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ABSTRACT:

This paper compares three value-at-risk approximation methods suggested in the literature: Cornish-Fisher (1937), Sillitto (1969), and Liu (2010). Simulation results are obtained for three families of distributions: student-*t*, skewed-normal, and skewed-*t*. We recommend the Sillitto approximation as the best method to evaluate the value at risk when the financial return has an unknown, skewed, and heavy-tailed distribution.

KEYWORDS: value at risk, Cornish-Fisher approximation, Sillitto approximation, Liu approximation

JEL Codes: G11, G24, G30, M2, Y1

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1. Introduction

The conventional parametric value-at-risk (VaR) is calculated under the normality assumption. In reality, this assumption is often invalid. To rectify, a better fitted distribution function must be specified. However, it is a difficult, if not impossible, task. Alternatively, an approximation method must be proposed. In this paper, we consider three approximation methods suggested in the literature in terms of accuracy and efficiency: Cornish-Fisher (1937), Sillitto (1969), and Liu (2010). For student-*t*, skewed-normal, and skewed-*t* distributions, we calculate the biases and standard deviations of each approximates. It is found that the Sillitto approximation provides the best performance whereas Liu approximation has the worst performance.

The remainder of this paper is organized as follows. In Section 2, we introduce the three approximation methods. Section 3 describes the simulation framework. The results are presented and discussed in Section 4. Effects of the truncation order are analyzed in Section 5. Finally, Section 6 concludes the paper.

2. Approximation Methods

Let R denote the random return of a given financial position and let r denote the standardized return: $r = (R - \mu)/\sigma$, where μ is the mean and σ is the standard deviation of the return. Let $F(.)$ denote the distribution function of the standardized return. Then the 100q% confidence level value-at-risk of the financial position is defined as the negative value of the 100q%-quantile of the financial return: $VaR = -G^{-1}(q)$, where $G(.)$ is the distribution of R. Alternatively, using the standardized return, we have

$$VaR = -\mu - \sigma F^{-1}(q) . \tag{1}$$

When the functional form of $F(.)$ is known, we can estimate the distribution parameters and then the value-at-risk accordingly. Otherwise, we need to search for the best fitted distributions among all possible candidates. This approach may produce a sub-optimal result if the true distribution is not one of the candidates.

As an alternative, we can rely upon distribution-free approximations of $F^{-1}(.)$ to calculate the value-at-risk. Zangari (1996) adopts the approximation method of Cornish and Fisher (1937):[1]

$$F^{-1}(q) = \alpha + \frac{\alpha^2 - 1}{6} s + \frac{\alpha^3 - 3\alpha}{24} \kappa - \frac{2\alpha^3 - 5\alpha}{36} s^2 , \tag{2}$$

where $\alpha = \Phi^{-1}(q)$ with $\Phi(.)$ being the distribution function of the standard normal random variable; $s = m_3 / \sqrt{m_2^3}$ is the skewness and $\kappa = (m_4 / m_2^2) - 3$ is the excessive kurtosis with m_i denoting the i^{th} order central moment. Since r is standardized, $m_2 = 1$. Thus,

$$F^{-1}(q) = \alpha + \frac{\alpha^2 - 1}{6} m_3 + \frac{\alpha^3 - 3\alpha}{24} m_4 - \frac{2\alpha^3 - 5\alpha}{36} m_3^2 . \tag{3}$$

Jaschke (2002) states that the approximation is suitable only when the return distribution is sufficiently close to being normal. Cavenaile and Lejeune (2010) suggest that the approximation should never be used when $|q| \le 0.0416$.

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[1] Jaschke (2002) points out that the monotonicity of the distribution and the convergence are not guaranteed for the Cornish-Fisher expansion. Thus, a contradicting case may occur where the estimated value-at-risk when q = 0.05 may be larger than when q = 0.01.

Liu (2010) applies *L*-moments proposed by Hosking (1990) to estimate the value–at-risk. Specifically, let l_i denote the i^{th} order *L*-moment.[2] We replace all the central moments in equation (3) by corresponding *L*-moments to derive the Hosking approximation:

$$F^{-1}(q) = \alpha + \frac{\alpha^2 - 1}{6} l_3 + \frac{\alpha^3 - 3\alpha}{24} l_4 - \frac{2\alpha^3 - 5\alpha}{36} l_3^2 . \tag{4}$$

While Liu (2010) documents empirical support, this method is ad hoc and lacking theoretical foundation.

Finally, Hosking (1990) characterizes the infinite series approximation of Sillitto (1969) by *L*-moments:

$$F^{-1}(q) = \sum_{i=1}^{\infty} \left[(2i-1) l_i \sum_{k=0}^{i-1} (-1)^{i-1+k} \binom{i-1}{k} \binom{i-1+k}{k} q^k \right] \tag{5}$$

For estimation purposes, the approximation must be truncated. In the simulation experiments, we choose the order to be 15 (i.e., $i = 15$).

3. Simulation Setups

To compare the three approximation methods, we conduct simulations assuming that the financial return is generated by one of the following three distributions: (1) Student *t*, (2) skewed-normal, and (3) skewed-*t*.[3] They are the most commonly considered robust non-normal distributions in the literature.

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[2] More precisely, Liu (2010) evaluates the value at risk of a portfolio using a combination of Cornish-Fisher approximation and *L*-comoments proposed by Serfling and Xiao (2007). Applying the backtesting methods from Kupiec (1995) and Christoffersen (1998), Liu (2010) finds the new approximation to outperform the original Cornish-Fisher approximation.

[3] Dokov, Stoyanov, and Rachev (2007) provides the exact formula for values at risk under the skewed-*t* distributions. Hu and Kercheval (2007) and Louzis, Xanthopoulos-Sisinis, and Refenes (2011) both suggest the skewed-*t* distribution to be an attractive model for value-at-risk estimation.

Student t Distribution: The corresponding density function is as follows:

$$t_n(x) = \left(\frac{1}{\sigma}\right)\frac{\Gamma((n+1)/2)}{\sqrt{n\pi}\Gamma(n/2)}\left(1+\frac{[(x-\mu)/\sigma]^2}{n}\right)^{-(n+1)/2}, \tag{6}$$

where n is the degree of freedom and $\Gamma(.)$ is the Gamma function.

Skewed-Normal Distribution: The corresponding density function is

$$f(x) = \left(\frac{2}{\sigma}\right)\phi\left(\frac{x-\mu}{\sigma}\right)\Phi\left(\frac{a(x-\mu)}{\sigma}\right), \tag{7}$$

where $\phi(.)$ is the density function of the standard normal random variable. The parameter a controls for the skewness. The distribution is right skewed when $a > 0$ and left skewed when $a < 0$.

Skewed-t Distribution: The corresponding density function is

$$f(x) = \left(\frac{2}{\sigma}\right)t_n\left(\frac{x-\mu}{\sigma}\right)T_{n+1}\left(\frac{a(x-\mu)}{\sigma}\left(\frac{n+1}{n+[(x-\mu)/\sigma]^2}\right)^{1/2}\right), \tag{8}$$

where $t_n(.)$ is defined in equation (6) and $T_{n+1}(.)$ is the Student t distribution with $(n+1)$ degrees of freedom. Once again, a controls for the skewness. The distribution is right skewed when $a > 0$ and left skewed when $a < 0$.

Parameter configurations are chosen as follows. For Student t distributions, we consider eight possible degrees of freedom, $n =$ 5, 6, 7, 8, 9, 10, 20, or 30. Four skewness parameter values are examined for skewed-normal distributions, $a =$-1, -1, 1, or 2. Four combinations of degree of freedom and skewness parameter values are adopted, $(n, a) =$ (7, -2), (8, -1), (9, 1) or (10, 2). In addition, without loss of generality, we set $\mu = 0$ and $\sigma = 1$.

In each simulation scenario, we consider eight quantiles: $q =$ 0.01, 0.05, 0.10, 0.20, 0.80, 0.90, 0.95, and 0.99. The first four cases correspond to the values at risk for long positions whereas the last four stand for short positions. For each quantile, we perform 200 simulations. In each simulation, we generate 10,000 samples from each distribution and calculate the values at risk based upon the three approximation methods, i.e., equations (3)-(5). For each approximation method, the means and standard deviations of the 200 estimated values are tabulated for comparison purposes.

4. Simulation Results

In all tables, the three approximation methods: Cornish-Fisher, Liu, and Sillittom are represented by (1), (2), and (3), respectively. Table 1 displays the simulation results for Student t distributions. Specifically, in Table 1(a), we present the approximate values and their corresponding standard deviations, along with the true values. In all cases, the Liu approximation has the smallest standard deviation.[4] The Cornish-Fisher approximation has very large standard deviations when $n = 5$ or 6. The standard deviation for the Sillitto approximation lies between the other two approximation methods. As expected, the standard deviations are largest for the two tails, $q = 0.01$ or 0.99 and tend to decrease with increasing degree of freedom.

Table 1(b) presents the approximation errors. Clearly, the Sillitto approximation dominates the other two approximation methods, particularly when n is less than 20. In addition, the Liu approximation performs better than the Cornish-Fisher approximation when $n \leq 7$. For sufficiently large n (i.e., $n \geq 20$), Student t

[4] This result is likely driven by the robustness of the L-moments.

distribution is very close to the normal distribution; consequently, Cornish-Fisher approximation performs very well. While both Liu and Cornish-Fisher approximations do not appropriately accommodate for the heavy tails, the Liu method performs better than the Cornish-Fisher method in these circumstances. Table 1(b) also indicates that, the Liu approximation always over-estimate the value at risk whereas the Cornish-Fisher approximation always under-estimate the value at risk. The Sillitto approximation also provides an under-estimate but the bias is ignorable.

Simulation results for skewed-normal distributions are summarized in Table 2. Once again, the Liu approximation has the smallest standard deviation in all cases. Table 2(b) indicate that, all three approximation methods over-estimate (under-estimate) the value at risk when the distribution is positively (negatively) skewed. Moreover, the bias is increased as the distribution becomes more skewed. When comparing the three methods, we find Cornish-Fisher and Sillitto methods to have similar performance and both dominate the Liu approximation. The relative performance of the Liu approximation deteriorates with a larger absolute skewness coefficient. In sum, the results suggest that the Liu approximation cannot appropriately take into account of the skewness of the distribution.

Finally, we turn to the skewed-*t* distributions. Table 3 retains the qualitative properties observed from Tables 1 and 2. Since the distribution is both skewed and heavy-tailed, we expect the Sillitto approximation to outperform the other two methods. Table 3(b) validates this conjecture. In addition, the dominance of the Sillitto approximation over the Cornish-Fisher approximation is more evident when the distribution is more heavy tailed whereas the dominance of the Sillitto approximation over the Liu approximation is more evident when the distribution is more skewed.

5. Effects of Truncation Orders

The previous results are based upon different truncation orders for both Cornish-Fisher and Sillitto approximations. As higher order terms for the Cornish-Fisher approximation are difficult to derive, the conventional practice considers only up to the fourth order. This concern does not apply to the Sillitto approximation. In this section, we compare the three approximations adopting the same truncation order. Specifically, we truncate at the fifth order of central moments for the Cornish-Fisher approximation and at the fifth order of *L*-moments for the Sillitto approximation. As a consequence, equation (3) is modified as follows:

$$F^{-1}(q) = \alpha + \frac{\alpha^2 - 1}{6} m_3 + \frac{\alpha^3 - 3\alpha}{24} m_4 - \frac{2\alpha^3 - 5\alpha}{36} m_3^2 + \frac{\alpha^4 - 6\alpha^2 + 3}{120} m_5 - \frac{\alpha^4 - 5\alpha^2 + 2}{24} m_3 m_4 + \frac{12\alpha^4 - 53\alpha^2 + 17}{324} m_3^3 . \quad (9)$$

Equation (4) becomes

$$F^{-1}(q) = \alpha + \frac{\alpha^2 - 1}{6} l_3 + \frac{\alpha^3 - 3\alpha}{24} l_4 - \frac{2\alpha^3 - 5\alpha}{36} l_3^2 + \frac{\alpha^4 - 6\alpha^2 + 3}{120} l_5 - \frac{\alpha^4 - 5\alpha^2 + 2}{24} l_3 l_4 + \frac{12\alpha^4 - 53\alpha^2 + 17}{324} l_3^3 . \quad (10)$$

Meanwhile, equation (5) is truncated at $i = 5$.

Simulation results are presented in Tables 4-6. The addition of the 5th order central moment affects the Cornish-Fisher approximate minimally. The Liu approximate actually become worse in most cases. Greater impacts are observed in the Sillitto approximation when the truncation order is reduced from 15 to 5.

Overall, for the Student *t* distributions, the Cornish-Fisher method has the best performance when $n \geq 7$. For the other two distributions, there is not a clear winner although on average the Cornish-Fisher method has the best performance while the Liu approximation has the worst performance. We, therefore, conclude that the dominant performance of the Sillitto approximation in the previous section is generated by the higher order truncation.

It is likely that the Cornish-Fisher approximation may perform equivalently or better with the same truncation order. However, this suggestion is infeasible in actual practices as the higher order terms for the Cornish-Fisher approximation are difficult to calculate. As a result, the higher order Sillitto approximation remains the best method to evaluate values-at-risk for unknown, skewed, and heavy-tailed distributions.

6. Conclusions

Based upon the simulation results, we recommend the higher order Sillitto approximation as the best method to evaluate the value-at-risk when the financial return has an unknown, skewed, and heavy-tailed distribution. The Cornish-Fisher approximation performs poorly in the presence of heavy tails, particularly when the confidence level for the value-at-risk is high (i.e., $|q|$ is sufficiently small). On the other hand, the Liu approximation is clearly inferior to the other two methods.

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Table 1a. Approximate quantile of Student *t* distribution

q		0.01	0.05	0.10	0.20	0.80	0.90	0.95	0.99
n=5		-2.61	-1.56	-1.14	-0.71	0.71	1.14	1.56	2.61
	(1)	-3.48(1.507)	-1.25(10.028)	-0.88(1.910)	-0.34(1.300)	0.47(0.298)	0.61(4.805)	1.4(1.464)	4.66(17.155)
	(2)	-2.35(0.003)	-1.64(0.001)	-1.27(0.000)	-0.83(0.000)	0.83(0.000)	1.27(0.000)	1.64(0.001)	2.35(0.003)
	(3)	-2.62(0.056)	-1.61(0.020)	-1.13(0.017)	-0.70(0.014)	0.70(0.013)	1.13(0.018)	1.62(0.02)	2.62(0.055)
n=6		-2.57	-1.59	-1.18	-0.74	0.74	1.18	1.59	2.57
	(1)	-2.94(0.221)	-1.64(0.607)	-0.92(2.220)	-0.61(0.235)	0.61(0.208)	1.15(0.592)	1.52(1.021)	3.04(0.612)
	(2)	-2.35(0.002)	-1.64(0.001)	-1.27(0.000)	-0.83(0.000)	0.83(0.000)	1.27(0.000)	1.64(0.001)	2.35(0.002)
	(3)	-2.58(0.049)	-1.63(0.018)	-1.17(0.017)	-0.73(0.012)	0.73(0.013)	1.17(0.015)	1.63(0.02)	2.58(0.047)
n=7		-2.53	-1.6	-1.2	-0.76	0.76	1.2	1.6	2.53
	(1)	-2.79(0.161)	-1.63(0.172)	-1.13(0.169)	-0.68(0.075)	0.69(0.058)	1.15(0.17)	1.58(0.385)	2.79(0.158)
	(2)	-2.35(0.002)	-1.64(0.001)	-1.27(0.000)	-0.83(0.000)	0.83(0.000)	1.27(0.000)	1.64(0.001)	2.35(0.002)
	(3)	-2.55(0.042)	-1.64(0.018)	-1.19(0.014)	-0.75(0.012)	0.75(0.011)	1.19(0.013)	1.64(0.016)	2.55(0.045)
n=8		-2.51	-1.61	-1.21	-0.77	0.77	1.21	1.61	2.51
	(1)	-2.67(0.099)	-1.61(0.121)	-1.16(0.13)	-0.72(0.059)	0.72(0.034)	1.20(0.205)	1.60(0.228)	2.68(0.128)
	(2)	-2.35(0.002)	-1.64(0.001)	-1.28(0.000)	-0.83(0.000)	0.83(0.000)	1.28(0.000)	1.64(0.001)	2.35(0.002)
	(3)	-2.52(0.044)	-1.64(0.016)	-1.20(0.012)	-0.76(0.010)	0.76(0.010)	1.20(0.012)	1.65(0.016)	2.52(0.038)
n=9		-2.49	-1.62	-1.22	-0.78	0.78	1.22	1.62	2.49
	(1)	-2.6(0.068)	-1.62(0.064)	-1.19(0.077)	-0.75(0.021)	0.75(0.029)	1.18(0.137)	1.63(0.165)	2.61(0.085)
	(2)	-2.35(0.002)	-1.64(0.001)	-1.28(0.000)	-0.83(0.000)	0.83(0.000)	1.28(0.000)	1.64(0.001)	2.35(0.002)
	(3)	-2.5(0.036)	-1.65(0.016)	-1.21(0.011)	-0.77(0.010)	0.77(0.010)	1.21(0.012)	1.65(0.016)	2.50(0.038)
n=10		-2.47	-1.62	-1.23	-0.79	0.79	1.23	1.62	2.47
	(1)	-2.56(0.058)	-1.62(0.06)	-1.21(0.067)	-0.76(0.022)	0.76(0.019)	1.21(0.043)	1.63(0.057)	2.55(0.05)
	(2)	-2.35(0.002)	-1.64(0.001)	-1.28(0.000)	-0.83(0.000)	0.83(0.000)	1.28(0.000)	1.64(0.001)	2.35(0.002)
	(3)	-2.49(0.039)	-1.65(0.015)	-1.22(0.012)	-0.78(0.009)	0.78(0.008)	1.22(0.012)	1.65(0.016)	2.48(0.036)
n=20		-2.4	-1.64	-1.26	-0.82	0.82	1.26	1.64	2.4
	(1)	-2.41(0.029)	-1.64(0.015)	-1.26(0.011)	-0.81(0.006)	0.81(0.007)	1.26(0.010)	1.64(0.014)	2.41(0.028)
	(2)	-2.34(0.002)	-1.64(0.001)	-1.28(0.000)	-0.84(0.000)	0.84(0.000)	1.28(0.000)	1.64(0.001)	2.34(0.002)
	(3)	-2.41(0.033)	-1.66(0.015)	-1.25(0.01)	-0.81(0.008)	0.81(0.008)	1.25(0.010)	1.66(0.014)	2.41(0.032)
n=30		-2.37	-1.64	-1.27	-0.82	0.82	1.27	1.64	2.37
	(1)	-2.38(0.023)	-1.64(0.011)	-1.26(0.009)	-0.82(0.006)	0.82(0.006)	1.27(0.008)	1.64(0.012)	2.38(0.025)
	(2)	-2.34(0.002)	-1.64(0.001)	-1.28(0.000)	-0.84(0.000)	0.84(0.000)	1.28(0.000)	1.64(0.001)	2.34(0.002)
	(3)	-2.38(0.03)	-1.66(0.015)	-1.26(0.009)	-0.82(0.008)	0.82(0.008)	1.26(0.011)	1.66(0.016)	2.38(0.031)

Table 1b. Difference between approximate and true quantiles

q		0.01	0.05	0.10	0.20	0.80	0.90	0.95	0.99
n=5									
	(1)	-0.87	0.31	0.26	0.37	-0.24	-0.53	-0.16	2.05
	(2)	0.26	-0.08	-0.13	-0.12	0.12	0.13	0.08	-0.26
	(3)	-0.01	-0.05	0.01	0.01	-0.01	-0.01	0.06	0.01
n=6									
	(1)	-0.37	-0.05	0.26	0.13	-0.13	-0.03	-0.07	0.47
	(2)	0.22	-0.05	-0.09	-0.09	0.09	0.09	0.05	-0.22
	(3)	-0.01	-0.04	0.01	0.01	-0.01	-0.01	0.04	0.01
n=7									
	(1)	-0.26	-0.03	0.07	0.08	-0.07	-0.05	-0.02	0.26
	(2)	0.18	-0.04	-0.07	-0.07	0.07	0.07	0.04	-0.18
	(3)	-0.02	-0.04	0.01	0.01	-0.01	-0.01	0.04	0.02
n=8									
	(1)	-0.16	0.00	0.05	0.05	-0.05	-0.01	-0.01	0.17
	(2)	0.16	-0.03	-0.07	-0.06	0.06	0.07	0.03	-0.16
	(3)	-0.01	-0.03	0.01	0.01	-0.01	-0.01	0.04	0.01
n=9									
	(1)	-0.11	0.00	0.03	0.03	-0.03	-0.04	0.01	0.12
	(2)	0.12	-0.02	-0.05	-0.04	0.04	0.05	0.02	-0.12
	(3)	-0.01	-0.03	0.01	0.01	-0.01	-0.01	0.03	0.01
n=10									
	(1)	-0.09	0.00	0.02	0.03	-0.03	-0.02	0.01	0.08
	(2)	0.12	-0.02	-0.05	-0.04	0.04	0.05	0.02	-0.12
	(3)	-0.02	-0.03	0.01	0.01	-0.01	-0.01	0.03	0.01
n=20									
	(1)	-0.01	0.00	0.00	0.01	-0.01	0.00	0.00	0.01
	(2)	0.06	0.00	-0.02	-0.02	0.02	0.02	0.00	-0.06
	(3)	-0.01	-0.02	0.01	0.01	-0.01	-0.01	0.02	0.01
n=30									
	(1)	-0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.01
	(2)	0.03	0.00	-0.01	-0.02	0.02	0.01	0.00	-0.03
	(3)	-0.01	-0.02	0.01	0.00	0.00	-0.01	0.02	0.01

Table 2a. Approximate quantile of skewed-normal distribution

q		0.01	0.05	0.10	0.20	0.80	0.90	0.95	0.99
A=-2		-2.96	-2.08	-1.63	-1.11	0.54	0.9	1.19	1.71
	(1)	-2.65(0.024)	-1.78(0.010)	-1.33(0.009)	-0.81(0.006)	0.85(0.005)	1.21(0.006)	1.49(0.010)	1.98(0.021)
	(2)	-2.37(0.002)	-1.66(0.001)	-1.28(0.000)	-0.83(0.000)	0.84(0.000)	1.27(0.000)	1.63(0.001)	2.31(0.002)
	(3)	-2.68(0.034)	-1.80(0.015)	-1.32(0.011)	-0.80(0.009)	0.84(0.007)	1.21(0.009)	1.51(0.011)	2.01(0.022)
A=-1		-2.55	-1.8	-1.41	-0.95	0.72	1.14	1.48	2.12
	(1)	-2.43(0.021)	-1.68(0.009)	-1.29(0.006)	-0.83(0.005)	0.85(0.004)	1.26(0.006)	1.6(0.0090)	2.24(0.020)
	(2)	-2.35(0.002)	-1.65(0.001)	-1.28(0.000)	-0.84(0.000)	0.84(0.000)	1.28(0.000)	1.64(0.001)	2.33(0.002)
	(3)	-2.45(0.03)	-1.70(0.014)	-1.29(0.010)	-0.83(0.008)	0.84(0.007)	1.26(0.010)	1.62(0.014)	2.24(0.026)
a=1		-2.12	-1.48	-1.14	-0.72	0.95	1.41	1.8	2.55
	(1)	-2.23(0.020)	-1.60(0.010)	-1.26(0.006)	-0.84(0.004)	0.83(0.005)	1.29(0.006)	1.68(0.009)	2.43(0.022)
	(2)	-2.33(0.002)	-1.64(0.001)	-1.28(0.000)	-0.84(0.000)	0.84(0.000)	1.28(0.000)	1.65(0.001)	2.35(0.002)
	(3)	-2.24(0.027)	-1.62(0.013)	-1.26(0.009)	-0.84(0.008)	0.83(0.008)	1.29(0.01)	1.70(0.015)	2.44(0.030)
a=2		-1.71	-1.19	-0.9	-0.54	1.11	1.63	2.08	2.96
	(1)	-1.99(0.020)	-1.49(0.011)	-1.21(0.008)	-0.85(0.005)	0.81(0.006)	1.33(0.008)	1.78(0.011)	2.65(0.023)
	(2)	-2.31(0.002)	-1.63(0.001)	-1.27(0.000)	-0.84(0.000)	0.83(0.000)	1.28(0.000)	1.66(0.001)	2.37(0.002)
	(3)	-2.02(0.020)	-1.51(0.012)	-1.21(0.009)	-0.85(0.007)	0.80(0.009)	1.32(0.012)	1.80(0.015)	2.67(0.031)

Table 2b. Difference between approximate and true quantiles

q		0.01	0.05	0.10	0.20	0.80	0.90	0.95	0.99
a=-2									
	(1)	0.31	0.30	0.30	0.30	0.31	0.31	0.30	0.27
	(2)	0.59	0.42	0.35	0.28	0.30	0.37	0.44	0.60
	(3)	0.28	0.28	0.31	0.31	0.30	0.31	0.32	0.30
a=-1									
	(1)	0.12	0.12	0.12	0.12	0.13	0.12	0.12	0.12
	(2)	0.20	0.15	0.13	0.11	0.12	0.14	0.16	0.21
	(3)	0.10	0.10	0.12	0.12	0.12	0.12	0.14	0.12
a=1									
	(1)	-0.11	-0.12	-0.12	-0.12	-0.12	-0.12	-0.12	-0.12
	(2)	-0.21	-0.16	-0.14	-0.12	-0.11	-0.13	-0.15	-0.20
	(3)	-0.12	-0.14	-0.12	-0.12	-0.12	-0.12	-0.10	-0.11
a=2									
	(1)	-0.28	-0.30	-0.31	-0.31	-0.30	-0.30	-0.30	-0.31
	(2)	-0.60	-0.44	-0.37	-0.30	-0.28	-0.35	-0.42	-0.59
	(3)	-0.31	-0.32	-0.31	-0.31	-0.31	-0.31	-0.28	-0.29

Table 3a. Approximate quantile of skewed-*t* distribution

q		0.01	0.05	0.10	0.20	0.80	0.90	0.95	0.99
n=7		-3.22	-1.92	-1.38	-0.82	0.66	0.96	1.22	1.74
a=-2	(1)	-3.12(0.060)	-1.26(2.007)	-0.68(4.611)	-0.59(0.135)	0.69(0.165)	1.25(0.926)	1.76(1.968)	2.40(0.927)
	(2)	-2.40(0.002)	-1.66(0.001)	-1.28(0.000)	-0.83(0.000)	0.84(0.000)	1.27(0.000)	1.62(0.001)	2.29(0.002)
	(3)	-3.14(0.051)	-1.86(0.020)	-1.24(0.020)	-0.68(0.014)	0.78(0.008)	1.09(0.010)	1.35(0.015)	1.85(0.032)
n=8		-2.82	-1.73	-1.26	-0.76	0.77	1.15	1.48	2.18
a=-1	(1)	-2.91(0.088)	-1.62(0.173)	-1.14(0.226)	-0.69(0.061)	0.74(0.023)	1.17(0.150)	1.58(0.216)	2.44(0.191)
	(2)	-2.37(0.002)	-1.65(0.001)	-1.28(0.000)	-0.83(0.000)	0.84(0.000)	1.27(0.000)	1.63(0.001)	2.32(0.002)
	(3)	-2.82(0.048)	-1.75(0.017)	-1.23(0.013)	-0.73(0.011)	0.78(0.008)	1.16(0.010)	1.52(0.016)	2.20(0.037)
n=9		-2.18	-1.48	-1.15	-0.76	0.79	1.28	1.74	2.79
a=1	(1)	-2.37(0.110)	-1.55(0.069)	-1.19(0.080)	-0.76(0.019)	0.72(0.049)	1.18(0.189)	1.63(0.367)	2.84(0.060)
	(2)	-2.32(0.002)	-1.63(0.001)	-1.27(0.000)	-0.84(0.000)	0.83(0.000)	1.28(0.000)	1.65(0.001)	2.37(0.002)
	(3)	-2.21(0.034)	-1.53(0.016)	-1.18(0.011)	-0.79(0.008)	0.74(0.012)	1.24(0.015)	1.75(0.021)	2.78(0.043)
n=10		-1.73	-1.21	-0.94	-0.62	0.92	1.47	1.99	3.15
a=2	(1)	-2.03(0.105)	-1.45(0.170)	-1.15(0.098)	-0.78(0.023)	0.71(0.040)	1.23(0.148)	1.72(0.229)	2.99(0.047)
	(2)	-2.30(0.002)	-1.62(0.001)	-1.27(0.000)	-0.84(0.000)	0.83(0.000)	1.28(0.000)	1.66(0.001)	2.39(0.002)
	(3)	-1.91(0.025)	-1.41(0.015)	-1.13(0.009)	-0.8(0.007)	0.72(0.013)	1.27(0.017)	1.85(0.018)	2.99(0.045)

Table 3b. Difference between approximate and true quantiles

q		0.01	0.05	0.10	0.20	0.80	0.90	0.95	0.99
n=7									
a=-2	(1)	0.10	0.66	0.70	0.23	0.03	0.29	0.54	0.66
	(2)	0.82	0.26	0.10	-0.01	0.18	0.31	0.40	0.55
	(3)	0.08	0.06	0.14	0.14	0.12	0.13	0.13	0.11
n=8									
a=-1	(1)	-0.09	0.11	0.12	0.07	-0.03	0.02	0.10	0.26
	(2)	0.45	0.08	-0.02	-0.07	0.07	0.12	0.15	0.14
	(3)	0.00	-0.02	0.03	0.03	0.01	0.01	0.04	0.02
n=9									
a=1	(1)	-0.19	-0.07	-0.04	0.00	-0.07	-0.10	-0.11	0.05
	(2)	-0.14	-0.15	-0.12	-0.08	0.04	0.00	-0.09	-0.42
	(3)	-0.03	-0.05	-0.03	-0.03	-0.05	-0.04	0.01	-0.01
n=10									
a=2	(1)	-0.30	-0.24	-0.21	-0.16	-0.21	-0.24	-0.27	-0.16
	(2)	-0.57	-0.41	-0.33	-0.22	-0.09	-0.19	-0.33	-0.76
	(3)	-0.18	-0.20	-0.19	-0.18	-0.20	-0.20	-0.14	-0.16

Table 4a. Approximate quantile of Student *t* distribution

Q		0.01	0.05	0.10	0.20	0.80	0.90	0.95	0.99
n=5		-2.61	-1.56	-1.14	-0.71	0.71	1.14	1.56	2.61
	(1)	-3.48(1.507)	-1.39(10.028)	-0.62(1.910)	-0.4(1.300)	-1.47(0.298)	1.02(4.805)	2.89(1.464)	3.46(17.155)
	(2)	-2.15(0.003)	-1.66(0.001)	-1.34(0.000)	-0.9(0.000)	0.9(0.000)	1.34(0.000)	1.66(0.001)	2.15(0.003)
	(3)	-2.22(0.056)	-1.79(0.020)	-1.35(0.017)	-0.7(0.014)	0.7(0.013)	1.34(0.018)	1.79(0.020)	2.22(0.055)
n=6		-2.57	-1.59	-1.18	-0.74	0.74	1.18	1.59	2.57
	(1)	-2.98(0.221)	-1.59(0.607)	-1.12(2.220)	-0.60(0.235)	0.62(0.208)	1.06(0.592)	1.54(1.021)	2.97(0.612)
	(2)	-2.14(0.002)	-1.66(0.001)	-1.34(0.000)	-0.91(0.000)	0.91(0.000)	1.34(0.000)	1.66(0.001)	2.14(0.002)
	(3)	-2.20(0.049)	-1.79(0.018)	-1.36(0.017)	-0.73(0.012)	0.73(0.013)	1.36(0.015)	1.79(0.020)	2.20(0.047)
n=7		-2.53	-1.6	-1.2	-0.76	0.76	1.2	1.6	2.53
	(1)	-2.78(0.161)	-1.59(0.172)	-1.15(0.169)	-0.69(0.075)	0.69(0.058)	1.13(0.170)	1.62(0.385)	2.80(0.158)
	(2)	-2.14(0.002)	-1.66(0.001)	-1.34(0.000)	-0.91(0.000)	0.91(0.000)	1.34(0.000)	1.66(0.001)	2.14(0.002)
	(3)	-2.19(0.042)	-1.79(0.018)	-1.37(0.014)	-0.75(0.012)	0.75(0.011)	1.37(0.013)	1.79(0.016)	2.19(0.045)
n=8		-2.51	-1.61	-1.21	-0.77	0.77	1.21	1.61	2.51
	(1)	-2.68(0.099)	-1.59(0.121)	-1.18(0.130)	-0.72(0.059)	0.72(0.034)	1.19(0.205)	1.30(0.228)	2.68(0.128)
	(2)	-2.13(0.002)	-1.66(0.001)	-1.34(0.000)	-0.91(0.000)	0.91(0.000)	1.34(0.000)	1.66(0.001)	2.13(0.002)
	(3)	-2.18(0.044)	-1.79(0.016)	-1.37(0.012)	-0.77(0.01)	0.77(0.010)	1.37(0.012)	1.79(0.016)	2.18(0.038)
n=9		-2.49	-1.62	-1.22	-0.78	0.78	1.22	1.62	2.49
	(1)	-2.60(0.068)	-1.64(0.064)	-1.19(0.077)	-0.75(0.021)	0.74(0.029)	1.2(0.137)	1.63(0.165)	2.60(0.085)
	(2)	-2.13(0.002)	-1.66(0.001)	-1.34(0.000)	-0.91(0.000)	0.91(0.000)	1.34(0.000)	1.66(0.001)	2.13(0.002)
	(3)	-2.16(0.036)	-1.78(0.016)	-1.38(0.011)	-0.78(0.01)	0.77(0.01)	1.38(0.012)	1.78(0.016)	2.17(0.038)
n=10		-2.47	-1.62	-1.23	-0.79	0.79	1.23	1.62	2.47
	(1)	-2.56(0.058)	-1.62(0.060)	-1.21(0.067)	-0.76(0.022)	0.76(0.019)	1.21(0.043)	1.64(0.057)	2.56(0.050)
	(2)	-2.13(0.002)	-1.66(0.001)	-1.34(0.000)	-0.91(0.000)	0.91(0.000)	1.34(0.000)	1.66(0.001)	2.13(0.002)
	(3)	-2.16(0.039)	-1.78(0.015)	-1.38(0.012)	-0.78(0.009)	0.78(0.008)	1.38(0.012)	1.78(0.016)	2.16(0.036)
n=20		-2.4	-1.64	-1.26	-0.82	0.82	1.26	1.64	2.4
	(1)	-2.41(0.029)	-1.64(0.015)	-1.25(0.011)	-0.81(0.006)	0.81(0.007)	1.26(0.010)	1.64(0.014)	2.42(0.028)
	(2)	-2.12(0.002)	-1.66(0.001)	-1.34(0.000)	-0.91(0.000)	0.91(0.000)	1.34(0.000)	1.66(0.001)	2.12(0.002)
	(3)	-2.12(0.033)	-1.77(0.015)	-1.39(0.010)	-0.81(0.008)	0.81(0.008)	1.39(0.010)	1.77(0.014)	2.12(0.032)
n=30		-2.37	-1.64	-1.27	-0.82	0.82	1.27	1.64	2.37
	(1)	-2.38(0.023)	-1.64(0.011)	-1.26(0.009)	-0.82(0.006)	0.82(0.006)	1.27(0.008)	1.64(0.012)	2.38(0.025)
	(2)	-2.12(0.002)	-1.66(0.001)	-1.34(0.000)	-0.91(0.000)	0.91(0.000)	1.34(0.000)	1.66(0.001)	2.12(0.002)
	(3)	-2.11(0.030)	-1.76(0.015)	-1.39(0.009)	-0.82(0.008)	0.82(0.008)	1.39(0.011)	1.76(0.016)	2.11(0.031)

Table 4b. Difference between approximate and true quantiles

q		0.01	0.05	0.10	0.20	0.80	0.90	0.95	0.99
n=5									
	(1)	-0.87	0.17	0.52	0.31	-2.18	-0.12	1.33	0.85
	(2)	0.46	-0.10	-0.20	-0.19	0.19	0.20	0.10	-0.46
	(3)	0.39	-0.23	-0.21	0.01	-0.01	0.20	0.23	-0.39
n=6									
	(1)	-0.41	0.00	0.06	0.14	-0.12	-0.12	-0.05	0.40
	(2)	0.43	-0.07	-0.16	-0.17	0.17	0.16	0.07	-0.43
	(3)	0.37	-0.20	-0.18	0.01	-0.01	0.18	0.20	-0.37
n=7									
	(1)	-0.25	0.01	0.05	0.07	-0.07	-0.07	0.02	0.27
	(2)	0.39	-0.06	-0.14	-0.15	0.15	0.14	0.06	-0.39
	(3)	0.34	-0.19	-0.17	0.01	-0.01	0.17	0.19	-0.34
n=8									
	(1)	-0.17	0.02	0.03	0.05	-0.05	-0.02	-0.31	0.17
	(2)	0.38	-0.05	-0.13	-0.14	0.14	0.13	0.05	-0.38
	(3)	0.33	-0.18	-0.16	0.00	0.00	0.16	0.18	-0.33
n=9									
	(1)	-0.11	-0.02	0.03	0.03	-0.04	-0.02	0.01	0.11
	(2)	0.36	-0.04	-0.12	-0.13	0.13	0.12	0.04	-0.36
	(3)	0.33	-0.16	-0.16	0.00	-0.01	0.16	0.16	-0.32
n=10									
	(1)	-0.09	0.00	0.02	0.03	-0.03	-0.02	0.02	0.09
	(2)	0.34	-0.04	-0.11	-0.12	0.12	0.11	0.04	-0.34
	(3)	0.31	-0.16	-0.15	0.01	-0.01	0.15	0.16	-0.31
n=20									
	(1)	-0.01	0.00	0.01	0.01	-0.01	0.00	0.00	0.02
	(2)	0.28	-0.02	-0.08	-0.09	0.09	0.08	0.02	-0.28
	(3)	0.28	-0.13	-0.13	0.01	-0.01	0.13	0.13	-0.28
n=30									
	(1)	-0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.01
	(2)	0.25	-0.02	-0.07	-0.09	0.09	0.07	0.02	-0.25
	(3)	0.26	-0.12	-0.12	0.00	0.00	0.12	0.12	-0.26

Table 5a. Approximate quantile of skewed-normal distribution

q		0.01	0.05	0.10	0.20	0.80	0.90	0.95	0.99
a=-2		-2.96	-2.08	-1.63	-1.11	0.54	0.90	1.19	1.71
	(1)	-2.65(0.024)	-1.78(0.010)	-1.33(0.009)	-0.81(0.006)	0.85(0.005)	1.21(0.006)	1.49(0.010)	1.99(0.021)
	(2)	-2.16(0.002)	-1.68(0.001)	-1.35(0.000)	-0.91(0.000)	0.91(0.000)	1.34(0.000)	1.65(0.001)	2.08(0.002)
	(3)	-2.38(0.034)	-1.92(0.015)	-1.46(0.011)	-0.8(0.009)	0.86(0.007)	1.31(0.009)	1.57(0.011)	1.80(0.022)
a=-1		-2.55	-1.80	-1.41	-0.95	0.72	1.14	1.48	2.12
	(1)	-2.43(0.021)	-1.68(0.009)	-1.29(0.006)	-0.83(0.005)	0.84(0.004)	1.26(0.006)	1.60(0.009)	2.23(0.020)
	(2)	-2.13(0.002)	-1.67(0.001)	-1.35(0.000)	-0.91(0.000)	0.91(0.000)	1.34(0.000)	1.66(0.001)	2.11(0.002)
	(3)	-2.18(0.030)	-1.81(0.014)	-1.41(0.010)	-0.83(0.008)	0.85(0.007)	1.37(0.010)	1.70(0.014)	2.00(0.026)
a=1		-2.12	-1.48	-1.14	-0.72	0.95	1.41	1.80	2.55
	(1)	-2.23(0.02)	-1.6(0.01)	-1.26(0.006)	-0.84(0.004)	0.83(0.005)	1.29(0.006)	1.68(0.009)	2.43(0.022)
	(2)	-2.11(0.002)	-1.66(0.001)	-1.34(0)	-0.91(0)	0.91(0)	1.35(0)	1.67(0.001)	2.13(0.002)
	(3)	-2(0.027)	-1.7(0.013)	-1.37(0.009)	-0.85(0.008)	0.83(0.008)	1.41(0.01)	1.81(0.015)	2.18(0.03)
a=2		-1.71	-1.19	-0.90	-0.54	1.11	1.63	2.08	2.96
	(1)	-1.99(0.02)	-1.49(0.011)	-1.21(0.008)	-0.85(0.005)	0.81(0.006)	1.33(0.008)	1.78(0.011)	2.65(0.023)
	(2)	-2.08(0.002)	-1.65(0.001)	-1.34(0)	-0.91(0)	0.91(0)	1.35(0)	1.68(0.001)	2.16(0.002)
	(3)	-1.8(0.02)	-1.57(0.012)	-1.31(0.009)	-0.86(0.007)	0.8(0.009)	1.46(0.012)	1.93(0.015)	2.38(0.031)

Table 5b. Difference between approximate and true quantiles

q		0.01	0.05	0.10	0.20	0.80	0.90	0.95	0.99
a=-2									
	(1)	0.31	0.30	0.30	0.30	0.31	0.31	0.30	0.28
	(2)	0.80	0.40	0.28	0.20	0.37	0.44	0.46	0.37
	(3)	0.58	0.16	0.17	0.31	0.32	0.41	0.38	0.09
a=-1									
	(1)	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.11
	(2)	0.42	0.13	0.06	0.04	0.19	0.20	0.18	-0.01
	(3)	0.37	-0.01	0.00	0.12	0.13	0.23	0.22	-0.12
a=1									
	(1)	-0.11	-0.12	-0.12	-0.12	-0.12	-0.12	-0.12	-0.12
	(2)	0.01	-0.18	-0.20	-0.19	-0.04	-0.06	-0.13	-0.42
	(3)	0.12	-0.22	-0.23	-0.13	-0.12	0.00	0.01	-0.37
a=2									
	(1)	-0.28	-0.30	-0.31	-0.31	-0.30	-0.30	-0.30	-0.31
	(2)	-0.37	-0.46	-0.44	-0.37	-0.20	-0.28	-0.40	-0.80
	(3)	-0.09	-0.38	-0.41	-0.32	-0.31	-0.17	-0.15	-0.58

Table 6a. Approximate quantile of skewed-*t* distribution

q		0.01	0.05	0.10	0.20	0.80	0.90	0.95	0.99
n=7		-3.22	-1.92	-1.38	-0.82	0.66	0.96	1.22	1.74
a=-2	(1)	-3.09(0.060)	-1.58(2.007)	-0.94(4.611)	-0.59(0.135)	0.69(0.165)	1.27(0.926)	1.84(1.968)	2.34(0.927)
	(2)	-2.21(0.002)	-1.69(0.001)	-1.35(0.000)	-0.90(0.000)	0.91(0.000)	1.32(0.000)	1.63(0.001)	2.08(0.002)
	(3)	-2.69(0.051)	-2.07(0.020)	-1.46(0.02)	-0.66(0.014)	0.81(0.008)	1.22(0.010)	1.43(0.015)	1.60(0.032)
n=8		-2.82	-1.73	-1.26	-0.76	0.77	1.15	1.48	2.18
a=-1	(1)	-2.91(0.088)	-1.62(0.173)	-1.14(0.226)	-0.69(0.061)	0.74(0.023)	1.18(0.150)	1.58(0.216)	2.61(0.191)
	(2)	-2.17(0.002)	-1.68(0.001)	-1.35(0.000)	-0.91(0.000)	0.91(0.000)	1.33(0.000)	1.65(0.001)	2.10(0.002)
	(3)	-2.43(0.048)	-1.93(0.017)	-1.42(0.013)	-0.72(0.011)	0.80(0.008)	1.31(0.010)	1.63(0.016)	1.91(0.037)
n=9		-2.18	-1.48	-1.15	-0.76	0.79	1.28	1.74	2.79
a=1	(1)	-2.38(0.110)	-1.59(0.069)	-1.19(0.080)	-0.76(0.019)	0.71(0.049)	1.19(0.189)	1.65(0.367)	2.84(0.060)
	(2)	-2.10(0.002)	-1.65(0.001)	-1.34(0.000)	-0.91(0.000)	0.91(0.000)	1.35(0.000)	1.67(0.001)	2.16(0.002)
	(3)	-1.92(0.034)	-1.63(0.016)	-1.32(0.011)	-0.81(0.008)	0.73(0.012)	1.42(0.015)	1.92(0.021)	2.40(0.043)
n=10		-1.73	-1.21	-0.94	-0.62	0.92	1.47	1.99	3.15
a=2	(1)	-2.05(0.105)	-1.45(0.170)	-1.16(0.098)	-0.78(0.023)	0.7(0.040)	1.22(0.148)	1.73(0.229)	2.99(0.047)
	(2)	-2.08(0.002)	-1.63(0.001)	-1.33(0.000)	-0.91(0.000)	0.9(0.000)	1.35(0.000)	1.69(0.001)	2.19(0.002)
	(3)	-1.67(0.025)	-1.48(0.015)	-1.25(0.009)	-0.83(0.007)	0.7(0.013)	1.47(0.017)	2.03(0.018)	2.59(0.045)

Table 6b. Difference between approximate and true quantiles

q		0.01	0.05	0.10	0.20	0.80	0.90	0.95	0.99
a=-2									
	(1)	0.13	0.34	0.44	0.23	0.03	0.31	0.62	0.60
	(2)	1.01	0.23	0.03	-0.08	0.25	0.36	0.41	0.34
	(3)	0.53	-0.15	-0.08	0.16	0.15	0.26	0.21	-0.14
a=-1									
	(1)	-0.09	0.11	0.12	0.07	-0.03	0.03	0.10	0.43
	(2)	0.65	0.05	-0.09	-0.15	0.14	0.18	0.17	-0.08
	(3)	0.39	-0.20	-0.16	0.04	0.03	0.16	0.15	-0.27
a=1									
	(1)	-0.20	-0.11	-0.04	0.00	-0.08	-0.09	-0.09	0.05
	(2)	0.08	-0.17	-0.19	-0.15	0.12	0.07	-0.07	-0.63
	(3)	0.26	-0.15	-0.17	-0.05	-0.06	0.14	0.18	-0.39
a=2									
	(1)	-0.32	-0.24	-0.22	-0.16	-0.22	-0.25	-0.26	-0.16
	(2)	-0.35	-0.42	-0.39	-0.29	-0.02	-0.12	-0.30	-0.96
	(3)	0.06	-0.27	-0.31	-0.21	-0.22	0.00	0.04	-0.56