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# Exchange Traded Funds, Liquidity, and Market Volatility

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## ABSTRACT

We propose a simple model of trading based on the Kyle (1985) framework for securities that are included in Exchange Traded Funds (ETFs). The model postulates that trading in ETFs will increase volatility in their component stocks, and that volatility spillovers will be increasing in liquidity and the relative proportion of each stock held by the fund. An empirical analysis of trading in the S&P 500 SPDR and three heavily traded industry ETFs confirms these hypotheses, using both Amihud's (2002) measure of illiquidity as well as stock turnover as proxies for liquidity. The results are consistent with a positive volume-volatility relation as well as trading-based explanations of volatility. The findings are relevant to market practitioners, regulators and investors in these increasingly popular products, since ETFs may in fact contribute to volatility in their underlying component stocks, and thus to the stock market in general.

JEL Classifications: G12, G14.

Keywords: Exchange Traded Fund, ETF, Volatility Spillover, Liquidity, Volume.

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## 1. INTRODUCTION

Over the past decade, exchange-traded funds (ETFs) have become the investment vehicle of choice for investors and traders seeking rapid, low-cost exposure to broad equity market indices, industry sectors, and other asset classes. Hedge fund managers, institutional investors, and individuals increasingly turn to ETFs to implement their investment strategies. Trading in these securities has become an important source of information dissemination in U.S. equity markets, and this paper examines how volatility information flows across broad market and industry ETFs and their largest component stocks.

Our simple theoretical model postulates that trading in ETFs increases volatility in their component stocks and predicts that the level of volatility spillovers is related to liquidity and the proportion of each stock in its respective ETF. When stocks are included as component stocks in ETFs, they are exposed to an additional source of volatility that is generated by trading activity in ETFs. To confirm these suppositions empirically, we conduct an analysis of volatility transmission among four of the most heavily traded ETFs in the U.S. using the recently developed spillover model of Diebold and Yilmaz (2009, 2012). We find that volatility spillovers flow bi-directionally among ETFs and their largest component stocks, but the effect is significantly stronger from ETFs to stocks than in the reverse direction. In addition, we find that the level of volatility spillovers from ETFs to component stocks is related to ETF liquidity and the proportion of each stock that is held in the ETFs. We document significant volatility spillovers from ETFs to their component stocks that are driven by the well-documented volume-volatility relation. The results are consistent with trading-based explanations of volatility, and the results are relevant to market practitioners, regulators and investors in these increasingly popular products, since ETFs may indeed be inducing additional volatility in U.S. equity markets.

Financial theory and the law of one price tell us that the prices of derivative instruments such as ETFs should be priced in a manner that is dependent on the value of their underlying securities. However, there is substantial research that documents the fact that derivative prices often lead spot prices, and one explanation for this phenomenon is the lower trading costs and higher liquidity that is frequently associated with derivatives markets. Chan (1992) finds that stock index futures lead cash market returns on an intraday basis, but only weak evidence of a relationship in the reverse direction. He credits this result to the greater ability of futures markets to process market-wide information, and he cites the model of Admati and Pfleiderer (1988), which posits that information dissemination is related to the level of trading intensity. Hasbrouck (2003) investigates the price discovery process for equity ETFs, floor-traded stock index futures, and electronically-traded stock index futures (eMini's) where the underlying asset is identical. He finds that most price discovery for the S&P 500 and Nasdaq-100 indexes occurs in the eMini market, even though contract sizes are much smaller than the floor-traded contracts. More recently, Roll, Schwartz, and Subrahmanyam (2010), and Johnson and So (2012) find that ratio of options/stock (O/S) trading volume provides useful information regarding future stock returns. They also note that the O/S ratio is positively correlated with firm size and therefore with liquidity in company shares.

Given that trading in derivatives such as ETFs may affect future returns and volatility of their underlying stocks, we also examine the effects of liquidity and the volume-volatility relation. Karpoff (1987) provides a survey of early work and Lamoureux and Lastrapes (1990) utilize a GARCH model that includes trading volume in the estimation of conditional volatility, finding it to be significant in the evolution of stock prices. Such models are consistent with the "mixture of distributions" hypothesis whereby the arrival of new information (as proxied by volume) affects future return distributions. The theoretical relation between the intensity of

information transmission and volatility are prominent in Kyle (1985) and Admati and Pfleiderer (1988). In both models, higher trading volumes increase the presence of informed traders such that the price impact of volume ( $\lambda$ ) is attenuated. DeLong, Shleifer, Summers, and Waldmann (1990) and Shleifer and Vishny (1997) develop models whereby noise traders contribute to price volatility and arbitrageurs may rationally choose not to undertake profitable arbitrage opportunities. Froot, Scharfstein, and Stein (1992) suggest that noise traders with short horizons may “herd” on information unrelated to economic fundamentals. Avramov, Chordia, and Goyal (2006) propose a theoretical model of trading where selling activity induces excess volatility. Finally, the model of Malinova and Park (2011) predicts that “higher participation and systematic improvements in the quality of traders’ information lead to higher volume... and higher price volatility.”

Empirical evidence regarding the relationship between liquidity and volatility is provided by Bessembinder and Seguin (1992), who find that equity market volatility is positively related to the “unexpected” component of futures volume (using an ARIMA (0,1,10) model). Jones, Kaul, and Lipson (1994) find that volatility is related to the number of transactions in NASDAQ stocks, while Chan and Fong (2000) find that size of trades and order imbalances drive the volume-volatility relation in NASDAQ and NYSE stocks. Chordia, Roll, and Subrahmanyam (2002) observe that “To explain volatility, it is imperative to account for order imbalance and volume.” French and Roll (1986) find compelling evidence that trading activity is related to stock volatility since returns are as much as 72 times more volatile when the market is open than when it is closed. They posit that this phenomenon is the result of differing rates of information transmission, but Haugen (2010) attributes this large difference in variance to trading activity itself. Amihud (2002), Chordia, Roll, and Subrahmanyam (2001), and Haugen and Baker (1996) document the negative relation between liquidity and expected return. Haugen, Talmor, and

Torous (1991) provide evidence of large shifts in volatility that are unrelated to economic conditions and/or events, leading them to conclude that “the noise component of volatility” stems from market microstructure itself. Finally, a recent paper by Ben-David, Franzoni, and Moussawi (2012) examines these issues relative to ETFs specifically. While the underlying intuition of their paper (that ETFs provide an additional source of volatility in component stocks) is quite similar to our study as well as consistent with our model, they employ different empirical techniques and high frequency data. They too find evidence of price shocks that stem from ETF trading activity and link these shocks to ETF order imbalances and bid-ask spreads.

The natural setting in which to examine the relationship among the volatilities of ETFs and their component stocks is the literature on volatility spillovers. Much of the literature in this area applies GARCH models to focus on the effects of negative returns, interdependence, and volatility “contagion.”<sup>1</sup> These studies and many others provide significant evidence of volatility spillovers across countries, asset classes, and securities. But our objective is to model the spillovers among large numbers of securities simultaneously over time, so we choose to utilize the recently developed model of Diebold and Yilmaz (2009, 2012), which provides an efficient and tractable estimation procedure. The model is similar in approach to the nonlinear multiplicative error models (MEM) developed by Engle (2002) and Engle, Gallo, and Velucchi (2008), and will be described further in Section 4.

In addition to the theoretical and empirical examinations of liquidity and volatility, the study is motivated by the dramatic increase in the popularity of ETFs over the past decade, both in terms of assets under management and trading volumes. According to BlackRock, one of the world’s largest asset managers and ETF issuers, U.S. ETF assets passed the \$1 trillion mark on

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<sup>1</sup> See, for example; Forbes and Rigobon (2002), Hamao, Masulis, and Ng (1990) and Lin, Engle, and Ito (1994).

December 16, 2010.<sup>2</sup> In terms of trading volume, U.S. ETF trading now accounts for 28.2% of total U.S. equity dollar volume, down slightly from 32.7% of volume in 2009.<sup>3</sup> In light of this exponential and continuing growth, concerns have begun to surface that these derivative securities may be adversely creating volatility in their underlying securities and in equity markets at large. Wurgler (2010) notes the recently rising correlations in equity markets and provides a link to the increase in index-linked investing. He estimates that at least \$8 trillion in investable assets are benchmarked to various U.S. broad-based indices alone, and suggests that this phenomenon is “distorting stock prices and risk-return tradeoffs.” A significant portion of this increase is necessarily related to the popularity of ETFs, at both the market and industry levels.

In addition to the exponential growth of ETFs as investment vehicles, the use of this particular data is motivated by the some observers who deem them a source of market instability. Bradley and Litan (2010) conduct an in-depth study of ETFs and conclude that they pose “unquantifiable but very real systemic risks of the kind that were manifested very briefly during the ‘Flash Crash’ of May 6, 2010.” They attribute increasing volatility feedback effects to ETF trading activity which exacerbates market declines. As evidence they point to the fact that on May 6, 2010, trading in the Russell 2000 Index ETF (IWM) amounted to over 56% of the total trading volume of its constituent stocks. The joint SEC-CFTC report “Findings Regarding the Market Events of May 6, 2010” addressed ETFs directly, noting that “equity-based ETFs were disproportionately affected by the extreme price volatilities of that afternoon.”<sup>4</sup> The report also states that the market for ETFs is dominated by professionals, so that a much larger proportion of liquidity is found near the last trade price than for typical equity securities. Thus when prices

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<sup>2</sup> Business Wire, “U.S. ETF and ETP Assets Break Through \$1 Trillion Milestone,” December 17, 2010.

<sup>3</sup> National Stock Exchange. “Monthly ETF Reports.” September 30, 2010.

<sup>4</sup> United States (2010), ‘Findings Regarding the Market Events of May 6, 2010’, *U.S. Commodity Futures Trading Commission and U. S. Securities Exchange Commission*, p.39.

exceed the “normal” levels supported by market liquidity, ETFs can be subject to “free falls” because a much larger proportion of resting limit orders are concentrated near the last price. The SEC-CFTC preliminary report on this matter noted that “Of the U.S.-listed securities with declines of 60% or more away from the 2:40 p.m. transaction prices, which resulted in their trades being cancelled by the exchanges, approximately 70% were ETFs.”<sup>5</sup> In essence, when the liquidity providers stepped away on May 6 there were relatively few resting limit orders below the last trade price (as opposed to many such orders on the much deeper books of common equities), so many trades were executed against “stub” quotes, at prices as little as \$0.01 and as high as \$100,000 per share. Approximately 160 ETFs traded at prices almost 100% lower than their closing price on the previous day (i.e. at or near \$0.01 per share). In summary, there appears to be substantial empirical and anecdotal evidence of a link between ETF liquidity and equity market volatility. Although ETFs have traditionally been seen as a cost-efficient and effective tool for asset allocation, there may be unintended consequences whose effects have not yet been fully realized.

The following section presents the main hypothesis and contribution of the study to the literature. Section 3 presents the data and summary statistics for the study. Section 4 presents the methodology and the results of generalized volatility spillover model and documents the relation of volatility spillovers to measures of liquidity. Section 5 contains some concluding remarks.

## 2. HYPOTHESIS DEVELOPMENT

The study follows in the long stream of literature that examines the efficient markets hypothesis. We seek to understand how trading in ETFs disseminates volatility information to their

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<sup>5</sup> United States. U.S. Commodity Futures Trading Commission and U. S. Securities Exchange Commission. *Preliminary Findings Regarding the Market Events of May 6, 2010*. Washington. GPO, 2010.



component stocks. It could reasonably be argued that lead-lag relationships in returns and volatility might exist in the direction from the underlying securities to the ETFs, since ETFs are derivative securities and information transmission is not instantaneous, as observed by Grossman and Stiglitz (1980). But mean-variance causality should not flow in the opposite direction as stock prices should be only indirectly influenced by the price of a market-based or industry-based ETF (perhaps due to new market-wide or industry-wide information). The results documented here are inconsistent with the EMH and indicate that trading in ETFs contributes to the volatility innovation process of their underlying securities. The other hypothesis tested here is whether or not ETFs are responsible for contributing to market volatility through the effects on their underlying securities. We extend the literature on volatility spillovers and their causes, lending credence to news reports and academic research that attributes some level of market volatility to the trading of ETFs.

We develop a theoretical framework based on the Kyle (1985) model of trading. Because large institutions and traders are able to exchange ETFs for an equivalent portfolios of stocks (through the mechanism of “creation” and “redemption” units), they provide efficient opportunities for arbitrage. Whenever a unit price becomes more (less) expensive than the value of the underlying portfolio, the arbitrageur can buy (short) the underlying stocks, create (redeem) a unit and short (buy) the ETF on the market. These positions offset and the arbitrageur does not face any financial or fundamental risk to remove the mispricing and he continues trading until prices reflect fundamental values. Any shock to the ETF price, whether created by new market or industry information or noise traders taking advantage of ETF liquidity, should be eliminated quickly with a correction in the price of ETF, the underlying stocks, or both. In summary, when a stock becomes a component in an ETF, it will be exposed to a new source of volatility.

We assume that when a stock is not included in an ETF, its price is determined by  $\tilde{P}^i \sim (\bar{P}^i, \sigma_{P_i}^2)$ , and that the stock price follows the linear form of Kyle (1985):  $\tilde{P}_t^i = \delta_t^i + \lambda_i(\tilde{\omega}_t^i)$ , where  $\delta_t^i$  is the available information for stock  $i$  at time  $t$ ,  $\lambda_i$  is price impact measure (illiquidity) for stock  $i$ , and  $\tilde{\omega}_t^i$  is the order flow. ETF prices under arbitrage are determined by  $\tilde{E}^t = \sum_{i=1}^n a_i \tilde{P}_i^t$ , where  $a_i$  is the proportion of stock  $i$  in ETF's portfolio and  $\tilde{P}_i^t$  the corresponding stock price. At any point of time, the ETF price may deviate from its fundamental value ( $\tilde{E}^{t+1} = \sum_{i=1}^n a_i \tilde{P}_i^t + \tilde{\varepsilon}$ ). Because the underlying prices are the same as before, we assume that the deviation is a supply/demand shock ( $\tilde{\varepsilon} \sim (0, \sigma_{\tilde{\varepsilon}}^2)$ ) to the price which creates an arbitrage opportunity. Arbitrageurs will take advantage of the mispricing until prices converge at  $t+2$ , when the price correction happens in the ETF and/or its underlying stocks.

In the Appendix we provide a proof to demonstrate that when a stock is included as a component of an ETF, price shocks to the ETF lead to a change of the underlying stocks' order flow and price. The magnitudes of these changes are:

$$E[\Delta \tilde{\omega}_i | \tilde{\varepsilon}] = \frac{a_i \tilde{\varepsilon}}{\sum_{j=1}^n a_j^2 \lambda_j + \lambda_{ETF}} \quad (1)$$

$$E[\Delta \tilde{P}_i | \tilde{\varepsilon}] = \frac{a_i \lambda_i \tilde{\varepsilon}}{\sum_{j=1}^n a_j^2 \lambda_j + \lambda_{ETF}} \quad (2)$$

Assuming that there is no correlation between price and the shock, individual stock volatility will now be:

$$\tilde{\sigma}_{\tilde{P}_i}^2 = \sigma_{P_i}^2 + \left[ \frac{a_i \lambda_i}{\sum_{j=1}^n a_j^2 \lambda_j + \lambda_{ETF}} \right]^2 \sigma_{\tilde{\varepsilon}}^2 \quad (3)$$

The second term captures ETF to stock volatility spillover, and the size of the spillover therefore depends on the illiquidity of the stock, the illiquidity of the ETF, the proportion of the stock in

each ETF, and the variance of the shock. In the later sections we use the above conjecture to examine the impact of each variable on volatility spillovers by using a linear regression:

$$Vol\ Spill_i = \alpha + \beta_1 \lambda_i + \beta_2 \lambda_{ETF} + \beta_3 a_i + u_i \quad (4)$$

where  $\lambda_i$  is our proxy for stock illiquidity,  $\lambda_{ETF}$  is our proxy for ETF illiquidity, and  $a_i$  is our proxy for the proportion of each stock held in their respective ETF. From Equation (3), we observe that individual stock volatility is decreasing in stock and ETF illiquidity (or increasing in liquidity), and increasing in the proportion of each stock that is held in its respective ETF. Thus we hypothesize that the signs for the coefficient  $\beta_1$  on stock illiquidity and  $\beta_2$  on ETF illiquidity and will be negative, while the sign for  $a_i$  (proportion) should be positive. We confirm these hypotheses empirically in Section 4.

### 3. DATA

The study utilizes daily ETF and component stock return and price data from January 5, 1999 to June 29, 2012, obtained from Bloomberg Professional®, for the S&P 500 ETF (symbol SPY) and three popular industry ETFs; the Energy Select Sector SPDR – XLE, the Financials Select Sector SPDR – XLF, and the Industrials Select Sector SPDR – XLI. The data includes high and low daily prices for each ETF as well as the ten largest component stocks that comprise the holdings of each ETF as of June 29, 2012. There are 3,396 observations for the each of the ETFs and their respective component stocks, with the exception of GS, MET, and UPS, which began trading subsequent to the inception of the industry ETFs. We present summary statistics for the returns of the ETFs and their largest component stocks in Table 1, where all returns are expressed in percentages, including dividends.

Although Berkshire Hathaway “B” shares are currently among the top ten component holdings in the Financials ETF, we exclude them from the study due to their relative illiquidity

prior to their 50:1 split in 2010. PNC Financial (PNC) is included as a substitute since it is the next largest component stock as of June 29, 2012. Although these companies were top holdings in the funds on that date, they were not necessarily top holdings for the entire period of the study. However, they were held in significant amounts by each of their respective ETFs for the entire period, and the fact that they may not have been in the top ten for the entire period decreases the likelihood of finding the volatility spillovers that we document below. For each of the ETFs and component stocks, we also collect daily dollar turnover, defined as share volume times price and calculated on an intra-day basis by Bloomberg. We present average daily turnover for the ETFs (by year) in Table 2, which we will discuss further in the results below.

#### 4. METHODOLOGY & RESULTS

##### *A. The Generalized Volatility Spillover Model*

In order to examine volatility spillovers among these ETFs and their largest component stocks, we implement the model of Diebold and Yilmaz (2009, 2012, hereafter DY), which relies on variance decompositions. The model is similar in approach to the nonlinear multiplicative error models (MEM) developed by Engle (2002) and Engle, Gallo, and Velucchi (2008), although it uses a least squares approach. Engle (2002) observes that even in the presence of non-negative data, least squares estimation remains consistent, and the advantage of the DY approach is that it enables us to generate a time series of spillover levels. We will utilize these time series to link volatility spillovers to measures of liquidity over the past decade.

The original spillover model in DY (2009) relies on Cholesky factorization to achieve orthogonality, making it sensitive to the ordering of variables. The authors compensate for this limitation by rotating and randomizing orderings to achieve robust results. In their 2012 paper,

however, DY adopt the generalized VAR framework of Koop, Pesaran, and Potter (1996) and Pesaran and Shin (1998), hereafter KPPS, which results in a model that is not sensitive to the ordering of variables. We utilize the more recent model specification to avoid the ordering of variables issue.

For each ETF and their ten largest component stock returns, we estimate eleven-variable vector autoregressions (VAR( $p$ )), using  $p$  equal to five lags to represent one week of trading activity:

$$x_t = \sum_{i=1}^5 \phi_i x_{t-i} + \varepsilon_t, \text{ where } \varepsilon \sim (0, \Sigma), i. i. d \quad (5)$$

Using a moving average representation, this expression becomes:

$$x_t = \sum_{i=0}^{\infty} A_i \varepsilon_{t-i}, \text{ where} \quad (6)$$

$$A_i = \phi_1 A_{i-1} + \phi_2 A_{i-2} + \dots + \phi_5 A_{i-5} \quad (7)$$

$A_0$  is an eleven by eleven identity matrix where  $A_i = 0$  for  $i < 0$ , and the moving average coefficients are used to construct variance decompositions. Thus we can calculate the fraction of the  $H$ -step-ahead error variance in a forecast of  $x_t$  that is generated by shocks to  $x_j \forall j \neq i$  for each  $i$ . In our estimations we set  $H = 10$  to generate 10-day ahead forecasts from the variance decompositions. DY define *own variances shares* as “the fractions of the  $H$ -step-ahead error variances in forecasting  $x_i$  that are due to shocks to  $x_i$  for  $i = 1, 2, \dots, N$ , and *cross variance shares*, or spillovers, as the fractions of the  $H$ -step-ahead error variances in forecasting  $x_i$  that are due to shocks to  $x_j$ , for  $i, j = 1, 2, \dots, N$ , such that  $i \neq j$ .” Thus each firm’s  $H$ -step-ahead variance decomposition is denoted by  $\theta_{ij}^g(H)$  for  $H = 1, 2, \dots, H$ :

$$\theta_{ij}^g(H) = \frac{\sigma_{jj}^{-1} \sum_{h=0}^{H-1} (e_i' A_h \Sigma e_j)^2}{\sum_{h=0}^{H-1} (e_i' A_h \Sigma A_h' e_j)} \quad (8)$$

The variance matrix for the error vector  $\varepsilon$  is denoted by  $\Sigma$  and the standard deviation of the error term for the  $j$ th equation is  $\sigma_{jj}$ . The selection vector  $e_i$  contains one as its  $i$ th element and zeros otherwise. Because the generalized variance decomposition framework of KPPS does not orthogonalize the innovations from the error term, the contributions to the variance of the forecast error may not sum to unity. Thus DY “normalize” each entry in the decomposition matrix (*own* and *cross* variance shares) by the row sum as follows:

$$\tilde{\theta}_{ij}^g(H) = \frac{\theta_{ij}^g(H)}{\sum_{j=1}^N \theta_{ij}^g(H)} \quad (9)$$

By definition, therefore,  $\sum_{j=1}^N \tilde{\theta}_{ij}^g(H) = 1$  and  $\sum_{i,j=1}^N \tilde{\theta}_{ij}^g(H) = N$ .

DY proceed to construct the *total volatility spillover index* using the volatility contributions from the preceding variance decomposition:

$$\begin{aligned} S^g(H) &= \frac{\sum_{i,j=1}^N \tilde{\theta}_{ij}^g(H)}{\sum_{i,j=1}^N \tilde{\theta}_{ij}^g(H)} \cdot 100 \\ &= \frac{\sum_{i,j=1}^N \tilde{\theta}_{ij}^g(H)}{N} \cdot 100 \end{aligned} \quad (10)$$

As DY observe in their 2012 paper, this “is the KPPS analog of the Cholesky factor based measure used by Diebold and Yilmaz (2009)” in which they measure global equity spillovers. They note that the index measures “the contribution of spillovers of volatility shocks... to the total forecast error variance.” The present study will focus on the individual directional spillover contributions from the ETFs to their component stocks and also in the reverse direction.

## B. Volatility Spillover Analysis

We utilize the DY framework to analyze volatility spillovers among four popular ETFs and their ten largest component stocks. To do this, we require daily estimates of price variance, and we choose the following variance estimate that is based on daily high and low prices. For each ETF and stock  $i$ , on day  $t$ , we calculate:

$$\tilde{\sigma}_{it}^2 = 0.361[\ln(P_{i,t}^{high}) - \ln(P_{i,t}^{low})]^2. \quad (11)$$

where  $P_{it}^{high}$  is the maximum (high) price observed for stock or ETF  $i$  on day  $t$ , and  $P_{it}^{low}$  is the minimum (low) price observation. Support for this measure of price variance is provided by Parkinson (1980), Alizadeh, Brandt, and Diebold (2002), and Chan and Lien (2003). In Table 3 we provide summary statistics for this calculation on an annualized percentage basis such that  $\hat{\sigma}_{it} = 100\sqrt{255 \cdot \tilde{\sigma}_{it}^2}$ . While the mean values for annualized standard deviation are generally in the 20 to 30 percent range for these high capitalization companies, there are clearly some extreme values observed during the financial crisis.

Utilizing the methodology outlined above, we calculate *total volatility spillover indexes* for each of the ETFs and their respective component stocks. We utilize a 200-day rolling estimation period, 5 lags in the VARs, and a 10-day forecast horizon, then plot the total volatility spillover indexes in Figure 1. Each of the spillover plots is characterized by the same periodic volatility “bursts” observed in DY (2009), who study global equity spillovers up to 2007. These spikes in volatility are clearly seen during the collapse of the internet bubble in 2001-2002, during the financial crisis in 2007, and surrounding the “flash crash” in May 2010. Our plots differ from those of DY (2009), however, in two ways. First, the general level of spillovers is higher (the average spillover for SPY is 63.9 percent, while those in DY09 averaged 39.5 percent), and the ETF spillovers reach almost 90 percent in recent years. This result is reasonable given that we study closely related securities that comprise their respective ETFs,

while DY (2009) study less closely related global equity markets. It is also consistent with rising levels of arbitrage activity in ETFs and their component stocks.

Additionally, while there was no observable trend in the spillover plots in DY (2009) (their sample is from 1992 – 2007), the plots in Figure 1 display clear upward trends since 2003, when spillovers were at relatively low levels. The inception of this trend coincides with the exponential growth in the trading volumes of the ETFs from 2003 – 2008 that can be seen in Table 2. This result provides the first indication of the volume-volatility relation among ETFs and their component stocks. The concurrent upward trends in volume and volatility are consistent with the trading-based explanations of Chan (1992), Chordia, Roll, and Subrahmanyam (2002) and Haugen (2010).

While the prior analysis is useful to examine the behavior of total volatility spillovers among all these securities, we are particularly interested in the two-way interaction of volatility spillovers among the ETFs and their largest component stocks. DY provide a method to examine these relationships through the calculation of “directional” volatility spillovers. They use the normalized forecast variance shares from Equation (9) to compute approximate directional volatility spillovers transmitted by ETF or stock  $i$  to ETF or stock  $j$ . These spillovers are approximate since the generalized variance decompositions may not sum to one, as noted above. DY normalize by row, so the directional spillovers “from others” sum to unity across rows, but the spillovers “to others” do not sum to one by columns. We apply this methodology to each of the four ETFs and their ten largest component stocks to compute total and directional volatility spillovers for these securities, and we present the results in Table 4.

First we note that the grand average of spillovers (plotted in Figure 1) among ETFs and stocks encompasses a fairly tight range, from 63.9 percent for SPY to 75.1 percent for XLE. As noted earlier, however, spillovers are significantly higher since 2007. Additionally, the total



directional spillovers from ETFs (contribution to others) are consistently higher than from ETFs to stocks. For example, the contribution from SPY to its largest component stocks is approximately 119 percent (contribution to others), but only 76 percent (contribution from others) in the reverse direction. Similar results obtain for the other three ETFs, where trading in ETFs is consistently providing more information regarding variance forecasts of the stocks than the stocks are providing about the variance forecasts for the ETFs. These results are consistent with our model that hypothesizes greater variance for stocks that are included in derivative instruments such as ETFs.

Further information regarding the volatility transmission process can be gleaned from Table 4 by looking at the individual directional volatility spillovers in the first columns and first rows of each sub-table (highlighted in gray). The largest spillovers consistently occur in the first column, reflecting spillovers from each ETF to its largest component stocks. It is also noteworthy that the spillovers in column one generally decline with the percentage of each stock in the ETF (the stocks are sorted so that the top holding is just below the ETF *own* spillover while the smallest holding is at the bottom of the column). This observation leads us to link the levels of volatility spillovers to measures of ETF and stock liquidity as well as the relative proportions of each stock held by the ETF.

As further evidence that ETF volatility spillovers play an important part in the variance innovations of component stocks, we present net pairwise volatility spillovers in Table 5. These spillovers are obtained simply by subtracting the stock to ETF spillovers in column 1 of each section from their respective ETF to stock spillovers in row 1 of each section of Table 4. In 42 out of 50 cases, ETF to stock spillovers are greater than spillovers in the reverse direction. The results are qualitatively similar when we normalize the variance shares by columns instead of by

rows. It is clear that ETF to stock spillovers play an important role in determining future errors in forecast variance of the largest component stocks.

### *C. Volatility Spillovers and Liquidity*

Given the observations of the previous section and their link to our theoretical model, we next examine how volatility spillovers are related to measures of liquidity and the proportion of each component stock held in each ETF. The first step in this process is to generate time series of the individual directional spillovers. The values in Table 4 are effectively average spillovers for each stock and ETF for each of the 200-day moving average estimations, so we accomplish this by extracting the daily spillover values as they are calculated on a daily basis in Equation (11). We then use these daily volatility spillover time series as dependent variables in a series of regressions designed to measure the impact of liquidity and the proportion of each stock in its respective ETF on spillover levels.

Our first potential proxy for liquidity is a time series analog to that provided by Amihud (2002), which we define as:

$$ILLIQ_{i,t} = \frac{1}{200} \frac{\sum_{t=1}^{200} |R_{i,t}|}{VOL_{i,t}} \quad (12)$$

where  $|R_{i,t}|$  is the absolute value of daily return for each stock and ETF and  $VOL_{i,t}$  represents daily ETF and stock dollar turnover. We employ a 200-day moving average of this figure so that it is comparable to the 200-day rolling estimates of volatility spillovers. In an extensive study of liquidity measures, Goyenko, Holden, and Trzcinka (2009) find that this measure provides an assessment of the price impact that is at least as effective as more recent and more complicated measures of liquidity. However, we cannot utilize this exact measure since volume

(and dollar turnover) is not stationary in general, and clearly not stationary during our sample period. To eliminate the effect of the rising trend in turnover, we adopt Amihud's (2002) mean-adjusted illiquidity that we modify by first calculating average illiquidity for each of the ten stocks in each ETF:

$$AILLIQ_t = \frac{1}{10} \sum_{i=1}^{10} ILLIQ_{i,t} \quad (13)$$

We then calculate the mean-adjusted illiquidity for each stock and ETF as follows:

$$ILLIQM_{i,t} = \frac{ILLIQ_{i,t}}{AILLIQ_t} \quad (14)$$

This measure of illiquidity is not sensitive to the overall level of turnover, and provides a good proxy for the relative illiquidities of the ETFs and their component stocks. Our model also postulates that the level of volatility spillovers is related to the proportion of each stock contained in the ETFs. We therefore estimate a proxy for the proportion of each stock in each ETF as if the ten component stocks are the only ETF holdings:

$$Proportion_{i,t} = \frac{MKTCAP_{i,t}}{\sum_{i=1}^{10} MKTCAP_{i,t}} \quad (15)$$

where  $MKTCAP_{i,t}$  is the closing daily market capitalization for each stock  $i$  at time  $t$ .

In order to estimate the relations among these variables and volatility spillovers, we estimate the following regression equation:

$$\ln(VolSpill_{i,t}) = \alpha + \beta_1 ILLIQM_{i,t} + \beta_2 ILLIQM_{ETF,t} + \beta_3 Proportion_{i,t} + \varepsilon_{i,t} \quad (16)$$

The regressions are estimated using robust standard errors that are clustered by 200-day periods, consistent with the 200-day rolling estimation period for the volatility spillovers, as suggested by Petersen (2009). The results of these regressions are contained in Table 6, and are conducted for two separate time periods. First, we estimate the results for the entire sample from 1999 to 2012, and also from 2003 to 2012. The estimation of the sub-period results are motivated by two

factors. First, the industry ETF volumes are relatively low during the early years of the sample, so the economic significance of these results may be low. Each of the industry ETFs did not reach \$10 million in average daily turnover until 2004. Also, the exponential growth of ETF trading volume begins in 2003, so a separate examination of the results during this sample period seems appropriate.

In Panel A of Table 6, we observe that ETF illiquidity is a significant driver of volatility spillovers for both the full sample and the sub-period for three of the four ETFs. The coefficients for ETF illiquidity are negative and significant as predicted by our model, and the negative coefficient for market illiquidity (SPY) is much larger than for the others. This provides an indication that illiquidity in this market-based ETF plays a more significant role in the volatility generating process for its component stocks, which is consistent with the fact that turnover is much larger for SPY than for the industry ETFs. It is also notable that the constants for all of the estimations are large and highly significant. Thus there seems to be some constant level of volatility spillover driving the forecast variance of each ETFs' component stocks that is unrelated to liquidity. The coefficients on stock illiquidity are negative and significant in two cases (XLE and XLF) during both the full sample and the sub-period. So in these two cases illiquidity in the component stocks actually contribute to the ability of the ETF to "spill over" volatility back to the components, which is consistent with a volatility feedback effect. The coefficients on proportion are positive and significant only in the full sample for two ETFs, and for none of them in the later period, although they remain positive. This may be the result of the fact that proportion and illiquidity are correlated, and the effects of proportion are being subsumed in the illiquidity coefficients.

The results for stock to ETF spillovers presented in Panel B of Table 6 are less conclusive. While there is still also a constant level of volatility spillovers from stocks to ETFs,

the effects of illiquidity and proportion are generally smaller, and only significant in the later period for SPY, which once again may be the result of its extremely high levels of turnover. However, proportion is significant and large for XLF in the both periods indicating that the largest financial firms are important to the volatility generating process for this ETF.

We also conduct an additional examination of the relationship between liquidity and volatility using raw dollar turnover as a proxy. Since turnover is not stationary, we de-trend turnover into its “expected” and “unexpected” components using a simple AR(1) process, in manner similar to those suggested by Bessembinder and Seguin (1992), Amihud (2002), and Lo and Wang (2010):

$$\ln(\textit{Turnover})_{i,t} = \alpha + \beta \ln(\textit{Turnover})_{i,t-1} + \varepsilon_{i,t} \quad (17)$$

We use the residual estimates from that equation to calculate 200-day moving averages of “unexpected” volume for both the ETFs and their component stocks, then utilize them as independent variables in the following regression:

$$\ln(\textit{Vol Spill}_{i,t}) = \alpha + \gamma_1 \varepsilon_{t,STK} + \gamma_2 \varepsilon_{t,ETF} + \gamma_3 \textit{Proportion}_{i,t} + u_t \quad (18)$$

Once again, the standard errors are clustered as recommended by Petersen (2009), and the results of these estimations are presented in Table 7. The results are generally similar to the results in Table 6. The volatility spillovers from ETFs to stocks are driven by ETF volume in three out of four cases for the later “high volume” period, which is consistent with the volume-volatility relation. The coefficients for proportion are significant and positive for all the ETFs in the full sample, and for two of the ETFs in the sub-sample. Positive coefficients are expected here because we are using volume as a proxy for liquidity, whereas in the previous results we proxied illiquidity. There is also a constant level of spillovers that is large and significant.

In Panel B we present the results for stock to ETF spillovers, where the results are once again slightly less conclusive than those in Panel A. ETF liquidity is positively related to spillovers only in half of the estimations, and the results for stock liquidity are mixed. Proportion is a significant factor in the full sample for all of the stock to ETF spillovers, but in only one ETF for the later period, indicating that the importance of this factor has attenuated as volume in the ETFs has grown much larger. The fact that the coefficient for ETF liquidity is positive and significant for SPY in the full sample but not in later years is another indication of the increased importance of trading in ETFs in the volatility generating process of their underlying component stocks. Finally, the large positive constant level of volatility spillover remains a significant feature of our results. These results demonstrate that the volume-volatility relation we document is robust to two different proxies for liquidity and provide an indication that trading in ETFs may in fact contribute to the volatility generating process of their largest component stocks.

## 5. CONCLUSION

We propose a simple theoretical model that seeks to explain the volatility generating process of ETFs and their largest component stocks. The model posits that shocks to ETF prices, which may be driven by new fundamental information, liquidity-seeking institutions and/or “noise” traders, increase the volatility of component stocks.

Using the recently developed volatility spillover model of Diebold and Yilmaz (2009, 2012), we examine volatility transmission among four of the most heavily traded ETFs and their largest component stocks. An examination of volatility spillover plots for the ETFs reveals the same volatility “bursts” found in DY (2009) that occur during periods of market instability. But

we also observe a clear upward trend in volatility spillovers since 2003, which is concurrent with the dramatic rise in ETF trading over this period.

We find that volatility spillovers in these securities flow bi-directionally, but the effect is stronger from ETFs to stocks than in the reverse direction, regardless of how the variance shares are normalized. In addition, using robust regression analysis, we demonstrate that the level of volatility spillovers from ETFs to component stocks is related to ETF liquidity and the proportion of each stock that is held in the ETFs. The results are strongest for the most heavily traded ETF, the S&P 500 SPDR, which indicates that price and volume shocks at the broad market level generate volatility spillovers in individual stock prices. We document significant volatility spillovers from ETFs to their component stocks that are driven by liquidity, using two different proxies to document a volume-volatility relation in these securities. The results are consistent with trading-based explanations of volatility, and they are relevant to market practitioners, regulators and investors in these increasingly popular products, since ETFs may indeed be inducing additional volatility in U.S. equity markets.

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**Table 1. Summary Statistics of Daily Returns for ETFs and 10 Largest Component Stocks.****January 5, 1999 – June 29, 2012**

**SPY: S&P 500 SPDR**

Symbol/Statistic	SPY	AAPL	XOM	MSFT	IBM	GE	CVX	T	JNJ	WFC	PG
N	3395	3395	3395	3395	3395	3395	3395	3395	3395	3395	3395
Mean	0.0099	0.1189	0.0341	0.0035	0.0269	-0.0027	0.0408	0.0063	0.0235	0.0256	0.0183
Median	0.0696	0.1120	0.0722	0.0000	0.0250	0.0000	0.0772	0.0251	0.0149	0.0000	0.0224
Std. Deviation	1.3625	3.1554	1.6963	2.1461	1.8768	2.1296	1.7416	1.8852	1.3559	2.7636	1.5509
Variance	1.8564	9.9566	2.8774	4.6058	3.5223	4.5351	3.0331	3.5540	1.8384	7.6373	2.4054
Skewness	0.0011	-3.7682	0.0435	-0.0342	-0.0974	0.0236	0.0861	0.1198	-0.3447	0.8102	-3.8357
Kurtosis	8.6065	87.2595	8.9635	7.7233	7.7864	6.8795	10.1630	5.2167	12.9505	21.2060	89.6674

**XLE: Energy Select Sector SPDR**

Symbol/Statistic	XLE	XOM	CVX	SLB	OXY	COP	APC	APA	NOV	HAL	EOG
N	3395	3395	3395	3395	3395	3395	3395	3395	3395	3395	3395
Mean	0.0371	0.0341	0.0408	0.0377	0.0784	0.0448	0.0453	0.0653	0.0720	0.0242	0.0707
Median	0.0881	0.0722	0.0772	0.0246	0.0874	0.0959	0.0979	0.1171	0.0782	0.0628	0.0836
Std. Deviation	1.8881	1.6963	1.7416	2.5416	2.2698	1.9445	2.6730	2.5281	3.3090	3.1339	2.6345
Variance	3.5650	2.8774	3.0331	6.4599	5.1518	3.7809	7.1447	6.3911	10.9498	9.8212	6.9408
Skewness	-0.4160	0.0435	0.0861	-0.2855	-0.2415	-0.3961	-0.5394	-0.1074	-0.2925	-1.5835	-0.1163
Kurtosis	8.3191	8.9635	10.1630	4.4238	8.0516	5.9274	7.6085	4.6691	6.2164	32.3485	4.1799

**XLF: Financials Select Sector SPDR**

Symbol/Statistic	XLF	WFC	JPM	BAC	C	USB	AXP	SPG	GS	MET	PNC
N	3395	3395	3395	3395	3395	3395	3395	3395	3313	3079	3395
Mean	-0.0058	0.0256	0.0029	-0.0253	-0.0545	0.0148	0.0249	0.0705	0.0213	0.0302	0.0169
Median	0.0000	0.0000	-0.0247	0.0000	0.0000	0.0297	0.0000	0.1088	0.0000	0.0302	0.0000
Std. Deviation	2.1738	2.7636	2.8553	3.3451	3.5628	2.5234	2.5917	2.4184	2.7306	0.0000	2.7057
Variance	4.7255	7.6373	8.1527	11.1898	12.6938	6.3677	6.7168	5.8485	7.4561	8.9919	7.3208
Skewness	-0.0071	0.8102	0.2749	-0.2883	-0.4668	-0.0495	0.0200	0.2466	0.6498	-0.3104	-1.3612
Kurtosis	10.9660	21.2060	10.2141	21.6220	32.7597	10.7950	7.2461	15.6273	12.3008	18.3857	57.3571

**XLI: Industrials Select Sector SPDR**

Symbol/Statistic	XLI	GE	UPS	UTX	MMM	UNP	BA	CAT	HON	EMR	DE
N	3395	3395	3180	3395	3395	3395	3395	3395	3395	3395	3395
Mean	0.0182	-0.0027	0.0227	0.0374	0.0353	0.0559	0.0317	0.0475	0.0165	0.0230	0.0546
Median	0.0760	0.0000	-0.0160	0.0326	0.0255	0.0318	0.0401	0.0558	0.0163	0.0000	0.0077
Std. Deviation	1.4749	2.1296	1.6658	1.9350	1.6280	1.9631	2.0973	2.2699	2.2866	1.9610	2.3777
Variance	2.1753	4.5351	2.7749	3.7442	2.6503	3.8539	4.3987	5.1525	5.2283	3.8455	5.6537
Skewness	-0.2117	0.0236	2.1666	-1.4309	0.0935	-0.2010	-0.1744	-0.1029	-0.2121	-0.0563	-0.1274
Kurtosis	4.7609	6.8795	42.9894	28.4974	4.0564	3.2747	4.9244	3.9788	10.2334	5.9124	4.3106

This table presents summary statistics of returns for the four ETFs and their ten largest component stocks for the sample period of January 5, 1999 to June 29, 2012. Three stocks (GS, MET, and UPS) were not publicly traded at the inception of the study, thus the number of observations is slightly lower than for the rest of the sample. Although Berkshire Hathaway “B” shares is currently a top ten component of XLF, we exclude that company from the study due to the relative illiquidity of those shares prior to the 50:1 split in 2010. PNC Financial (PNC) is included as the next largest substitute.

**Table 2. ETF Average Daily Turnover (\$ millions)**

	SPY	XLE	XLF	XLI
Year	Average Daily Turnover	Average Daily Turnover	Average Daily Turnover	Average Daily Turnover
1999	955.00	4.80	5.60	1.30
2000	1,090.00	10.70	13.50	1.82
2001	1,610.00	11.90	19.60	1.72
2002	3,220.00	8.40	55.10	4.36
2003	3,920.00	9.96	53.90	7.54
2004	4,870.00	65.10	97.00	18.60
2005	7,420.00	680.00	220.00	26.70
2006	9,160.00	1,280.00	263.00	47.30
2007	23,400.00	1,520.00	1,430.00	152.00
2008	35,100.00	2,460.00	3,520.00	295.00
2009	22,500.00	1,230.00	1,730.00	258.00
2010	23,800.00	1,040.00	1,450.00	484.00
2011	27,300.00	1,510.00	1,300.00	712.00
2012	21,000.00	1,050.00	1,160.00	558.00
Average	13,238.93	777.20	808.41	183.45

This table presents average daily dollar turnover, by year, for the four ETFs during the sample period of January 5, 1999 to June 29, 2012. The data is calculated as price times volume on an intraday basis by the Bloomberg Professional ® service, and is presented in millions of U.S. dollars.

**Table 3. Summary Statistics of Annualized Volatility for ETFs and 10 Largest Component Stocks (in Percent).**

**SPY: S&P 500 SPDR**

Symbol/Statistic	SPY	AAPL	XOM	MSFT	IBM	GE	CVX	T	JNJ	WFC	PG
N	3396	3396	3396	3396	3396	3396	3396	3396	3396	3396	3396
Mean	15.18	34.80	20.00	24.05	21.05	24.22	20.62	23.45	16.17	27.11	17.36
Std. Dev.	10.78	21.30	12.27	14.68	13.60	18.03	12.56	15.15	10.25	25.18	13.94
Min.	2.28	3.98	4.39	4.50	2.81	3.12	4.50	3.92	2.89	3.47	3.43
Max.	114.51	248.85	156.87	121.63	120.93	226.10	162.66	164.93	120.07	247.28	446.03

**XLE: Energy Select Sector SPDR**

Symbol/Statistic	XLE	XOM	CVX	SLB	OXY	COP	APC	APA	NOV	HAL	EOG
N	3396	3396	3396	3396	3396	3396	3396	3396	3396	3396	3396
Mean	20.92	20.00	20.62	31.28	26.68	23.22	31.61	29.96	39.20	36.96	32.54
Std. Dev.	13.58	12.27	12.56	16.62	16.14	13.99	18.57	17.26	24.34	23.91	18.78
Min.	4.29	4.39	4.50	5.15	3.66	4.20	5.86	5.51	3.53	7.88	5.73
Max.	170.15	156.87	162.66	185.80	169.44	161.57	235.96	204.12	306.72	618.32	220.31

**XLF: Financials Select Sector SPDR**

Symbol/Statistic	XLF	WFC	JPM	BAC	C	USB	AXP	SPG	GS	MET	PNC
N	3396	3396	3396	3396	3396	3396	3396	3396	3315	3081	3396
Mean	21.15	27.11	30.08	29.67	32.85	27.87	28.50	24.23	29.69	29.67	26.65
Std. Dev.	17.79	25.18	23.09	29.94	33.76	23.20	21.78	22.15	22.63	26.95	24.86
Min.	2.42	3.47	4.10	3.88	3.41	3.70	2.41	1.69	0.00	0.00	2.99
Max.	169.23	247.28	252.29	460.85	570.92	322.25	257.28	246.86	320.98	262.33	512.63

**XLI: Industrials Select Sector SPDR**

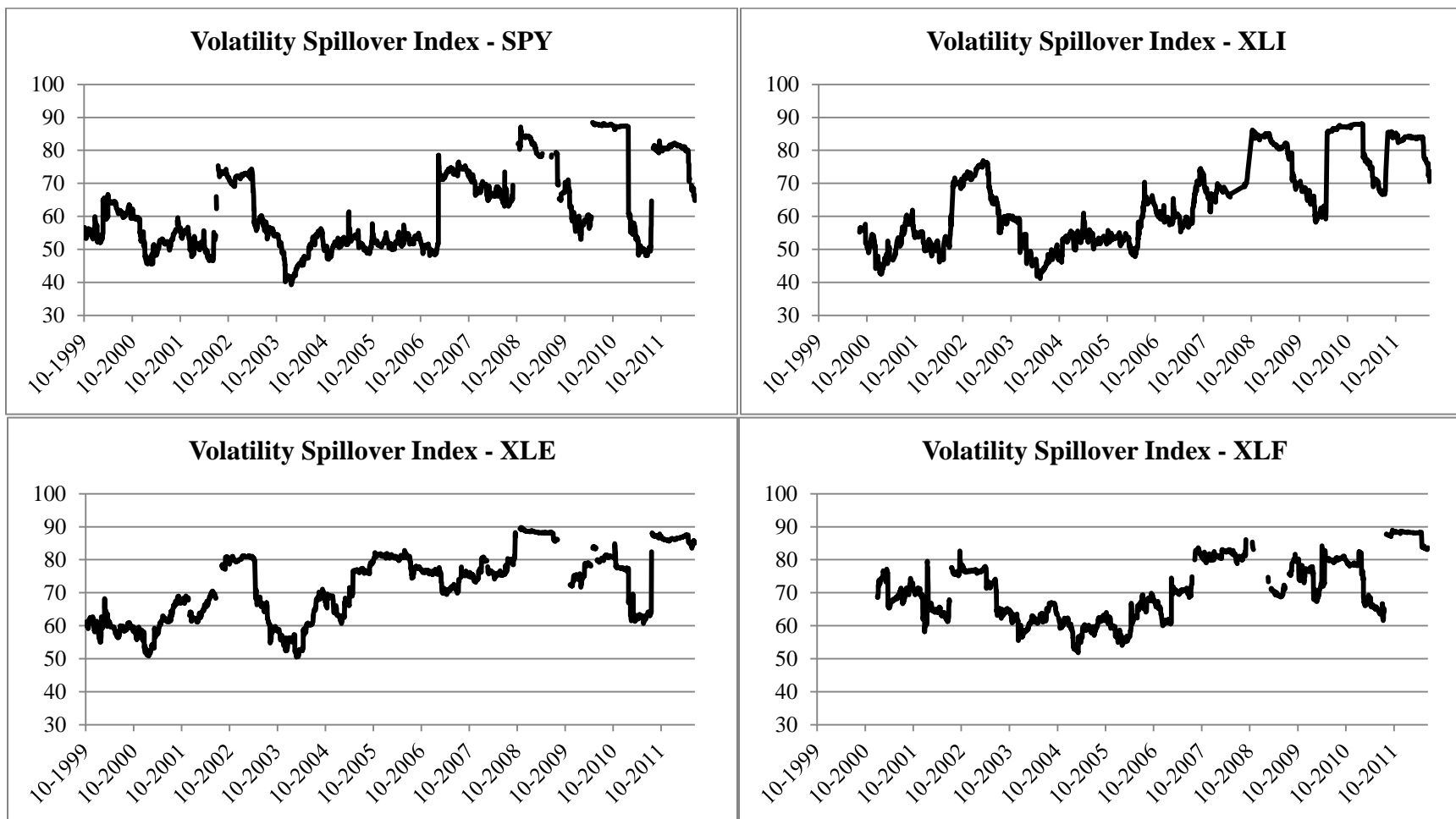
Symbol/Statistic	XLI	GE	UPS	UTX	MMM	UNP	BA	CAT	HON	EMR	DE
N	3396	3396	3181	3396	3396	3396	3396	3396	3396	3396	3396
Mean	16.36	24.22	17.91	22.45	19.93	23.74	25.13	27.01	26.71	23.84	28.76
Std. Dev.	11.15	18.03	12.70	14.12	12.53	15.47	14.44	15.67	17.04	14.47	17.87
Min.	2.02	3.12	0.00	4.08	4.18	3.76	4.67	5.91	5.24	4.43	4.12
Max.	111.27	226.10	200.71	163.07	274.75	148.93	136.65	203.11	239.02	148.31	211.77

This table presents summary statistics for our volatility estimates for the four ETFs and their ten largest component stocks for the sample period of January 5, 1999 to June 29, 2012. Daily volatility is estimated using daily high and low prices using the method suggested by Parkinson (1980) and others. For each ETF and stock  $i$ , on day  $t$ , we calculate:

$$\tilde{\sigma}_{it}^2 = 0.361[\ln(P_{i,t}^{high}) - \ln(P_{i,t}^{low})]^2. \quad (11)$$

where  $P_{it}^{high}$  is the maximum (high) price observed for stock or ETF  $i$  on day  $t$ , and  $P_{it}^{low}$  is the minimum (low) price observation. We provide summary statistics for this calculation on an annualized percentage basis such that  $\hat{\sigma}_{it} = 100\sqrt{255 \cdot \tilde{\sigma}_{it}^2}$ .

Figure 1. – Total Volatility Spillover Indices



Utilizing the methodology of Diebold and Yilmaz (2012), we calculate *total volatility spillover indexes* for each of the ETFs and their respective component stocks. The time series estimates are presented here using a 200-day rolling estimation period, 5 lags in the VARs, and a 10-day forecast horizon. Each of the spillover plots is characterized by the same periodic volatility “bursts” observed in DY (2009), who study global equity spillovers up to 2007. These spikes in volatility are clearly seen during the collapse of the internet bubble in 2001-2002, during the financial crisis in 2007, and surrounding the “flash crash” in May 2010.

**Table 4. Directional Volatility Spillovers.**

Panel A: S&P 500 SPDR (SPY)

		CONTRIBUTION FROM												
	Symbol	SPY	AAPL	XOM	MSFT	IBM	GE	CVX	T	JNJ	WFC	PG	From Others	
TO	SPY	24.3	3.3	12.9	4.7	7.0	8.2	13.9	7.4	8.6	7.0	2.5	76	
	AAPL	7.6	44.5	4.7	4.0	8.7	5.9	4.7	2.7	3.8	1.4	12.0	56	
	XOM	15.6	2.6	24.7	2.7	4.6	5.1	20.1	7.0	11.0	4.4	2.2	75	
	MSFT	11.7	4.2	7.6	35.6	11.2	5.9	7.1	5.8	7.2	3.3	0.5	64	
	IBM	12.2	7.2	6.1	9.0	35.1	6.6	6.4	6.4	6.0	3.1	1.8	65	
	GE	11.3	3.9	5.6	2.8	5.5	37.3	4.4	5.0	5.9	13.7	4.8	63	
	CVX	17.1	2.4	19.5	2.4	4.8	5.3	27.8	5.7	9.1	4.5	1.4	72	
	T	13.1	2.4	9.1	3.5	7.9	5.9	7.7	34.3	10.2	5.4	0.5	66	
	JNJ	12.9	2.6	12.7	4.0	6.0	6.8	10.4	8.7	30.3	3.7	1.9	70	
	WFC	9.5	1.0	3.4	2.0	4.8	15.5	3.0	2.4	2.6	55.2	0.5	45	
	PG	7.8	14.8	6.7	0.8	3.3	8.0	4.7	0.9	4.0	1.0	48.1	52	
		Contr. to others	119	44	88	36	64	73	82	52	68	47	28	703
		Contr. including own	143	89	113	72	99	111	110	86	99	103	76	63.90%

Panel B: Energy Select Sector SPDR (XLE)

		CONTRIBUTION FROM												
	Symbol	XLE	XOM	CVX	SLB	OXY	COP	APC	APA	NOV	HAL	EOG	From Others	
TO	XLE	17.7	8.9	9.5	9.9	9.9	12.8	7.3	9.2	8.5	0.9	5.3	82	
	XOM	13.9	17.7	11.8	9.5	8.2	12.2	5.6	9.2	7.1	0.8	4.0	82	
	CVX	13.6	11.3	15.6	9.2	8.9	13.0	6.2	8.9	8.0	0.6	4.6	84	
	SLB	11.6	7.2	7.4	22.2	9.7	10.4	6.6	9.0	8.1	1.0	6.8	78	
	OXY	12.5	6.9	8.4	10.6	18.0	12.7	7.1	8.3	7.7	0.5	7.2	82	
	COP	13.0	8.8	9.3	10.1	10.2	19.9	6.6	8.8	7.4	0.5	5.5	80	
	APC	10.5	5.7	6.4	9.7	8.5	9.8	23.9	9.3	7.1	0.6	8.5	76	
	APA	11.8	7.1	7.4	10.3	8.3	10.5	7.3	20.2	8.9	0.7	7.4	80	
	NOV	10.9	6.0	7.0	11.1	7.8	10.2	6.8	10.1	22.6	0.8	6.6	77	
	HAL	4.5	2.9	2.2	4.7	2.2	2.7	1.9	2.3	2.8	71.9	1.9	28	
	EOG	9.4	5.7	6.5	9.6	9.0	9.5	7.9	10.5	7.0	0.5	24.5	75	
		Contr. to others	112	70	76	95	83	104	63	86	73	7	58	826
		Contr. including own	129	88	91	117	101	124	87	106	95	79	82	75.10%

**Table 4 (continued). Directional Volatility Spillovers.**

Panel C: Financials Select Sector SPDR (XLF)

		CONTRIBUTION FROM												
	Symbol	XLF	WFC	JPM	BAC	C	USB	AXP	SPG	GS	MET	PNC	From Others	
TO	XLF	21.2	12.6	11.1	11.7	4.1	6.3	7.1	6.2	8.1	7.7	3.8	79	
	WFC	12.8	25.1	8.2	16.1	7.2	9.3	4.4	2.8	4.3	6.4	3.6	75	
	JPM	13.1	11.0	25.7	10.6	7.1	7.2	6.8	3.5	4.5	6.4	4.1	74	
	BAC	10.0	15.4	6.1	31.3	6.1	9.6	3.9	2.2	1.6	4.2	9.6	69	
	C	6.7	9.5	9.6	12.8	39.4	2.9	5.1	2.8	1.7	7.4	2.2	61	
	USB	10.1	14.1	7.2	14.4	5.7	26.9	4.6	2.7	3.1	5.8	5.4	73	
	AXP	13.1	10.2	7.3	9.7	1.9	5.1	28.9	5.9	6.4	8.6	2.9	71	
	SPG	11.6	5.5	7.1	8.0	5.5	3.1	7.2	27.1	4.3	14.5	6.1	73	
	GS	15.3	7.1	10.5	6.0	2.5	2.5	6.0	7.3	32.5	8.9	1.3	68	
	MET	6.4	7.4	3.5	10.0	3.7	3.1	9.1	4.4	5.0	44.3	3.0	56	
	PNC	7.2	9.6	5.4	16.6	3.2	10.5	3.8	3.0	2.4	6.3	32.1	68	
		Contr. to others	106	102	76	116	47	60	58	41	41	76	42	766
		Contr. including own	128	127	102	147	86	87	87	68	74	120	74	69.60%

Panel D: Industrials Select Sector SPDR (XLI)

		CONTRIBUTION FROM												
	Symbol	XLI	GE	UPS	UTX	MMM	UNP	BA	CAT	HON	EMR	DE	From Others	
TO	XLI	24.0	8.3	8.1	8.0	6.8	9.0	7.1	9.4	4.3	7.3	7.8	76	
	GE	10.9	44.0	5.2	4.6	7.0	3.0	4.6	8.6	3.4	3.6	5.1	56	
	UPS	9.4	5.1	29.3	9.4	3.6	9.4	3.3	10.3	5.5	5.5	9.1	71	
	UTX	10.2	5.0	10.9	25.5	5.3	7.1	7.9	7.8	7.2	6.5	6.5	74	
	MMM	12.3	7.8	5.2	9.2	35.6	3.9	8.9	6.9	4.0	3.1	3.0	64	
	UNP	11.5	5.8	7.9	6.4	2.5	31.0	4.2	8.0	3.1	5.1	14.4	69	
	BA	11.0	5.4	7.0	11.2	6.5	6.0	29.5	7.1	3.9	5.1	7.2	71	
	CAT	10.5	8.2	9.8	7.1	4.6	6.9	4.3	28.9	3.8	5.0	10.9	71	
	HON	7.8	4.5	8.1	11.3	3.8	4.9	4.6	6.2	40.2	3.8	4.8	60	
	EMR	11.8	5.8	9.2	8.7	2.6	9.2	5.3	8.0	4.8	26.4	8.2	74	
	DE	9.2	4.6	8.0	6.2	1.8	9.0	3.2	9.7	2.9	3.9	41.5	59	
		Contr. to others	105	61	79	82	45	68	53	82	43	49	77	744
		Contr. including own	129	105	109	108	80	99	83	111	83	75	118	67.60%

The table contains approximate directional volatility spillovers transmitted by ETF or stock  $i$  (contained in the top rows) to ETF or stock  $j$  (contained in the left-most columns). The “from” spillovers are approximate since the generalized variance decompositions may not sum to one since they are normalized by row sum and not column sum, as in Diebold and Yilmaz (2012). The results are qualitatively similar when normalizing by column instead of by row. The component stocks are sorted such that the stock that comprises the greatest percentage of each ETF (as of June 29, 2012) is just below the ETF in the left-most column, and then in descending order of proportion.



**Table 5. Net Pairwise Volatility Spillovers for ETFs and 10 Largest Component Stocks.**

	FROM		FROM		FROM		FROM
Symbol	SPY		XLE		XLF		XLI
AAPL	4.30	XOM	10.60	WFC	9.50	GE	7.60
XOM	2.70	CVX	0.70	JPM	0.20	UPS	-3.50
MSFT	7.00	SLB	6.90	BAC	5.30	UTX	5.50
IBM	5.20	OXY	5.50	C	-0.30	MMM	5.30
TO GE	3.10	COP	4.80	USB	1.90	UNP	3.30
CVX	3.20	APC	-3.40	AXP	-0.80	BA	-2.90
T	5.70	APA	4.40	SPG	4.20	CAT	3.10
JNJ	4.30	NOV	2.30	GS	6.70	HON	-0.80
WFC	2.50	HAL	-2.50	MET	-0.60	EMR	4.80
PG	5.30	EOG	6.90	PNC	4.70	DE	6.70
Mean	4.33		3.62		3.08		2.91

This table contains net pairwise volatility spillovers that are calculated by subtracting the stock to ETF spillovers in column one of each Panel of Table 4 from their respective ETF to stock spillovers in row one of each Panel of Table 4. Thus these figures represent the volatility spillover from each ETF to their respective component stocks in excess of the volatility spillover in the opposite direction (stock to ETF).

**Table 6. Directional Volatility Spillovers and Illiquidity**

$$\ln(\text{Vol Spill}_{i,t}) = \alpha + \beta_1 \text{ILLIQM}_{i,t} + \beta_2 \text{ILLIQM}_{ETF,t} + \beta_3 \text{Proportion}_{i,t} + \varepsilon_{i,t}$$

$$\text{ILLIQ}_{i,t} = \frac{1}{200} \sum_{t=1}^{200} |R_{i,t}| / \text{VOL}_{i,t} \quad \text{AILLIQ}_t = \frac{1}{10} \sum_{i=1}^{10} \text{ILLIQ}_{i,t} \quad \text{ILLIQM}_{i,t} = \frac{\text{ILLIQ}_{i,t}}{\text{AILLIQ}_t}$$

**Panel A. ETF to Stock Volatility Spillovers**

VARIABLES	1999-2012				2003-2012			
	SPY	XLE	XLF	XLI	SPY	XLE	XLF	XLI
Stock Illiquidity	-0.07 (-1.14)	-0.06*** (-4.07)	-0.20*** (-3.83)	0.05 (1.68)	-0.02 (-0.25)	-0.05** (-2.76)	-0.17* (-2.16)	0.00 (0.08)
ETF Illiquidity	-1.10*** (-5.79)	-0.04* (-1.92)	-0.02 (-0.80)	-0.00*** (-3.64)	-1.17*** (-8.50)	-0.01 (-0.53)	-0.06*** (-4.45)	-0.01*** (-3.39)
Proportion	0.67 (1.48)	0.38** (2.28)	-0.00 (-0.01)	0.85** (2.88)	0.76 (1.22)	0.13 (0.97)	0.01 (0.03)	0.35 (0.90)
Constant	2.26*** (22.77)	2.32*** (34.08)	2.49*** (32.65)	1.97*** (15.56)	2.21*** (14.50)	2.35*** (38.00)	2.48*** (21.36)	2.16*** (14.72)
Observations	29,879	30,290	26,330	28,050	22,050	22,590	21,580	22,370
Adj. R-squared	0.06	0.12	0.20	0.20	0.06	0.02	0.16	0.10

**Panel B. Stock to ETF Volatility Spillovers**

VARIABLES	1999-2012				2003-2012			
	SPY	XLE	XLF	XLI	SPY	XLE	XLF	XLI
Stock Illiquidity	-0.05 (-0.60)	-0.07*** (-3.67)	-0.13 (-1.53)	0.05 (0.47)	-0.02 (-0.11)	-0.09** (-2.34)	-0.11 (-0.88)	-0.03 (-0.30)
ETF Illiquidity	-0.60** (-2.60)	-0.03*** (-3.30)	-0.02** (-2.64)	-0.00*** (-5.47)	-0.46*** (-3.39)	-0.03 (-1.67)	-0.06 (-1.52)	-0.00 (-1.25)
Proportion	1.59 (1.70)	0.60** (2.72)	1.50*** (3.33)	0.62 (1.53)	0.88 (0.74)	0.25 (0.96)	1.66** (2.22)	0.01 (0.01)
Constant	1.72*** (9.98)	2.04*** (48.47)	1.95*** (15.97)	1.75*** (10.50)	1.76*** (7.42)	2.10*** (41.78)	1.93*** (10.11)	1.93*** (10.81)
Observations	29,879	30,290	26,330	28,050	22,050	22,590	21,580	22,370
Adj. R-squared	0.04	0.09	0.13	0.07	0.01	0.05	0.14	0.03

This table presents the results of robust regressions of volatility spillovers on illiquidity and a proxy for the proportion of each stock held in its respective ETF. Illiquidity is measured using Amihud's (2002) mean-adjusted measure (*ILLIQM*) to account for the rising trend in volume over this period. Robust t-statistics (in parentheses) are estimated using 200-day periods as suggested by Petersen (2009).

\*\*\* p<0.01, \*\* p<0.05, \* p<0.10.

**Table 7. Directional Volatility Spillovers and Turnover**

$$\ln(\text{Turnover})_{i,t} = \alpha + \beta \ln(\text{Turnover})_{i,t-1} + \varepsilon_{i,t}$$

$$\ln(\text{Vol Spill}_{i,t}) = \alpha + \gamma_1 \varepsilon_{t,STK} + \gamma_2 \varepsilon_{t,ETF} + \gamma_3 \text{Proportion}_{i,t} + u_t$$

**Panel A. ETF to Stock Volatility Spillovers**

VARIABLES	1999-2012				2003-2012			
	SPY	XLE	XLF	XLI	SPY	XLE	XLF	XLI
$\varepsilon_{STK}$	1.21*** (2.98)	-0.32 (-0.14)	0.83 (0.81)	0.42 (0.55)	0.55 (0.60)	-4.05*** (-3.41)	1.29 (0.95)	1.42 (1.55)
$\varepsilon_{ETF}$	0.70 (1.67)	1.51 (1.36)	-0.02 (-0.02)	2.27*** (4.71)	1.29** (2.82)	4.24*** (4.14)	-0.60 (-0.56)	1.72*** (3.91)
Proportion	1.02*** (3.10)	0.57*** (3.09)	1.60*** (5.93)	0.73** (2.83)	0.54 (1.51)	0.33* (2.09)	1.66*** (6.29)	0.36 (0.96)
Constant	2.06*** (50.49)	2.14*** (32.75)	2.09*** (43.61)	1.83*** (29.43)	2.13*** (39.43)	2.26*** (48.12)	2.12*** (42.49)	2.09*** (39.18)
Observations	29,879	30,290	26,330	28,050	20,060	20,600	19,590	20,380
Adj. R-squared	0.06	0.14	0.11	0.38	0.08	0.09	0.11	0.32

**Panel B. Stock to ETF Volatility Spillovers**

VARIABLES	1999-2012				2003-2012			
	SPY	XLE	XLF	XLI	SPY	XLE	XLF	XLI
$\varepsilon_{STK}$	1.07* (1.83)	-0.69 (-0.40)	2.37** (2.56)	0.95 (1.13)	0.63 (0.47)	-4.09* (-2.11)	3.84** (2.45)	-0.05 (-0.10)
$\varepsilon_{ETF}$	0.61* (2.04)	1.32 (1.51)	-0.71 (-1.16)	0.86** (2.38)	0.96 (1.09)	5.01** (2.92)	-1.57 (-1.33)	1.78*** (8.14)
Proportion	1.85** (2.35)	0.82*** (3.47)	2.54*** (6.54)	0.48* (2.02)	0.09 (0.10)	0.46 (1.53)	2.77*** (6.89)	0.03 (0.11)
Constant	1.60*** (18.99)	1.87*** (55.03)	1.67*** (35.02)	1.72*** (50.63)	1.79*** (16.51)	1.95*** (41.45)	1.67*** (36.97)	1.85*** (50.87)
Observations	29,879	30,290	26,330	28,050	20,060	20,600	19,590	20,380
Adj. R-squared	0.05	0.09	0.14	0.15	0.03	0.07	0.20	0.19

This table presents the results of robust regressions of volatility spillovers on liquidity and a proxy for the proportion of each stock held in its respective ETF. Liquidity is measured We de-trend turnover into its “expected” and “unexpected” components using a simple AR(1) process, in manner similar to those suggested by Bessembinder and Seguin (1992), Amihud (2002), and Lo and Wang (2010). Robust t-statistics (in parentheses) are estimated using 200-day periods as suggested by Petersen (2009).

\*\*\* p<0.01, \*\* p<0.05, \* p<0.10.

## Appendix – Proofs of Equations (1) and (2).

Prior to a price shock generated by new fundamental information or liquidity seeking traders, the ETF price is:

$$P_{ETF_B} = \sum_{i=1}^n a_i P_i \quad (19)$$

which is the portfolio's fundamental value. After a shock to the price of the ETF we have:

$$P_{ETF_A} = \sum_{i=1}^n a_i P_i + \varepsilon \quad (20)$$

Assuming no market frictions or new information, the mispricing will be eliminated by a change in price of the ETF and/or the underlying stocks. Thus, the following equality should hold:

$$P_{ETF_A} - \Delta P_{ETF_A} = \sum_{i=1}^n a_i P_i + \varepsilon - \Delta P_{ETF_A} = \sum_{i=1}^n a_i (P_i + \Delta P_{i_A}) \quad (21)$$

or:

$$\sum_{i=1}^n a_i \Delta P_{i_A} + \Delta P_{ETF_A} = \varepsilon \quad (22)$$

In the Kyle (1985) linear pricing framework, price is a function of information and order flow:  $\tilde{P}_t^i = \tilde{\delta}_t^i + \lambda_i(\tilde{\omega}_t^i)$ . However, we assume that information has not changed and the price correction will be only a function of order flow ( $\Delta\omega_i$ ). Replacing ( $\Delta P_i$ ) in (22) yields:

$$\sum_{i=1}^n a_i \lambda_i \Delta\omega_{i_A} + \lambda_{ETF} \Delta\omega_{ETF_A} = \varepsilon \quad (23)$$

The arbitrageur takes no risk and is able to create and/or redeem the exact proportion of each underlying stock to remove the mispricing, so the order flow for each stock is proportional to the holding of that stock in the portfolio:

$$\frac{\Delta\omega_{n_A}}{\Delta\omega_{1_A}} = \frac{a_n}{a_1}, \dots, \frac{\Delta\omega_{n_A}}{\Delta\omega_{n-1_A}} = \frac{a_n}{a_{n-1}} \quad (24)$$

Rewriting (23) results in:

$$a_1\lambda_1\Delta\omega_{1_A} + a_2\lambda_2\Delta\omega_{2_A} + \cdots a_{n-1}\lambda_{n-1}\Delta\omega_{n-1_A} + a_n\lambda_n\Delta\omega_{n_A} + \lambda_{ETF}\Delta\omega_{ETF_A} = \varepsilon \quad (25)$$

Substituting for every  $\Delta\omega_{i_A}$  as a function of  $\Delta\omega_{n_A}$  by using (24) gives:

$$\begin{aligned} a_1\lambda_1\Delta\omega_{n_A}\left(\frac{a_1}{a_n}\right) + a_2\lambda_2\Delta\omega_{n_A}\left(\frac{a_2}{a_n}\right) + \cdots a_{n-1}\lambda_{n-1}\Delta\omega_{n_A}\left(\frac{a_{n-1}}{a_n}\right) + a_n\lambda_n\Delta\omega_{n_A} \\ + \lambda_{ETF}\Delta\omega_{n_A}\left(\frac{1}{a_n}\right) = \varepsilon \end{aligned} \quad (26)$$

Solving for  $\Delta\omega_{n_A}$ :

$$\Delta\omega_{n_A} = \frac{a_n\varepsilon}{\sum_{j=1}^n a_j^2\lambda_j + \lambda_{ETF}} \quad (27)$$

which is the ex-post change in flow when a shock of size  $\varepsilon$  is realized. Thus the ex-ante expected changes in flow and price, conditional on shock  $\tilde{\varepsilon}$  are:

$$E[\Delta\tilde{\omega}_i|\tilde{\varepsilon}] = \frac{a_i\tilde{\varepsilon}}{\sum_{j=1}^n a_j^2\lambda_j + \lambda_{ETF}} \quad (28)$$

$$E[\Delta\tilde{P}_i|\tilde{\varepsilon}] = \frac{a_i\lambda_i\tilde{\varepsilon}}{\sum_{j=1}^n a_j^2\lambda_j + \lambda_{ETF}} \quad (29)$$