\pi} \sigma^{\frac{1 - \theta}{\theta}} \left((1 - \rho)^{(1 - \rho)} a \rho^\rho (\alpha A)^\rho \right)^{\frac{1 - \theta}{\theta}} \frac{1 - \theta}{\theta}$

Furthermore, differentiating (12) with respect to σ :

$$\frac{\partial w(k_y)}{\partial \sigma} = (1 - \alpha) \alpha A k_y^{\alpha - 1} \frac{\partial k_y}{\partial \sigma} \quad (58)$$

Using the expression for γ and by substituting for the expressions of (56), (57), and (58) into $\Psi'(\sigma)$ we get:

$$\Psi'(\sigma) = \frac{(1-\theta)(1-\alpha)Ak_y^\alpha w^{-\theta}}{\gamma^\theta} \left\{ [\alpha - (1-\gamma)(1-\alpha)\rho] \frac{1}{k_y} \frac{\partial k_y}{\partial \sigma} + [1-\gamma]\sigma^{-1} \right\} \quad (59)$$

Finally, substitute for $\Psi(\sigma)$ and $\Psi'(\sigma)$ into $\omega'(\sigma)$ to get (47). This completes the derivation of (47).

6. Proof of Proposition 6: The sign of $\omega'(\sigma)$ depends on the sign of the term in brackets. In particular, an interior solution occurs when $\frac{\sigma}{k_y} \frac{\partial k_y}{\partial \sigma} = \frac{\gamma}{[\alpha - (1-\gamma)(1-\alpha)\rho]}$. Suppose $\theta = \frac{1}{2}$. Using (53), the steady-state value of Q is:

$$Q^*(\pi, \sigma) = \frac{\left[(1-\rho) + \frac{\alpha}{(1-\alpha)} \right] + \sqrt{\left[(1-\rho) + \frac{\alpha}{(1-\alpha)} \right]^2 + 4 \frac{1}{\sigma} \frac{\alpha}{(1-\alpha)} \frac{\pi}{1-\pi}}}{2} \quad (60)$$

Taking the derivative with respect to σ :

$$\frac{\sigma}{Q} \frac{\partial Q}{\partial \sigma} = \frac{-2 \frac{\alpha}{(1-\alpha)}}{\left[(1-\rho) + \frac{\alpha}{(1-\alpha)} \right] \tau(\sigma, \pi) + \frac{1-\pi}{\pi} \sigma \left[(1-\rho) + \frac{\alpha}{(1-\alpha)} \right]^2 + 4 \frac{\alpha}{(1-\alpha)}}$$

where $\tau(\sigma, \pi) = \left(\left[(1-\rho) + \frac{\alpha}{(1-\alpha)} \right]^2 \left(\frac{1-\pi}{\pi} \right)^2 + 4 \frac{1}{\sigma} \frac{\alpha}{(1-\alpha)} \frac{1-\pi}{\pi} \right)^{\frac{1}{2}}$, with $\frac{\partial \tau(\sigma, \pi)}{\partial \sigma} < 0$.

Next, we can solve for k_y from (56) and take the derivative with respect to money growth:

$$\frac{\sigma}{k_y} \frac{\partial k_y}{\partial \sigma} = - \frac{1}{(1-\alpha)\rho} \frac{\sigma}{Q} \frac{\partial Q}{\partial \sigma}$$

Moreover, we substitute the expression for $\frac{\sigma}{Q} \frac{\partial Q}{\partial \sigma}$:

$$\frac{\sigma}{k_y} \frac{\partial k_y}{\partial \sigma} = \frac{2 \frac{\alpha}{\rho(1-\alpha)^2}}{\left[(1-\rho) + \frac{\alpha}{(1-\alpha)} \right] \tau(\sigma, \pi) + \frac{1-\pi}{\pi} \sigma \left[(1-\rho) + \frac{\alpha}{(1-\alpha)} \right]^2 + 4 \frac{\alpha}{(1-\alpha)}} \quad (61)$$

We proceed to find an expression for $V \equiv \frac{1}{\left[\frac{\alpha - (1-\gamma)}{\gamma} (1-\alpha)\rho \right]}$. By definition of γ , and using (42) and (43):

$$V \equiv \frac{1}{\alpha + [\alpha - (1-\alpha)\rho] \frac{1-\pi}{\pi} Q^* \sigma} \quad (62)$$

Finally, substituting for (60), (61), and (62) into the optimality condition and rearranging terms, we get:

$$LHS(\sigma^*, \pi) = RHS(\sigma^*, \pi) \quad (63)$$

where $LHS(\sigma) = 2\alpha + [\alpha - (1 - \alpha)\rho] \left[\left[(1 - \rho) + \frac{\alpha}{(1 - \alpha)} \right] \frac{1 - \pi}{\pi} + \tau(\sigma, \pi) \right]$ and $RHS(\sigma) = \left(\left[(1 - \rho) + \frac{\alpha}{(1 - \alpha)} \right] \frac{1 - \pi}{\pi} + \tau(\sigma, \pi) \right) \left[(1 - \rho) + \frac{\alpha}{(1 - \alpha)} \right] \frac{\rho(1 - \alpha)^2}{\alpha} \sigma + 4\rho(1 - \alpha)$

The optimal rate of inflation, σ^* solves (63). In this manner we distinguish between two different cases. First, suppose $\alpha - (1 - \alpha)\rho > 0$. Under this condition, $\frac{\partial LHS(\sigma, \pi)}{\partial \sigma} < 0$ and $\lim_{\sigma \rightarrow \infty} LHS(\sigma) \rightarrow 2\alpha + 2[\alpha - (1 - \alpha)\rho] \left[(1 - \rho) + \frac{\alpha}{(1 - \alpha)} \right] \frac{1 - \pi}{\pi}$. Moreover, $\lim_{\sigma \rightarrow 0} LHS(\sigma, \pi) \rightarrow \infty$ since $\tau(\sigma, \pi) \rightarrow \infty$. Furthermore, $\frac{\partial RHS(\sigma, \pi)}{\partial \sigma} > 0$. In addition, $\lim_{\sigma \rightarrow \infty} RHS(\sigma, \pi) \rightarrow \infty$ and $\lim_{\sigma \rightarrow 0} RHS(\sigma, \pi) \rightarrow 0$. In this case, an interior solution always exist as both curves always intersect. However, optimal monetary policy generally exceeds the Friedman rule.

Next, suppose $\alpha - (1 - \alpha)\rho < 0$. Under this condition, $\frac{\partial LHS(\sigma, \pi)}{\partial \sigma} > 0$. In addition, $\lim_{\sigma \rightarrow 0} LHS(\sigma, \pi) \rightarrow -\infty$ and $\lim_{\sigma \rightarrow \infty} LHS(\sigma, \pi) \rightarrow 2\alpha + 2[\alpha - (1 - \alpha)\rho] \left[(1 - \rho) + \frac{\alpha}{(1 - \alpha)} \right] \frac{1 - \pi}{\pi}$. An interior solution does not exist in this case. In particular, $\omega'(\sigma) < 0$. As welfare is falling with inflation rate, the Friedman rule where $Q\sigma^* \simeq 1$ is optimal. This completes the proof of Proposition 6.

7. Proof of Proposition 7. It is clear from (63) that technology parameters have no effect on optimal monetary policy. This completes the proof of Proposition 7.

8. Deriving (50) and (51). The problem under CES production function can be solved using the system of equations, (11), (14), (31), (32), (39), (48), (49), and $k_k + k_y = k$, with 8 unknowns, k_y , k_k , k , P_k , r , Q , m , and w . The first step is to reduce this system into a 3×3 system by writing r and P_k as a function of Q , using the expression for wages, and the capital market clearing condition. In particular, we use (31), (32), (39), (49), and $k_k + k_y = k$ into (11), (14), and (48) to obtain:

$$Q^{\frac{1}{\rho}} = A\alpha(1 - \rho)^{\frac{(1 - \rho)}{\rho}} a^{\frac{1}{\rho}} \rho (k - k_k)^{\epsilon - 1} \left[\alpha (k - k_k)^{\epsilon} + (1 - \alpha) \right]^{\frac{1 - \epsilon}{\epsilon}} \quad (64)$$

$$k_F = \frac{1}{(1 - \rho)} Q k_k \quad (65)$$

and

$$k_B = (1 - \gamma(Q, \sigma)) \frac{(1 - \rho)^{\frac{(1 - \rho)}{\rho}} a^{\frac{1}{\rho}} \rho}{Q^{\frac{(1 - \rho)}{\rho}}} A(1 - \alpha) \left[\alpha (k - k_k)^{\epsilon} + (1 - \alpha) \right]^{\frac{1 - \epsilon}{\epsilon}} \quad (66)$$

Finally, equations (50) and (51) are generated by using (65) into (64) and (66). This completes the derivation of equations (50) and (51).

9. Proof of Proposition 8: We first combine (50) and (51) to obtain:

$$k = \left(\frac{1-\alpha}{\alpha} \right)^{\frac{1}{\epsilon}} (1-\gamma(Q, \sigma))^{\frac{1}{\epsilon}} \left(1 - (1-\rho) \frac{1}{Q} \right)^{\frac{1-\epsilon}{\epsilon}} Q^{\frac{1}{\epsilon}} \quad (67)$$

Moreover, by substituting this expression into (50), we get:

$$Q^{\frac{\epsilon}{\rho(1-\epsilon)}} = \left(\alpha^{\frac{1}{\epsilon}} \psi \right)^{\frac{\epsilon}{(1-\epsilon)}} \left[1 + \frac{1}{(Q - (1-\rho)) (1-\gamma(Q, \sigma))} \right] \quad (68)$$

This polynomial yields the steady-state solution for Q .

Define $G(Q) = Q^{\frac{\epsilon}{\rho(1-\epsilon)}}$ and $J(Q) = \left(\alpha^{\frac{1}{\epsilon}} \psi \right)^{\frac{\epsilon}{(1-\epsilon)}} \left[1 + \frac{1}{(Q - (1-\rho)) (1-\gamma(Q, \sigma))} \right]$. It is clear that $G'(Q) > 0$, $G(0) = 0$, and $\lim_{Q \rightarrow \infty} G(Q) \rightarrow \infty$. Furthermore, $J'(Q) < 0$ since $\frac{\partial \gamma(Q, \sigma)}{\partial Q} < 0$. In addition, $\lim_{Q \rightarrow \infty} J(Q) \rightarrow \left(\alpha^{\frac{1}{\epsilon}} \psi \right)^{\frac{\epsilon}{(1-\epsilon)}}$ since $\gamma \rightarrow 0$ and $\lim_{Q \rightarrow (1-\rho)} J(Q) \rightarrow \infty$. In this manner, both curves always intersect once and the polynomial has a unique positive root. Consequently, a steady state exists if at Q^* money is dominated in rate of return. This is achieved if $J\left(\frac{1}{\sigma}\right) > G\left(\frac{1}{\sigma}\right)$. This condition is equivalent to $\frac{1}{\left(\frac{1}{\sigma} - (1-\rho)\right)(1-\pi)} > \frac{1}{\left(\alpha^{\frac{1}{\epsilon}} \psi\right)^{\frac{\epsilon}{(1-\epsilon)}}} \left(\frac{1}{\sigma}\right)^{\frac{\epsilon}{\rho(1-\epsilon)}} - 1$. This completes the proof of Proposition 8.

10. Proof of Corollary 3: For a given rate of return to capital, $J(Q)$ is rising in either technology parameter. This occurs because $\psi = (1-\rho)^{\frac{(1-\rho)}{\rho}} a^{\frac{1}{\rho}} \rho A$. Consequently, the steady-state value of Q increases under higher levels of technology in either sector. Moreover, it is easy to show that the capital stock rises as well. This is done by differentiating (67) with respect to A where:

$$\epsilon k^{\epsilon-1} \frac{\partial k}{\partial A} = \left\{ -\frac{\partial \gamma}{\partial Q} Q + \frac{(1-\gamma(Q, \sigma))(1-\epsilon)(1-\rho)}{(1-(1-\rho)\frac{1}{Q})} \frac{1}{Q} + (1-\gamma(Q, \sigma)) \right\} \left(1 - (1-\rho) \frac{1}{Q} \right)^{(1-\epsilon)} \left(\frac{1-\alpha}{\alpha} \right) \frac{\partial Q}{\partial A}$$

Clearly, $\frac{\partial k}{\partial A} > 0$ since $\frac{\partial \gamma}{\partial Q} < 0$ and $\frac{\partial Q}{\partial A} > 0$. A similar proof can be established for the effects of a . We next need to show that Q rises more under investment-specific technological change. This is achieved by holding Q fixed and showing that $J(Q)$ shifts by more as a rises relative to an increase in A . As we are holding Q fixed, we can ignore constant terms. In this manner, Q rises by more under sector specific growth if the condition in Corollary 3 holds.

Finally, we show the effects of different technology parameters on r and P_k . As discussed previously, the steady-state rate of return to capital is generated by (68). Furthermore, using (31), (32), and the expression for ψ into (68):

$$V(P_k) = Z(P_k)$$

$$\text{Where } V(P_k) = \left[(1-\rho) \rho^{\frac{\rho}{(1-\rho)}} P_k^{\frac{\rho}{(1-\rho)}} \right]^{\frac{\epsilon}{\rho(1-\epsilon)}} \text{ and } Z(P_k) = \frac{\left[1 + \frac{1}{(Q(a, P_k) - (1-\rho)) (1-\gamma(Q(a, P_k), \sigma))} \right]}{\left(\alpha^{\frac{1}{\epsilon}} (1-\rho)^{\frac{(1-\rho)}{\rho}} \rho A \right)^{-\frac{\epsilon}{(1-\epsilon)}}}$$

Clearly, $V'(P_k) > 0$ and $Z'(P_k) < 0$. Moreover, under higher A , $Z(P_k)$ increases for

a given P_k . As a result, the steady-state P_k also rises under higher A . In contrast, under higher levels of a , $Z(P_k)$ falls for a given P_k . Consequently P_k is declining in a . As r and P_k are positively related by (31), this result also applies for r . This completes the proof of Corollary 3.

11. Proof of Proposition 9: By (45), welfare is maximized when $\frac{(1-\theta)}{\sigma}\Psi(\sigma) = \Psi'(\sigma)$. Substituting for the expressions of $\Psi(\sigma)$ and $\Psi'(\sigma)$, this condition becomes:

$$\frac{1}{\sigma} = \frac{dw/d\sigma}{w(\sigma)} - \frac{\theta}{(1-\theta)} \frac{d\gamma/d\sigma}{\gamma(\sigma)} \quad (69)$$

We next need to find an expression for each term in (69).

Differentiating (49) and (50) with respect to σ , and using (67) generates:

$$\frac{dw/d\sigma}{w(\sigma)} = \frac{-k_y^\epsilon}{(1-\alpha)} \frac{1}{\rho} \frac{1}{Q} \frac{\partial Q}{\partial \sigma} \quad (70)$$

Moreover, totally differentiating γ with respect to σ and using the fact that $\frac{1-\pi}{\pi} (Q\sigma)^{\frac{1-\theta}{\theta}} = \frac{(1-\gamma)}{\gamma}$:

$$\frac{\theta}{(1-\theta)} \frac{d\gamma/d\sigma}{\gamma(\sigma)} = -\frac{1-\gamma}{\sigma} \left[1 + \frac{\sigma}{Q} \frac{\partial Q}{\partial \sigma} \right] \quad (71)$$

Finally, we can solve for k_y^ϵ from (50). Using the expression for k_y^ϵ , (70), (71), and the derivative of (68) with respect to σ into (69), the optimality condition becomes:

$$\zeta(Q(\sigma), \sigma) = \Gamma(Q(\sigma), \sigma) \quad (72)$$

where $\zeta(Q(\sigma), \sigma) = \frac{1}{(1-\frac{(1-\rho)}{Q})} + \frac{\epsilon}{\rho(1-\epsilon)} \frac{1}{\left(1 - \frac{\psi \frac{\epsilon}{(1-\epsilon)} \frac{1}{\alpha} \frac{1-\epsilon}{Q \rho(1-\epsilon)}\right)}$ and $\Gamma(Q(\sigma), \sigma) =$

$$\frac{1-\theta}{\theta} \frac{1}{\alpha \rho} \left[\frac{1}{\frac{Q \frac{\epsilon}{(1-\epsilon)} \rho}{\psi \frac{\epsilon}{(1-\epsilon)} \alpha \frac{1-\epsilon}{1-\epsilon}} - 1 \right].$$

First, it is clear that $\Gamma(Q(\sigma), \sigma)$ is strictly increasing in σ and passes by the origin since $\frac{\partial Q}{\partial \sigma} < 0$. Moreover, $\lim_{\sigma \rightarrow \infty} \Gamma(Q(\sigma), \sigma) \rightarrow \infty$ since $Q \rightarrow 0$. Furthermore, $\frac{d\zeta(Q(\sigma), \sigma)}{d\sigma} > 0$, $\lim_{\sigma \rightarrow \infty} \zeta(Q(\sigma), \sigma) \rightarrow \frac{1-\theta}{\theta}$, and $\lim_{\sigma \rightarrow 0} \zeta(Q(\sigma), \sigma) \rightarrow \frac{1}{\theta} + \frac{\epsilon}{\rho(1-\epsilon)}$.

Suppose an interior solution exists. In this case, the optimal rate of money growth falls under higher a if $\frac{a}{Q^*} \frac{\partial Q^*}{\partial a} < 1$. This occurs because $\frac{d\Gamma(Q(\sigma), \sigma)}{da} |_{\sigma=\sigma_0} > 0$ and $\frac{d\zeta(Q(\sigma), \sigma)}{da} |_{\sigma=\sigma_0} \leq 0$. Similarly, the optimal rate of money growth falls under higher A if $\frac{A}{Q^*} \frac{\partial Q^*}{\partial A} < 2\rho$.

Finally, if welfare is falling with inflation ($\omega'(\sigma) < 0$), an interior solution does not exist. Optimal monetary policy implies that $\sigma_{FR}^* Q^* = 1$. Since Q rises under higher levels of technology, then σ_{FR}^* must fall. This must occur as nominal equity returns are rising with inflation. That is, the direct effect of higher money growth on $\sigma_{FR}^* Q^*$ dominates the indirect effect through lower Q . Moreover, as $\frac{\partial Q^*}{\partial a} |_{\sigma=\sigma_0} > \frac{\partial Q^*}{\partial A} |_{\sigma=\sigma_0}$, the optimal inflation rate under investment specific growth is lower relative to neutral change. This completes the proof of Proposition 9.