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Hamid Beladi_
Department of Economics
University of Texas at San Antonio

Reza Oladi
Department of Applied Economics
Utah State University

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Hamid Beladi*

Department of Economics, University of Texas at San Antonio

Reza Oladi

Department of Applied Economics, Utah State University

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JEL: O14, O18

Keywords: Urban unemployment, rural-urban migration, firm heterogeneity

*Correspondence address: Hamid Beladi, Department of Economics, University of Texas at San Antonio, One UTSA Circle, San Antonio, Texas 78249-0633, Tel: 210-458-7038, Fax: 210-458-7040, Email: hamid.beladi@utsa.edu.

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Abstract

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1 Introduction

The steady and increasing pace of industrialization of major developing economies such as China and India has brought major social and economic problems in the past few decades. Two of such intertwined issues are massive rural-urban migration and ensuing urban unemployment. While the literature on rural-urban migration and urban unemployment is decades old and goes back to the celebrated Harris and Todaro (1970) and its extensions such as Khan (1980, 1982),¹ the recent industrialization trends in these developing economies as well as recent development of the theory of heterogeneous firms warrant a new look at this well-established theory of urban unemployment.

¹See also Batra and Lahiri (1988), Marjit (1991), among others.

A central element of the literature on rural-urban migration and urban unemployment is the assumption of perfect competitive markets. The purpose of our current paper with its new look at this issue is twofold. First, we construct a new model that combines a perfectly competitive rural sector with an imperfectly competitive urban sector. Second, and more importantly, we assume that the urban firms are heterogeneous.

Although much of the theories of the past decades in development economics (and other fields of economics such as international economics) rely heavily on firm homogeneity, there are ample anecdotal evidence suggesting that firms are heterogeneous. Significant empirical studies also confirm the heterogeneity of firms in both developed and developing economies. For example, see Clerides et al. (1998) for Colombia, Mexico, and Morocco and Bernard and Jensen (1999) for the United States. While theory of heterogeneous firms have been increasingly employed in some areas of economics (for example, see Melitz (2003) and more recently Beladi and Oladi (2011) for firm heterogeneity and the new trade theory), it is surprising to see that such new theoretical development is missing in an important and timely area such as rural-urban migration and urban unemployment. As stated earlier, we intend to fill this important gap in the literature.

We build upon Dixit and Stiglitz (1977) and construct a general equilibrium model of an economy with a rural and an urban sector. Rural sector produces an identical good in a perfectly competitive market whereas the urban sector produces a continuum of differentiated goods in a monopolistic competitive market. Firms in the urban sector enjoy increasing returns to scale as in Krugman (1980). In contrast to Dixit and Stiglitz (1977) and Krugman (1980), urban firms are heterogeneous in their productivities. As in Harris and Todaro (1970) and its extensions such as Khan (1980, 1982), we assume that an institutionally fixed and binding minimum wage in the urban sector gives rise to rural-urban migration and is the cause of urban unemployment. However, it is worth re-emphasizing that our points of departure from these works is the assumption of monopolistic competitive urban sector with heterogeneous firms.

We show that an increase in binding urban minimum wage will increase the aggregate price of the urban sector and lowers its aggregate output. We define a measure of intra-industry competitive position of urban firms as in Beladi and Oladi (2011) and show that such a measure will not be affected by changes in the binding urban minimum wage. Since firms are not identical in our model, each firm experiences a different level of output cut in response to an increase in the minimum

wage. Moreover, the decrease in output for each firm is proportional to the change in aggregate urban output. Nevertheless, this proportional decrease in output for each firm is different and is a function of its competitive position. In addition, an increase in the binding minimum wage will reduce welfare if the urban labor demand is inelastic.

We also study the effects of technical progress in the rural sector of developing economies. We show that rural technical progress has no effect on urban aggregate price index. However, through its positive effect on national income, rural technical progress leads to an increase in aggregate urban production. While any firm's competitive position remains unchanged, its output increase proportionally to the increase in aggregate urban output in response to the rural technical progress. The proportion is however different for each firm and depends on its competitive position in the urban industry. The higher is a firm's competitive position, the bigger is its portion of the increase in urban output. Most importantly, rural technical progress can be immiserizing if the urban employment is elastic with respect to rural real wage.

While studying technical progress in rural sector of developing economies is interesting and relevant for many developing economies, it is perhaps more interesting and relevant to many developing economies to analyze the effects of urban technical progress. To study the effects of urban technical progress we introduce a notion of *pattern of technical progress* that arises from our introduction of heterogeneous urban firms.² We show that this pattern plays a crucial role in the effects of urban technical progress in developing countries. If the pattern of technical progress is such that the productivity growth for the least productive firms is less than that of industry average, then technical progress leads to weakening of their competitive position. Thus, they exit the industry. As a result, if enough of these firms exit, the unemployment rate will rise and national income will fall. It is conceivable that with this pattern of urban technical progress welfare falls. It is noteworthy that the reason for the possibility of immiserizing technical progress is entirely different from the existing literature, where for example inter-sectoral mobility of capital plays a crucial role (for example, see Khan (1982)).³

The structure of the rest of the paper is as follows. We construct our model in the following

²Although the notion of sectoral biased technical progress has been addressed in the literature (for example, see Beladi and Oladi (2011)), the notion of (intra-sectoral) pattern of technical progress has not been introduced in the literature in our knowledge.

³See also Marjit (1991) and Chao and Yu (1993).

section. We analyze the effects of a change in binding urban minimum wage in section 3. Sections 4 and 5 study the effects of technical progress in the rural and urban sectors, respectively. Section 6 concludes the paper.

2 The setup

Consider a developing economy with a possible range of differentiated manufacturing goods denoted by $\Lambda \equiv [0, \bar{\lambda}]$ that are produced in urban regions and an identical rural good. The consumer's preferences are represented by the following utility function:

$$U = X \left[\int_{\lambda \in \Lambda} [q(\lambda)]^\rho d\lambda \right]^{\frac{1}{\rho}} \quad (1)$$

where $\rho \in (0, 1)$ and $q(\lambda)$ and X are quantities of good $\lambda \in \Lambda$ and of rural good. It follows from Dixit and Stiglitz (1977) that the urban sector aggregate price and the aggregate industry output are, respectively, given by:

$$P_Q = \left[\int_{\lambda \in \Lambda} [P(\lambda)]^{1-\sigma} d\lambda \right]^{\frac{1}{1-\sigma}} \quad (2)$$

$$Q = \left[\int_{\lambda \in \Lambda} [q(\lambda)]^\rho d\lambda \right]^{\frac{1}{\rho}} \quad (3)$$

where $P(\lambda)$ is the price of good $\lambda \in \Lambda$ and $\sigma = 1/(1-\rho) > 1$ is the demand elasticity of substitution. Following Beladi and Oladi (2011) the equilibrium consumption of good $\lambda \in \Lambda$ is given by:

$$q(\lambda) = Q [\xi(\lambda)]^{-\sigma} \quad (4)$$

where $\xi(\omega) \equiv P(\lambda)/P_Q$ is the intra-industry relative price of variety λ . As in Beladi and Oladi (2011), we call $\xi(\lambda)$ the competitive position of firm $\lambda \in \Lambda$. The lower ξ is for a firm, the better it is positioned within the industry. Equation (4) states that the optimal consumption of any good (i.e., the production level for any firm) is an increasing (decreasing) function of aggregate industry output (its intra-industry relative price).

We assume, as in Krugman (1980), that the production technologies for all urban firms use only labor as input and exhibit increasing returns to scale. However, unlike Krugman (1980), we

maintain that urban firms are heterogeneous in their production technologies. The labor usage for good $\lambda \in \Lambda$ is given by:

$$l(\lambda) = \delta + \frac{q(\lambda)}{\gamma(\lambda)} \quad (5)$$

where $\gamma(\lambda) > 0$ is the measure of productivity for the firm producing good $\lambda \in \Lambda$ and δ is the fixed cost of production.⁴ We assume that productivity is increasing in good (or firm) index, i.e., $\gamma' > 0$. That is, we sort urban firms such that the higher indexed firms are more productive. We further assume that there exists an institutionally fixed and binding minimum wage in the urban sector and denoted by \bar{W} .

The market in the rural sector is perfectly competitive and its Ricardian production technology is given by:

$$l_X = aX \quad (6)$$

where $a > 0$ is the fixed unit labor requirement in rural sector and $l(X)$ is the quantity of labor usage in rural sector. We assume, as in Harris and Todaro (1970), that there exists unemployment in the urban sector. Therefore, the resource constraint is given by:

$$l_X + l_u + \int_{\lambda \in \Lambda} [l(\lambda)] d\lambda = \bar{l} \quad (7)$$

where l_u is the level of unemployment in urban sector. The competitive equilibrium in the rural sector requires that:

$$W_X = \frac{P_X}{a} \quad (8)$$

where P_X is the price of the rural good. Assuming a binding minimum wage in the urban sector, the rural-urban migration results in:

$$W_X = e\bar{W} \quad (9)$$

where $e \equiv (\bar{l} - l_X - l_u)/(\bar{l} - l_X)$ is the rate of urban employment and \bar{W} is the binding urban minimum wage. Therefore, urban unemployment rate is $1 - e$. Equation (9) states that the rural wage must equate the expected urban wage at equilibrium.

Our general equilibrium model entails a two-stage budgeting problem. The equilibrium of the

⁴Since there is a one-to-one relationship between goods and firms (see Dixit and Stiglitz 1977), we also identify firms with $\lambda \in \Lambda$.

first stage requires market clearing conditions $X = 0.5I/P_X$ and $Q = 0.5I/P_Q$ for rural and urban sectors, respectively. The right-hand sides of these clearing conditions are demand functions for X and the aggregate urban sector and I is the aggregate income. We also define aggregate income by:

$$I = \bar{l}W_X + \int_{\lambda \in \Lambda} \pi(\lambda) d\lambda \quad (10)$$

That is, the aggregate income amounts to the sum of labor income and aggregate firms' profit.⁵

Turning now to the equilibrium of the second stage, profit maximizing behavior by urban firms results in the following pricing rule:

$$P(\lambda) = \frac{\bar{W}}{\rho\gamma(\lambda)} \quad (11)$$

That is, more productive firms have lower prices and therefore better competitive position, i.e. lower ξ . Similarly, it can be shown that the profit for firm $\lambda \in \Lambda$ is given by:

$$\pi(\lambda) = \frac{[\frac{\rho\gamma(\lambda)P_Q}{\bar{W}}]^{\sigma-1}}{\sigma} E - \delta \quad (12)$$

where $E \equiv PQ$ is the aggregate urban industry expenditure. Urban firms continue their operations so long as they maintain non-negative profit. Equation (12) can be used to derive the marginal firm as:

$$\pi(\underline{\lambda}) = \frac{1}{\sigma} P(\underline{\lambda})Q[\xi(\underline{\lambda})]^{-\sigma} - \delta = 0 \quad (13)$$

where $\underline{\lambda}$ is the marginal firm that earns zero-profit. This implies that all goods $\lambda \in [0, \underline{\lambda}]$ will not be produced.

Note that our model can be fully determined given its two stage budgeting structure. Equation (11) determines $P(\lambda)$. Given Q , equations (2) and (13) determine P_Q and $\underline{\lambda}$ simultaneously. Then, equations (3) and (4) determine $q(\lambda)$ and Q , since we have already determined $\underline{\lambda}$, and $P(\lambda)$ (hence, $\xi(\lambda)$). Equation (12) determines $\pi(\lambda)$, which completes the determination of the second stage of our model. Turning now to the first stage, equations (6)-(10) as well as the demand function for the rural good and the definition of e (a total of 7 equations) simultaneously determine the remaining 7 variables of the first stage, i.e., l_X, l_u, X, I, P_X, W_X and e . In the rest of the paper, we express

⁵Note that we used equation (9) and the definition of e to derive labor income, i.e., the first term in equation (10).

all good prices and wages in terms of the urban good and denote them by lower case letters. Thus, we maintain that the price of rural good is unity. for the sake of notational simplicity, we drop the subscript for the aggregate urban industry relative price, denoting it by p .

3 The effects of minimum wage

We first start with the effects of an increase in minimum wage on our developing economy. An increase in binding minimum wage results in an increase in urban unemployment rate. To see this, assume an increase in the real urban minimum wage \bar{w} , in terms of the rural good. As the real rural wage is pinned down by our Ricardian set-up, equations (8) and (9) imply that the urban employment rate e must decrease which, in turn, implies an increase in urban unemployment rate.

We shall now determine the effects of an increase in \bar{w} on intra-industry relative price of the marginal urban firm, the range of goods produced in the urban sector, and the aggregate urban good price simultaneously. To do this, differentiate equation (2) and simplify to obtain:

$$\frac{dp}{d\bar{w}} = \frac{p}{\bar{w}} + \frac{p^\sigma}{\sigma - 1} \left[\frac{1}{\rho\gamma(\lambda)} \right]^{1-\sigma} \frac{d\lambda}{d\bar{w}} \quad (14)$$

Next, by differentiating the intra-industry relative price for the marginal firm $\xi(\lambda) = p(\lambda)/p$ and using equation (14) to simplify the result, we get:

$$\frac{d\xi(\lambda)}{d\bar{w}} = -\xi(\lambda) \frac{p^{\sigma-1}}{\sigma - 1} \left[\frac{1}{\rho\gamma(\lambda)} \right]^{1-\sigma} \frac{d\lambda}{d\bar{w}} \quad (15)$$

Similarly, differentiate the zero-profit condition (13), and noting the unitary elastic property of aggregate urban sector demand to simplify the result, we obtain:

$$\frac{dp}{d\bar{w}} + \sigma p [\xi(\lambda)]^{-1} \frac{d\xi(\lambda)}{d\bar{w}} - \Phi \frac{d\lambda}{d\bar{w}} = \frac{p}{\bar{w}} \quad (16)$$

where $\Phi \equiv (\sigma [\xi(\lambda)]^{\sigma+1} d\pi(\lambda)/d\lambda)/Q > 0$ By solving equations (14)-(16) simultaneously, we obtain that $d\lambda/d\bar{w} = d\xi(\lambda)/d\bar{w} = 0$ and $dp/d\bar{w} = p/\bar{w} > 0$. That is, an increase in urban minimum wage has no effect on the range of produced urban goods (as well as the number of urban firms) and the intra-industry relative price of marginal urban firm (i.e., its competitive position). However, the

aggregate price of the urban sector increases as a result of an increase in the minimum wage. The reason is intuitive. Since an increase in urban minimum wage increases the price of all urban firms with the same percentage rate (i.e., $(dp(\lambda)/d\bar{w})/p(\lambda) = 1/\bar{w}, \forall \lambda \in \Lambda$), the competitive position of all firms in the urban sector remain unchanged. Thus, the range of urban goods/firms remains unchanged. However, the urban sector aggregate price goes up as all urban produced varieties are now more expensive.

It is interesting to study the effect of a change in urban minimum wage on national income. To do so, we first study its effects on profit of urban firms. Differentiate equation (12) and simplify the result to obtain:

$$\frac{d\pi(\lambda)}{d\bar{w}} = \Delta \left(-\frac{dp}{d\bar{w}} - \frac{\sigma p}{\xi(\lambda)} \frac{d\xi(\lambda)}{d\bar{w}} + \frac{p}{\bar{w}} \right) \quad \forall \lambda \in \Lambda \quad (17)$$

where $\Delta = (\bar{w}Q[\xi(\lambda)]^{-\sigma})/(\sigma p p \gamma(\lambda))$. Although, we verified earlier that the intra-industry relative price for the marginal firm remains unchanged, it is straightforward that this claim is also true for all other firms.⁶ Therefore, it follows from equation (17) that $d\pi(\lambda)/d\bar{w} = 0, \forall \lambda \in \Lambda$ since we established earlier that $dp/d\bar{w} = p/\bar{w}$. It then follows that $d/d\bar{w} \int_{\lambda \in \Lambda} \pi(\lambda) d\lambda = 0$. Recall also that we have verified that $d\lambda/d\bar{w} = 0$. Since we also have that $dw_X/d\bar{w} = 0$, we conclude that $dI/d\bar{w} = 0$.

The effect of an increase in urban minimum wage on the aggregate urban output is negative since $dQ_p/dp < 0$ and we already established that p increases. This also implies from equation (4) that $dq(\lambda)/d\bar{w} = [\xi(\lambda)]^{-\sigma} dQ_p/d\bar{w} < 0, \forall \lambda \in \Lambda$. That is, the output of all urban firms will fall. Moreover, each firm share of industry output cut is proportionate, i.e., $dq(\lambda)/dQ_p = 1/[\xi(\lambda)]^\sigma \forall \lambda \in \Lambda$. A firm with higher intra-industry relative price will experience a lower share of the urban industry output cut. In other words, urban firms with higher productivity that are also bigger will face a bigger share of the industry output cut.

Next, we study the effects of a change in minimum wage on the employment and output of the rural sector as well as the level of urban unemployment. Differentiate equation (7) to obtain:

$$\frac{dl_X}{d\bar{w}} + \frac{dl_u}{d\bar{w}} = -\frac{dl_\Lambda}{d\bar{w}} \quad (18)$$

where $l_\Lambda \equiv \int_{\lambda \in \Lambda} [l(\lambda)] d\lambda$. Next, substitute equation (8) in equation (9), use the definition of urban

⁶ $d\xi(\lambda)/d\bar{w} = \xi(\lambda)[1/\bar{w} - (dp/d\bar{w})/p] = 0, \forall \lambda \in \Lambda$ since we showed that $dp/d\bar{w} = p/\bar{w}$.

employment rate, and differentiate the resulting equation to get:

$$\frac{dl_X}{d\bar{w}} = -al_\Lambda - a\bar{w}\frac{dl_\Lambda}{d\bar{w}} \quad (19)$$

By solving equations (18) and (19) simultaneously, we obtain:

$$\frac{dl_X}{d\bar{w}} = -al_\lambda(1 + \eta_\Lambda) \quad (20)$$

$$\frac{dl_u}{d\bar{w}} = -\frac{dl_\Lambda}{d\bar{w}} + al_\Lambda(1 + \eta_\Lambda) \quad (21)$$

where $\eta_\Lambda \equiv \frac{dl_\Lambda}{d\bar{w}} \frac{\bar{w}}{l_\Lambda}$ is the elasticity of labor demand in the urban sector. Therefore, employment in rural sector falls as a result of an increase in urban minimum wage if and only if the urban labor demand is inelastic. In turn, we conclude that rural output falls as a result of a rise in \bar{w} if and only if urban labor demand is inelastic. Finally, equation (21) reveals that urban unemployment level rises as a result of an increase in urban minimum wage if urban labor demand is inelastic. Recall that we already established that a increase in minimum wage will always increase the unemployment rate.

Last but not the least, it is also interesting to study the effect of a change in urban minimum wage on welfare. We have already established that the real income does not change. What is the welfare implication of a rise in urban minimum wage? Recall that the urban output Q always falls as a result of such a policy. On the other hand rural output falls as a result of such a policy if and only if urban labor demand is inelastic. We therefore conclude that an increase in urban minimum wage can be welfare reducing. A reduction in welfare takes place if urban labor demand is inelastic. The condition on urban elasticity of labor is very crucial. Clearly, if the urban labor demand is elastic, the rural employment and output increases as a result of an increase in the minimum wage. This states that under such a scenario reverse migration takes place since elastic urban labor demand leads to enough reduction in urban employment level (and rate) that lowers the urban expected wage. To achieve an equilibrium some workers will move back to rural sector. In fact, if such elasticity is sufficiently high, the urban unemployment level can fall. It is noteworthy that under elastic urban labor demand scenario, the policy of increasing urban minimum wage conceivably can increase welfare.

4 Rural technical progress

Next consider the effects of technical progress in rural sector. In our model this happens through a decrease in a , the unit labor requirement in the rural sector. To conduct such a comparative static analysis, we have to first study the effect of rural technical progress on the urban sector aggregate price, the range of produced urban goods, the intra-industry relative price for the urban marginal firm, and the aggregate national income. Therefore, differentiate equations (2), (10), (13) and the definition of intra-industry relative price for the urban marginal firm (i.e, $\xi(\underline{\lambda})$) to obtain, respectively:

$$\frac{dp}{da} - \frac{\bar{w}p^\sigma[\rho\gamma(\underline{\lambda})]^{\sigma-1}}{\sigma-1} \frac{d\underline{\lambda}}{da} = 0 \quad (22)$$

$$\frac{dI}{da} + \pi(\underline{\lambda}) \frac{d\underline{\lambda}}{da} + \frac{(\sigma-1)p^{\sigma-1}Q\Omega}{\sigma} \frac{dp}{da} = \frac{-\bar{l}}{a^2} \quad (23)$$

$$\frac{dp}{da} + \sigma p[\xi(\underline{\lambda})]^{-1} \frac{d\xi(\underline{\lambda})}{da} - \Phi \frac{d\underline{\lambda}}{da} = 0 \quad (24)$$

$$\frac{d\xi(\underline{\lambda})}{da} + \frac{\xi(\underline{\lambda})}{p} \frac{dp}{da} = 0 \quad (25)$$

where $\Omega = \int_{\underline{\lambda}}^{\bar{\lambda}} [\rho\gamma(\lambda)]^{\sigma-1} d\lambda$ and Φ is defined as in the preceding section. By solving equations (22)-(25) simultaneously, we obtain that $dp/da = d\underline{\lambda}/da = d\xi(\underline{\lambda})/da = 0$ and $dI/da = -\bar{l}/a^2 < 0$. That is, the rural technical progress has no effect on urban aggregate price, the marginal urban firm (or the range of produced urban goods), and the intra-industry relative price of the marginal urban firm. However, the rural technical progress increases the national income. As national income grows as a result of technical progress in rural sector, it follows from the aggregate demand for the urban sector that the aggregate urban output must increase. In turn, it follows that each urban firm's output must increase. However, the extent of increase in output of each urban firm will be different. In particular, we have $dq(\lambda)/da = [\xi(\lambda)]^{-\sigma} dQ/da, \forall \lambda \in \Lambda$. That is, the increase in each firm's output is proportional to the change in aggregate urban output. Moreover, such a proportion is an increasing function of a firm's competitive position. The higher is a firm's competitive position (i.e., the lower is its intra-industry relative price), the more its output grows as a result of technical progress in rural sector.

It is left to study the effect of technical progress in rural sector on rural production as well as sectoral employments and the urban unemployment rate. Differentiate equation (7) and equation

(9) with respect to a by using equation (8) and the definition of urban employment rate in equation (9). Then, solve the resulting system of equations to obtain:

$$\frac{dl_X}{da} = -\bar{w}l_\Lambda - a\bar{w}\frac{dl_\Lambda}{da} \quad (26)$$

$$\frac{dl_u}{da} = \bar{w}l_\Lambda + \left(1 - \frac{\bar{w}}{w}\right)\frac{dl_\Lambda}{da} \quad (27)$$

Since we already established that $dq(\lambda)/da < 0, \forall \lambda \in \Lambda$, thus $dl_\Lambda/da < 0$, equation (26) implies that it is conceivable to have $dl_X/da > 0$. That is, technical progress in the rural sector can lead to a reduction in rural employment. To shed some more light on this, note from equation (8) that $da = -a^2dw$. Substituting this in equation (26) and with some simplification we can obtain that $dl_X/da = \bar{w}l_\Lambda(\eta_{\Lambda w} - 1)$, where $\eta_{\Lambda w} \equiv (dl_\Lambda/dw)(w/l_\Lambda)$ is the elasticity of urban employment with respect to rural wage. This implies that $dl_X/da < 0$ if and only if $\eta_{\Lambda w} < 1$. In turn, we conclude that $dX/da < 0$ if and only if $\eta_{\Lambda w} < 1$, i.e., the rural sector technical progress increases rural production if and only if $\eta_{\Lambda w} < 1$. Note that the welfare implication of this last result is enormous. As the output of rural sector falls as a result of technical progress in rural sector if $\eta_{\Lambda w} > 1$, we conclude that under this condition rural technical progress can be immiserizing as it would be conceivable that $du/da < 0$ if $\eta_{\Lambda w} > 1$.

Similarly, we can use equation (8) in equation (27) to obtain $dl_u/da = \bar{w}l_\Lambda[1 - (1 - w/\bar{w})\eta_{\Lambda w}]$. Since it follows from equation (9) that that $w/\bar{w} < 1$, this implies that the unemployment level falls as a result of urban technical progress if $\eta_{\Lambda w} \in [0, 1]$. As for the urban unemployment rate, it is straightforward to see from equations (8) and (9) that e must rise as a result of rural technical progress, implying a decrease in urban unemployment rate.

The reader can readily verify that the effects of population growth is similar to the technical progress in rural area.⁷ It can be shown that $dp/d\bar{l} = d\lambda/d\bar{l} = d\xi(\lambda)/d\bar{l} = 0$ and $dI/\bar{l} = w > 0$. As our earlier results, $dI > 0$ and $dp = 0$ imply that $dQ > 0$. Then, the output of each firm $\lambda \in [\underline{\lambda}, \bar{\lambda}]$ increases proportionally. However, the proportional increase for any firm is a function of the firm's competitive position in its industry. That is, the more competitive firms in the industry

⁷Equations (22), (24) and (25) will remain the same, except all the derivatives will be with respect to \bar{l} . However, equation (23) changes to:

$$\frac{dI}{d\bar{l}} + \pi(\lambda)\frac{d\lambda}{d\bar{l}} + \frac{(\sigma - 1)p^{\sigma-1}Q\Omega}{\sigma}\frac{dp}{d\bar{l}} = w.$$

will experience a higher share of growth rate in urban output.

5 Urban technical progress

While the questions we raised in the preceding section are certainly relevant and important in the context of developing economies, it is perhaps more likely that technical progress takes place in the urban manufacturing sector in these economies. The anecdotal evidence of the past few decades seems to support this claim. Therefore, a more interesting set of questions would be to study the effects of technical progress in the urban sectors on various aspects of our developing economy on which we focus in the remainder of the paper. The only modification of our setup would be to introduce a parameter for technical progress for urban firms. Redefine the measure of productivity by $\gamma(\lambda, \beta)$ for any firm $\lambda \in \Lambda$, where $\beta > 0$ is an index of technical progress. Assume that $\gamma_\lambda > 0$ as before and that $\gamma_\beta > 0$. Moreover, the sign of $\gamma_{\beta\lambda}$ identifies the pattern of technical progress across the heterogeneous firms within the urban sector. As we will show shortly, such a pattern of technical progress in urban sector is consequential for its effects on our developing economy.

Now, having replaced $\gamma(\lambda)$ by $\gamma(\lambda, \beta)$ in equations (2), (10) and (13) (albeit all prices expressed in real term) as well as the definition of intra-industry relative price of the marginal urban firm, we differentiate these equations to obtain, respectively:

$$\frac{dp}{d\beta} - \frac{p^\sigma}{\sigma - 1} [p(\underline{\lambda})]^{1-\sigma} \frac{d\lambda}{d\beta} = -\frac{\rho p^\sigma}{\bar{w}} \int_{\underline{\lambda}}^{\bar{\lambda}} [p(\lambda)]^{2-\sigma} \gamma_\beta(\lambda, \cdot) d\lambda \quad (28)$$

$$\frac{dI}{d\beta} + \pi(\underline{\lambda}) \frac{d\lambda}{d\beta} = \frac{(\sigma - 1) Q p^\sigma}{\sigma} \int_{\underline{\lambda}}^{\bar{\lambda}} [\rho \gamma(\lambda, \cdot)]^{\sigma-2} \gamma_\beta(\lambda, \cdot) d\lambda \quad (29)$$

$$\xi(\underline{\lambda}) \frac{dp}{d\beta} - \sigma \pi_\lambda(\cdot) + \sigma p \frac{d\xi(\underline{\lambda})}{d\beta} = -\frac{\bar{w} \gamma_\beta(\underline{\lambda}, \cdot)}{\rho [\gamma(\underline{\lambda})]^2} \quad (30)$$

$$\frac{\xi(\underline{\lambda})}{p} \frac{dp}{d\beta} + \frac{d\xi(\underline{\lambda})}{d\beta} = -\xi(\underline{\lambda}) \frac{\gamma_\beta(\underline{\lambda}, \cdot)}{\gamma(\underline{\lambda})} \quad (31)$$

Solve the system of equations (28), (30) and (31) simultaneously for $dp/d\beta$ to obtain:

$$\frac{dp}{d\beta} = -\frac{P^\sigma}{p[\xi(\underline{\lambda})]^{2-\sigma} + \sigma \pi_\lambda} \left((\sigma - 1) [p(\lambda)]^{1-\sigma} p(\underline{\lambda}) \frac{\gamma_\beta(\underline{\lambda})}{\gamma(\underline{\lambda})} + \sigma \rho \pi_\lambda \int_{\underline{\lambda}}^{\bar{\lambda}} [p(\lambda)]^{1-\sigma} \frac{\gamma_\beta(\lambda, \cdot)}{\gamma(\lambda)} d\lambda \right) \quad (32)$$

Since $\pi_\lambda > 0$, it follows from equation (32) that $dp/d\beta < 0$. That is, technical progress in the

urban sector always lowers the urban relative aggregate price regardless of the pattern of technical progress in the sector. Another interesting question one might ask is what effect urban technical progress has on the marginal urban firm. Solving the above system of equations for $d\underline{\lambda}/d\beta$ and $d\xi(\underline{\lambda})/d\beta$, we obtain:

$$\frac{d\underline{\lambda}}{d\beta} = \frac{(\sigma - 1)p(\underline{\lambda})}{p[\xi(\underline{\lambda})]^{2-\sigma} + \sigma\pi_\lambda} \left(\alpha - \frac{\gamma_\beta(\underline{\lambda}, \cdot)}{\gamma(\underline{\lambda}, \cdot)} \right) \quad (33)$$

$$\frac{d\xi(\underline{\lambda})}{d\beta} = \frac{\sigma + \pi_\lambda \xi(\underline{\lambda})}{p[\xi(\underline{\lambda})]^{2-\sigma} + \sigma\pi_\lambda} \left(\alpha - \frac{\gamma_\beta(\underline{\lambda}, \cdot)}{\gamma(\underline{\lambda}, \cdot)} \right) \quad (34)$$

where $\alpha \equiv \int_{\underline{\lambda}}^{\bar{\lambda}} [1/\xi(\lambda)]^{\sigma-1} [\gamma_\beta(\lambda, \cdot)/\gamma(\lambda, \cdot)] d\lambda$ is an aggregate index of productivity growth due to technical progress. The weights for each good in this index depend on competitive position of the firm producing the good. The higher the competitive position of a firm is (i.e., the lower its intra-industry relative price is), the bigger its weight in this index is. It is interesting that the pattern of technical progress plays a crucial role here. It is evident from equation (33) that $d\underline{\lambda}/d\beta > 0$ if and only if $\alpha > \gamma_\beta(\underline{\lambda}, \cdot)/\gamma(\underline{\lambda}, \cdot)$. That is, technical progress forces the marginal urban firm to exit if and only if its productivity growth rate is less than our industry index α . Obviously this result is symmetric. That is, potential urban firms enter the industry if and only if the productivity growth rate due to technical progress for the (initial) marginal firm is larger than the industry index. This can also be seen by studying the effect of urban technical progress and its pattern on intra-industry relative price of the (initial) marginal firm. Equation (34) shows that $d\xi(\underline{\lambda})/d\beta > 0$ if and only if $\alpha > \gamma_\beta(\underline{\lambda}, \cdot)/\gamma(\underline{\lambda}, \cdot)$. That is, the competitive position of the marginal firm will be weakened if and only if its rate of productivity growth due to technical progress is less than the index for the industry. In fact, it is for this reason that such a firm will exit. In other words, if the pattern of urban technical progress is biased toward more productive firms, then it results in weakening the competitive position of less productive firms. This in turn leads to reduction of profits for these firms. Consequently some of these firms exit the market. It is also noteworthy that such type of upward biased technical progress reduces the number of manufactured urban goods as well.

Turning now to the effects of urban technical progress on unemployment, national income and welfare, the pattern of urban technical progress plays a key role again. Assume that urban technical progress is biased toward the more productive urban firms so that $d\underline{\lambda}/d\beta > 0$ as discussed above. It

then follows from equation (29) that national income can fall. It then follows that market clearing condition of rural good that X can fall, which results in $dl_X < 0$. On the other hand, it follows from equations (8) and (9), both stated in real terms, that $de = 0$. Finally, by differentiating the definition of e and maintaining that $de = 0$, we obtain that $dl_u = (e - 1)dl_X$. Therefore, $dl_u > 0$ if $dl_X < 0$. All these imply that urban technical progress can lead to an increase in unemployment if pattern of urban technical progress is biased toward more productive urban firms. Under this scenario technical progress can be immiserizing. The intuition is interesting. Given the heterogeneity of urban firms, if the pattern of technical progress is biased toward the bigger and more productive urban firms, then some least productive urban firms will be force out of the market. Now, if such pattern of progress is biased enough that a large number of these firms/goods disappear, then unemployment rises and national income falls. The reduction in welfare will then be a possibility. Conversely, it is evident from the above argument that none of these adverse outcomes will be possible if the pattern of technical progress is biased toward the less productive firms.

6 Conclusion

Rapid industrialization and urbanization of major developing economies such as China and India in the past few decades as well as the recent developments in the theory of heterogeneous firms motivated us to reconsider a well-established theory in development economics that addresses urban-rural migration and urban unemployment. The celebrated Harris and Todaro's (1970) theory of urban unemployment and all its extensions maintain a central assumption of perfect competitive markets and, therefore, firm homogeneity. However, most industrial goods as well as firms are not identical. This paper fills this important gap in the literature.

We constructed upon Dixit and Stiglitz (1979) by introducing the possibility of rural-urban migration, institutionally set urban minimum wage and ensuing urban unemployment. In addition, and most importantly, we depart from the literature (both the strand of literature instigated by seminal papers of Dixit and Stiglitz (1977) and Krugman (1980) and the branch that followed Harris and Todaro (1970) by assuming a continuum of heterogeneous urban firms. In the context of our model, we studied the effects of a change in the binding urban minimum wage on developing

economies. We showed that an increase in the urban minimum wage will increase the urban aggregate price index and reduces the aggregate urban output. While such a change does not influence the competitive positions of the urban firms, each firm will experience an output cut proportional to the change in aggregate urban output. It is interesting that such a proportional output cut for each urban firm is different and depends on its initial competitive position: a firm with a better competitive position will see its output cut by a bigger proportion. We also showed that an increase in the urban minimum wage can lead to an increase (decrease) in welfare if urban labor demand is elastic (inelastic).

Another interesting set of questions to study are the effects of rural technical progress on urban aggregate price, competitive positions of urban firms, and the rural sector. We showed that rural technical progress has no effect on urban aggregate price and competitive positions of heterogeneous urban firms. However, it has a positive effect on national income which in turn increases the aggregate urban output. Therefore, each urban firm will experience an increase in its output due to rural technical progress and such an increase is proportional to the increase in aggregate urban output. As before, each firm's proportion of increase in its output is different and depends on its initial competitive position. We also show a very interesting result that rural technical progress can be immiserizing.

Notwithstanding the importance of rural technical progress in developing countries, it is more crucial to study the effects of urban technical progress in these economies. To do this, we introduced an aggregate industry index of urban technical progress. We showed that the pattern of urban technical progress plays a crucial role. Accordingly, urban technical progress will force marginal urban firm out of the market if and only if such firm's productivity growth rate is less than the industry index. That is, urban technical progress which is biased toward more productive firms in the industry will lead to disappearance of less productive urban firms and thus a reduction in the range of produced urban goods. In contrast to the literature, the pattern of urban technical progress (a notion that is absent in the literature in our knowledge) plays also an important role in the effects of technical progress on unemployment, national income as well as welfare. Accordingly, if urban technical progress is biased toward more productive urban firms, then it can increase unemployment and decrease national income and welfare.

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