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Evaluating the Effectiveness of Futures Hedging

Donald Lien

University of Texas at San Antonio

Guel Lee

University of New South Wales

Li Yang

University of New South Wales

Chunyang Zhou

Shanghai Jiaotung University

COLLEGE OF BUSINESS KNOWLEDGE FOR A NEW WORLD

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Donald Lien

University of Texas at San Antonio

Guel Lee

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1. INTRODUCTION

Futures market provides a useful tool for hedgers to reduce the overall risk. The extent of the usefulness is, however, determined by the hedging strategy adopted by the hedger. In this regard, the hedging effectiveness measure proposed by Ederington (1979) has been the most popular criterion to evaluate the usefulness. Different hedging strategies are compared in terms of Ederington hedging effectiveness (EHE). The strategy possessed with the greatest EHE is deemed the best strategy.

Specifically, EHE is the percentage reduction in the return variance of the hedged portfolio relative to the return variance of the unhedged portfolio. While the variance could be conditional or unconditional, in empirical studies EHE is always calculated on the basis of unconditional variance. This is natural as Ederington (1979) considers only unconditional constant hedge strategies. Further development in futures hedging literature focuses on conditional dynamic hedge strategies. However, EHE remains the major criterion to evaluate the usefulness of these strategies. This approach is inappropriate since the conditional hedge strategy is constructed to minimize conditional variance but its usefulness is measured by unconditional variance. As long as there is not a linear relationship between conditional and unconditional variances, the EHE should not serve as a benchmark to evaluate the conditional hedge strategy.

This paper examines the EHE comparisons between the OLS hedge strategy (i.e., the unconditional strategy) with various conditional hedge strategies, assuming spot and futures returns are described by different statistical framework. It is shown that, for most statistical models, the OLS hedge strategy is most likely to outperform the optimal conditional hedge strategy. For example, in a vector error correction model (VECM), the optimal conditional hedge ratio should take into account the cointegration relationship. The resulting EHE from this optimal

ECM hedge ratio, however, underperforms the OLS hedge ratio (where the cointegration relationship is ignored). Similarly, when spot and futures returns follow a multivariate generalized autoregressive conditional heteroskedasticity (MGARCH) model, the GARCH hedge ratio is likely to be inferior to the OLS hedge ratio in terms of EHE.

The above results are not surprising as OLS hedge ratio is chosen to minimize the unconditional variance whereas ECM and GARCH hedge ratios minimize their corresponding conditional variances. By definition, EHE is biased in favor of OLS hedge ratio over other hedge ratios. The only possible exception is the regime switching (RS) hedge ratio. It is analytically shown that RS-OLS hedge ratio would outperform the conventional OLS hedge ratio under certain assumptions.

Besides EHE, another popular hedging effectiveness measure is certainty equivalent derived from expected utility comparisons from hedged and unhedged portfolios. It is shown the sample certainly equivalent estimator, similar to the sample EHE estimator, is biased. On the other hand, this utility-based effectiveness measure does not necessarily favor the OLS hedge ratio except when the futures price is a martingale or when the hedger is extremely risk averse.

The remaining of the paper is organized as follows. In Section 2, we discuss the Ederington hedging effectiveness measure and demonstrate the superiority of the OLS hedge ratio. The next sections consider two specific dynamic hedging strategies; Section 3 examines the GARCH specifications and Section 4 the regime switching models. The two models provide contradicting conclusions regarding the relative performance to the OLS hedge ratio. In Section 5, we analyze the utility-based hedging effectiveness. Finally, conclusions are provided in Section6.

2. EDERINGTON HEDGING EFFECTIVENESS

The fundamental idea of Ederington (1979) originates from Johnson (1960) and Stein (1961) which introduce portfolio theory into the area of hedging. Most of the previous hedging theories consider only the `naive' hedging, which is done by trading the hedging instrument in same amount as the asset being hedged. Ederington shows that the hedge ratio, which is the ratio of the amount of the hedging instrument being used relative to the amount of the asset being hedged, must be adjusted to obtain the maximum hedging effectiveness. To derive this result, Ederington proves that there exists an optimal hedge ratio which minimizes the variance of the portfolio value.

2.1 Definition

Ederington shows that if we construct a hedged portfolio *P* which consists of the asset being hedged, *S*, and a hedging instrument, *F*, the optimal hedge ratio is the value where the partial derivative of the portfolio return variance with respect to the hedge ratio becomes zero. This partial derivative is given by

$$\frac{\partial Var(p)}{\partial h} = X_s^2[2hVar(f) - 2Cov(s, f)], \tag{1}$$

where X_s and X_f are the positions of the asset and the hedging instrument, respectively; $h = -X_f / X_s$ is the hedge ratio; *s* and *f* are the price changes in *S* and *F*, respectively; p is the return of the portfolio: $p = X_s s + X_f f = X_s(s - hf)$. $Var(.)$ and $Cov(.,.)$ are the variance and covariance operators, respectively. Given this formula, the optimal hedge ratio h^* can be easily derived by setting equation (1) equal to zero, i.e.,

$$h^* = Cov(s, f)/Var(f). \tag{2}$$

As shown in equations (1) and (2), Ederington regards hedging as an act of "minimizing variance." When devising his measure of hedging effectiveness, he also takes this property as the main criteria. Specifically, the Ederington hedging effectiveness (EHE hereafter) is defined as

$$H = 1 - \frac{Var(p)}{Var(s)}. \tag{3}$$

Equation (3) shows that EHE is directly related to the percentage reduction of the variance in the asset return after hedging.

2.2 Some properties

The most evident characteristic of EHE is its simplicity. The variance and covariance in Equation (3) are both unconditional and are assumed to be constant over time. While this aspect of EHE is one of the reasons why it is being widely used, it also has caused some controversies about its appropriateness. It was argued that, given the information set, the hedger is concerned with the conditional variance of the portfolio return. Accordingly, unconditional variance and covariance in equation (3) should be replaced by their conditional counterparts. Various variables were taken into account to derive conditional variance and covariance, including past prices and inventories. In addition, recent research emphasizes the non-constancy nature of conditional variance and covariance and recommends time-varying hedge ratios.

In empirical implementation, the complete sample is divided into two sub-samples. The first sub-sample is applied to construct the most appropriate (within-sample) statistical models for conditional variance and covariance. Based upon the estimated model, optimal hedge ratios are obtained for the second sub-sample. Returns for the hedged portfolio are calculated for this sub-sample. The unconditional variance of the return series is adopted to calculate the so-called post-sample EHE which serves as a benchmark to compare various hedge strategies. Thus, the within-sample model is chosen to minimize conditional variance whereas the post-sample hedging effectiveness is evaluated at unconditional variance. The inconsistency in criteria raises a concern for the appropriateness of EHE to be used as the criteria to compare conditional hedge strategies. Moreover, one would suspect the hedge strategy constructed by minimizing the within-sample unconditional variance may have the best out-of-sample EHE.

To address this question, Lien (2005a) introduces several assumptions:

(1) The size of the estimation sample is sufficiently large.

(2) The size of the evaluation sample is sufficiently large.

(3) There is no structural change between the estimation sample and the evaluation sample.

Under these conditions, it is shown that the ratio of unconditional covariance to the conditional variance provides the best EHE. This ratio can be obtained by the ordinary least squares (OLS) method when regressing the spot price change on the futures price change.

The first two assumptions ensure sample unconditional variance and sample unconditional covariance both to be close enough to their population counterparts. The third assumption requires the estimation and evaluation samples to be drawn from the same population such that the hedge ratio derived from the former is applicable to the latter. When there is a structural change across the two samples, nothing can be guaranteed. However, Lien (2005a, 2005b) warns that it is a tautology to prove the superiority of OLS hedge ratio to other hedge ratios with EHE. Since the OLS hedge ratio is the hedge ratio which produces the minimum unconditional variance, it cannot be inferior to any other hedge ratios when compared in terms of EHE, which measures the unconditional variance reduction. Alternatively, one can argue that it is not appropriate to compare conditional hedge strategies on the basis of EHE.

Lien (2005b) illustrates the superiority of the OLS hedge ratio with a simple example:

$$f_t = \alpha_0 + \alpha_1 f_{t-1} + u_t \ , \tag{4}$$

$$s_t = \beta_0 + \beta_1 s_{t-1} + v_t; \tag{5}$$

where both $\{u_t\}$ and $\{v_t\}$ are white noises. Let σ_u^2 and σ_v^2 denote the variances of u_t and v_t, respectively, and let σ_{uv} denote the covariance between u_t and v_t. In addition, to ensure stationarity, we require $|\alpha_1| < 1$ and $|\beta_1| < 1$. The unconditional hedge ratio (i.e., OLS hedge ratio) is

$$h_u = \left(\frac{1-\alpha_1^2}{1-\alpha_1\beta_1}\right)\left(\frac{\sigma_{uv}}{\sigma_u^2}\right), \tag{6}$$

whereas the conditional hedge ratio is simply $h_c = \sigma_{uv} / \sigma_u^2$. By construction, h_u performs better than h_c in terms of EHE.

Upon incorporating the cointegration relationship between spot and futures prices into equations (4) and (5), Lien (2005a) demonstrates the superiority of the OLS hedge ratio over the error correction hedge ratio. For the importance of the error correction term for futures hedging and further comparisons between the two hedge ratios, see Lien (1996, 2004).

2.3 Estimation bias

In Lien (2006), it is shown that, the usual EHE estimator is downward biased and, therefore, tends to underestimate the true hedging performance, even when the estimator for optimal hedge ratio is unbiased. This is because the estimated optimal hedge ratio itself is a random variable so that its variance affects the expected EHE. Lien explains this by decomposing the EHE formula (3) as follows.

Let $M = I_k - (e_k e_k' / k)$ where I_k is a $(k \times k)$-dimensional identity matrix and e_k is a *k*-dimensional vector such that all elements are equal to 1. Then equation (3) can be decomposed to:

$$H = 1 - \frac{w'Mw}{p'Mp}, \tag{7}$$

where p and w are k-dimensional vectors consisting of k unhedged asset returns and hedged portfolio returns, respectively. Because the estimated hedge ratio $\hat{h}$ substitutes the optimal hedge ratio h^*, The EHE one calculates based on $\hat{h}$ is also in fact an estimated EHE, $\hat{H}$. That is,

$$\hat{H} = 1 - \frac{\hat{w}'M\hat{w}}{p'Mp}, \tag{8}$$

where $\hat{w}$ is a k-dimensional vector consisting of the portfolio returns which are hedged with $\hat{h}$.

Since $\hat{w} = w + (h^* - \hat{h}) f$, equation (7) can be rewritten as

$$1 - \hat{H} = \frac{w'Mw}{p'Mp} + 2(h^* - \hat{h})\left[\frac{f'Mw}{p'Mp}\right] + (h^* - \hat{h})^2\left[\frac{f'Mf}{p'Mp}\right], \tag{9}$$

and therefore

$$\hat{H} = H - 2(h^* - \hat{h})\left[\frac{f'Mw}{p'Mp}\right] - (h^* - \hat{h})^2\left[\frac{f'Mf}{p'Mp}\right]. \tag{10}$$

As a consequence,

$$E(\hat{H}) = H + 2b\left[\frac{f'Mw}{p'Mp}\right] - [b^2 + Var(\hat{h})]\left[\frac{f'Mf}{p'Mp}\right], \tag{11}$$

where $b = E(\hat{h} - h^*)$ and $b^2 + Var(\hat{h})$ are the estimation bias and mean squared error of $\hat{h}$, respectively. If $\hat{h}$ is an unbiased estimator of h^*, b becomes zero and equation (11) is reduced to

$$E(\hat{H}) = H - Var(\hat{h})\left[\frac{f'Mf}{p'Mp}\right], \tag{12}$$

This shows that $\hat{H}$ is a downward biased estimator of H, even when $\hat{h}$ is an unbiased estimator of h^*.

We provide two further remarks. First, Chen and Sutcliffe (2007) examine the benefits of a composite hedge where multiple hedging instruments are adopted over a simple hedge where only one hedging instrument is adopted. The benefit is measured by the improvement in EHE. Lien (2008) demonstrates the empirical estimator is biased. Secondly, through empirical studies, Lien (2007) concludes that the downward bias of the EHE estimator is negligible and therefore bias correction seems to be redundant.

3. GARCH HEDGING STRETAGY

The previous analysis assumes the conditional second moments of spot and futures returns are constant over time. This assumption is frequently rejected through empirical data analysis. To describe time varying second moments, researchers rely upon different versions of multivariate GARCH (Generalized Autoregressive Conditional Heteroskedasticity) models. The standard univariate GARCH model is an extension of the ARCH (Autoregressive Conditional Heteroskedasticity) model proposed by Engle (1982).

3.1 GARCH specification

Specifically, consider a time series $\{y_t\}$ such that

$$y_t = b'x_t + \varepsilon_t \,, \tag{13}$$

where x_t is the vector of exogenous variables contained in the information set previous to time t -1, ϑ_{t-1}. The error term is normally distributed conditional on the information set; i.e., $\varepsilon_t \big| \vartheta_{t-1} \sim N(0, \sigma_t^2)$. Bollerslev (1986) proposed the following process for the conditional variance:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \ . \tag{14}$$

The result is termed GARCH(p,q) process. If we set p = 0 and $q \neq 0$, the process is reduced to an ARCH(q) process. When $p = q = 0$, it is further reduced to a process with a constant variance.

For futures hedging purpose, we need to consider multivariate GARCH models which specify the process for the conditional covariance as well. Various specifications are available in the literature, e.g., constant correlation, BEKK, and DCC (Dynamic Conditional Correlation) models. Different dynamic hedge ratios are then generated and compared on the basis of EHE; for example, see Baillie and Myers (1991), Myers (1991), Kroner and Sultan (1993), Dawson et al. (2000), and Kavussanos and Visvikis (2008).

3.2 GARCH hedging strategy

Underlying GARCH models, the optimal hedge ratio is determined by the ratio of the conditional covariance to the conditional variance,

$$h_{t-1}^{c} = \frac{Cov_{t-1}(s_t, f_t)}{Var_{t-1}(f_t)}, \tag{15}$$

where $Cov_{t-1}(s_t, f_t)$ is the conditional covariance between spot and futures returns at time t based upon information available at time $t-1$ and $Var_{t-1}(f_t)$ is the conditional variance of the futures return at time t based upon information available at time $t-1$. As both conditional moments are time varying, the conditional hedge ratio is expected to change over time as well.

That is, although OLS and GARCH hedge ratios have the same object of variance minimization, they differ in terms of the target variance. While OLS hedge ratio considers the unconditional variance, GARCH hedge ratio focuses on conditional variance under the GARCH assumptions. This difference suggests a concern about the appropriate procedure of assessing and comparing their effectiveness. Since their objectives are different, the relative superiority of one hedge ratio over the other can vary when one applies a different effectiveness measure. In particular, since EHE depends upon the reduction in the unconditional variance, OLS hedge ratio is naturally favored. Lien (2009) explains this result as follows.

3.3 EHE and GARCH hedging strategy

Let us assume that there are two portfolios P0 and P1, both consisting of an asset S and a hedging instrument F. The first portfolio is constructed from the OLS hedge ratio, $h_0 = Cov(s_t, f_t)/Var(f_t)$, whereas the second portfolio is constructed from the GARCH hedge ratio, $h_{t-1}^c = Cov_{t-1}(s_t, f_t)/Var_{t-1}(f_t)$. We can decompose the unconditional variance of the return from portfolio P1 as:

$$\begin{aligned} Var(p1) &= Var(s_t - h_{t-1}^c f_t) \\ &= E[Var_{t-1}(s_t - h_{t-1}^c f_t)] + Var[E_{t-1}(s_t - h_{t-1}^c f_t)], \end{aligned} \tag{16}$$

where $E(.)$ is the unconditional expectation operator and $E_{t-1}(.)$ is the conditional expectation operator based upon information available at time $t-1$. We can rewrite the first term of equation (16) as follows:

$$E\left[Var_{t-1}(s_t) - \frac{Cov_{t-1}^2(s_t, f_t)}{Var_{t-1}(f_t)}\right] = Var(s_t) - E\left[\frac{Cov_{t-1}^2(s_t, f_t)}{Var_{t-1}(f_t)}\right], \tag{17}$$

using the definition of h_{t-1}^c. Suppose that the sample size is sufficiently large, we can approximate the second term of equation (17) by

$$E\left[\frac{Cov_{t-1}^2(s_t, f_t)}{Var_{t-1}(f_t)}\right] \approx \frac{E[Cov_{t-1}^2(s_t, f_t)}{E[Var_{t-1}(f_t)]} = \frac{Cov^2(s_t, f_t)}{Var(f_t)}. \tag{18}$$

Consequently,

$$Var(p1) \approx Var(s_t) - \frac{Cov^2(s_t, f_t)}{Var(f_t)} + Var[E_{t-1}(s_t - h_{t-1} f_t)]. \tag{19}$$

On the other hand, by the definition of h_0, the unconditional variance of the return from portfolio P0 is:

$$Var(p0) = Var(s_t - h_0 f_t) = Var(s_t) - \frac{Cov^2(s_t, f_t)}{Var(f_t)}. \quad (20)$$

Therefore,

$$Var(p1) \approx Var(p0) + Var[E_{t-1}(s_t - h_{t-1} f_t)], \quad (21)$$

implying $Var(p1)$ tends to be larger than $Var(p0)$. That is, the OLS hedge ratio is likely to have a greater hedging effectiveness than the GARCH hedge ratio, in terms of EHE.

Note that, the derivation of equations (16)-(21) do not rely on any specific properties of the GARCH model. The above conclusion, therefore, applies to any general dynamic hedge strategy that aims at minimizing the conditional variance; see, also, Lien (2010). In other words, when adopting EHE as the effectiveness measure, the OLS hedge ratio is likely to outperform any dynamic hedge ratio. However, we should be careful when interpreting this result. As Lien (2005a) points out, EHE is focused on the unconditional variance and it would be an abuse to use EHE to assess a conditional variance minimization strategy.

Kavussanos and Nomikos (2000) suggest that, for the GARCH hedge strategy to outperform the OLS hedge strategy, the variability of the resulting GARCH ratio must be sufficiently large. On the other hand, Park and Jei (2010) find an inverse relationship between the variability of the GARCH hedge ratio and corresponding hedging effectiveness (i.e., EHE).

4. REGIME SWITCHING HEDGING STRETAGY

Lien (2010) provides a theoretical analysis on the relationship between the variability of the hedge ratio and hedging performance in support of the finding from Park and Jei (2010). Extending the result to general dynamic hedge strategy, there is a small window for the strategy to outperform the OLS strategy, that is, when the variability of the hedge ratio cannot be too small or too large. We therefore turn to regime switching hedge strategies.

4.1 Definition of regime switching

Both GARCH and regime switching models belong to the family of non-linear time series. Hamilton (1988, 1989) characterizes the concept of "regime switching" (RS hereafter) and proposes an approach to model the RS process. The simplest RS model specification of RS is the first-order Markov process with two states. If $S_t \in \{0,1\}$ denotes the (not directly observable) state of the system in which the source of the time-series data exists, the transition between two states is driven by the following first-order Markov process:

$$\Pr ob(S_t = 1 | S_{t-1} = 1) = p\,,\ \Pr ob(S_t = 0, | S_{t-1} = 1) = 1 - p\,; \tag{22a}$$

$$\Pr ob(S_t = 0 | S_{t-1} = 0) = q\,,\ \Pr ob(S_t = 1, | S_{t-1} = 0) = 1 - q\,. \tag{22b}$$

Thus, the probability of state transition depends only upon the state of the previous period.

In each state, the spot and futures returns can be described by linear models such as ECM, or non-linear models such as GARCH processes. For the former case, there will be two constant hedge ratios each pertaining to one state; for the latter case, there will be two dynamic hedge ratios instead. The literature on RS hedge strategies began with the former case and recently extended to the latter case. For example, Sarno and Valente (2000) and Alizadeh et al. (2008) combine RS with ECM; Alizadeh and Nomikos (2004) and Lee and Yoder (2007a, b) add RS into the GARCH models; Lee (2010) combines RS with dynamic conditional correlation (DCC) models. In most cases, it is shown that the hedging performance is improved when regime switching is incorporated into the econometric framework.

4.2 RS hedging strategy

Although RS can be introduced in various ways, it is most understandable when we combine RS with the OLS hedging strategy. Lien (2012b) explains the basic framework of the RS-OLS strategy as follows. Suppose that RS process is given as (22). When $S_{t-1} = 1$, the OLS hedge ratio in equation (2) is modified to

$$h_1^* = \frac{pCov_1(s_t, f_t) + (1-p)Cov_0(s_t, f_t)}{pVar_1(f_t) + (1-p)Var_0(f_t)}, \tag{23}$$

where $Var_n(.)$ and $Cov_n(.,.)$ denote the variance and covariance operators in state *n*, respectively; *n* = 0, 1. Similarly, when $S_{t-1} = 0$, the corresponding OLS hedge ratio is

$$h_0^* = \frac{qCov_0(s_t, f_t) + (1-q)Cov_1(s_t, f_t)}{qVar_0(f_t) + (1-q)Var_1(f_t)}. \tag{24}$$

The pair of hedge ratios (h_0^*, h_1^*) constitutes the optimal RS-OLS hedge ratio. To apply this hedge strategy, it requires the hedger to be able to identify the state at the moment of making the hedging decision.

Lien (2012b) compares the RS-OLS hedge strategy to the conventional OLS hedge strategy. To calculate the conventional OLS hedge ratio under the RS framework, we first derive the steady-state probability for each state. Let α and $1-\alpha$ denote the steady-state probability of state 1 and 0, respectively. Thus,

$$(1-\alpha)p + \alpha(1-q) = \alpha, \tag{25}$$

or, equivalently

$$\alpha = \frac{1-q}{2-p-q}. \tag{26}$$

Given the steady-state probability of each state, we can obtain the conventional OLS hedge ratio as follows:

$$h^* = \frac{\alpha Cov_1(s_t, f_t) + (1-\alpha)Cov_0(s_t, f_t)}{\alpha Var_1(f_t) + (1-\alpha)Var_0(f_t)}. \tag{27}$$

Let $h_{RS}^* = \alpha h_1^* + (1-\alpha)h_0^*$, the expected RS-OLS hedge ratio. Lien (2012b) shows the expected RS-OLS hedge ratio exceeds the conventional OLS hedge ratio:

$$h_{RS}^* \geq h^*. \tag{28}$$

Thus, more transaction cost is incurred when implementing the RS-OLS hedge strategy.

4.3 Hedging effectiveness

To compare the hedging effectiveness, let $V(h)$ denote the variance of the return from the hedged portfolio, where $h = h_0^*, h_1^*$, or h^*. The expected variance of the RS-OLS hedged portfolio is then $V_{RS} = \alpha V(h_1^*) + (1-\alpha)V(h_0^*)$. Lien (2012b) demonstrates that

$$V_{RS} \leq V(h^*); \tag{29}$$

that is, the RS-OLS hedged portfolio has a smaller variance than the conventional OLS hedged portfolio. Consequently, the RS-OLS strategy outperforms the OLS strategy in terms of EHE.

While the RS-OLS seems to be very promising, a serious problem with this result is that, as Lien (2011a) points out, the superiority of the RS-OLS strategy is based on the assumption that a hedger can always correctly identify the prevailing state at the decision time correctly. To successfully conduct the above hedging strategy, we must succeed in at least three tasks to complete the correct identification:

(1) We must identify the entire set of possible states.

(2) We must identify the prevailing state.

(3) We must identify the relationship between spot and futures returns in each state.

In reality, it is unlikely to complete any of these tasks without errors. Hamilton (1989) is well aware of these issues and emphasizes the importance of "optimal probabilistic inference" to find the turning points. One may try to go around this problem by a weighted average strategy such that the optimal hedge ratio is chosen to be

$$\hat{h}^* = \beta h_1^* + (1-\beta) h_0^*, \tag{30}$$

where β is the estimated probability that the prevailing state is state 1. However, as Lien (2012b) points out, this again dilutes the relative superiority of RS-OLS strategy, since at least one of the states is false at any time *t*. We can conclude that, therefore, one must succeed in the structural definition of possible states and correct identification of the current state to fully take advantage of the RS framework.

5. UTILITY-BASED HEDGING EFFECTIVENESS

Up to now, we assume that the sole objective of hedging is variance reduction, and correspondingly the optimal hedge ratio is the one that minimizes variance. This is quite intuitive because risk minimization is the most important reason that hedging is actually being done. In the real world, however, the variance-minimizing hedge ratio is not always the optimal one. To understand why this is true, we must know that there are some other factors than variance minimization about which a hedger should consider. For example, if a hedger assumes that a price process is sub-martingale and wants to take advantage of positive expected return, she will try to afford some risk by a non-perfect hedging. In this situation, variance reduction cannot be the perfect measure for hedging effectiveness.

5.1 Definition of utility-based hedging

Given these restrictions, we can adopt a multivariable function as the alternative and consider additional factors other than variance to measure the hedging effectiveness. In particular, we can consider how large the expected return will be after hedging cost is offset, how much risk a hedger can afford to retain a certain amount of expected return, as well as how large the variance will be. Many previous studies, e.g. Kroner and Sultan (1993), Gagnon et al. (1998), Follmer and Leukert (1999, 2000), and Monoyios (2004) introduce the idea of utility function to construct a framework for this multivariate relationship.

A basic framework of utility-based hedging effectiveness measure is provided in Lien (2012a). Consider a two-date one-period model. The expected utility of an unhedged portfolio can be defined as

$$E[U(w_{1,u})] = E[U(w_0 + s_1 - s_0)], \tag{31}$$

where w_0 is the initial wealth (i.e., the wealth at time 0), s_1 is the random value of the spot asset at time 1, s_0 is the value of spot asset at time 0, and $w_{1,u}$ is the random value of wealth at time 1 when there is no hedging conducted. If we adopt a hedging strategy, the expected utility of the hedged portfolio is

$$E[U(w_{1,h})] = E[U(w_0 + s_1 - s_0 - h(f_1 - f_0))], \tag{32}$$

where f_1 is the random value of the hedging instrument at time 1, f_0 is the value of the hedging instrument at time 0, and $w_{1,h}$ is the random value of wealth at time 1 when hedging is conducted. Hedging performance is measured by the certainty equivalent *C*:

$$E]U(w_{1,u} + C)] = E[U(w_{1,h})]. \tag{33}$$

5.2 Utility function and risk aversion

One of the simplest types of utility function, which can be used as a hedging effectiveness measure, is the expected mean-variance utility function. It is also quite popular since it can consider all the factors above within a simple framework; see, for example, Kroner and Sultan (1993), Gagnon et al. (1998), and Lafuente and Novales (2003). Suppose that a hedger is endowed with a strictly increasing and twice-differentiable concave utility function $U(x)$, such that $U'(x) > 0$ and $U''(x) < 0$. Then the expected utility of the hedged portfolio P at time t -1 can be defined as

$$E_{t-1}[U(P)] = E_{t-1}(p_t) - \lambda Var_{t-1}(p_t), \quad (34)$$

where p is the return of the portfolio P and λ is a positive risk aversion parameter.

The existence of the risk aversion parameter is suggested by Merton (1973). Chou (1988) explains that there exists a linear relationship between the equity premium π and return variance in the inter-temporal CAPM model of Merton (1973), such that

$$\pi_t = \lambda_m Var(M_t), \quad (35)$$

where M_t is the instantaneous market return and λ_m is the harmonic mean of individual investor's risk-aversion parameter. Various studies, e.g. Grossman and Shiller (1981) and Pindyck (1986), show that the idea of premium can explain much of the stock price changes beyond changes in dividends and interest rates. Also, their estimation results show that λ ranges approximately from 3 to 4.5.

One thing we should note is that, the estimation of λ relies on the variance estimation method. Poterba and Summers (1986) employ a two-stage OLS procedure to estimate the variance, and conclude that shocks to the volatility decay rapidly so that it is skeptical to claim that fluctuations in risk premia account for much of the variation in prices. On the other hand, Chou (1988) introduces GARCH-M model and argues that the persistence of volatility shocks is significant such that fluctuations in risk premia can explain much of the price changes. Given that the other aspects of both researches are quite similar, this observation implies that different variance estimation method will lead to different estimates for λ.

5.3 Utility-based hedging effectiveness

Similar to the EHE case, Lien (2012a) shows that the sample utility-based hedging effectiveness estimator is downward biased and therefore tends to underestimate the true hedging performance, even when the sample estimator for the optimal hedge ratio is unbiased. To explain in detail why this happens, Lien first assumes that a hedger is endowed with a mean-variance expected utility function, i.e., equation (34). Given equations (33) and (34), we obtain

$$E[U(w_{1,u}+C)] = w_0 + C + E(\Delta s) - \lambda Var(\Delta s)\,, \tag{36}$$

$$E[U(w_{1,h})] = w_0 + E(\Delta s - h\Delta f) - \lambda Var(\Delta s - h\Delta f)\,, \tag{37}$$

where $\Delta s = s_1 - s_0$ and $\Delta f = f_1 - f_0$. From the above two equations, we derive

$$C = -hE(\Delta f) + \lambda[Var(\Delta s) - Var(\Delta s - h\Delta f)]. \tag{38}$$

The sample estimator of *C* is then

$$\hat{C} = -\hat{h}E(\Delta f) + \lambda[Var(\Delta s) - Var(\Delta s - \hat{h}\Delta f)]. \tag{39}$$

From equations (38) and (39), we obtain

$$\hat{C} = C - (h-\hat{h})E(\Delta f) + \lambda[Var(\Delta s - h\Delta f) - Var(\Delta s - \hat{h}\Delta f)]. \tag{40}$$

After algebraic manipulations, equation (40) becomes

$$\hat{C} = C - (h-\hat{h})E(\Delta f) + 2\lambda(\hat{h}-h)Cov(\Delta s, \Delta f) - \lambda(\hat{h}^2 - h^2)Var(\Delta f)\,. \tag{41}$$

Suppose that $\hat{h}$ is an unbiased estimator of *h*, i.e., $E(\hat{h}) = h$, the

$$E(\hat{C}) = C - \lambda E(\hat{h}^2 - h^2)Var(\Delta f) = C - \lambda Var(\hat{h})Var(\Delta f) < C\,. \tag{42}$$

That is, the expected value of $\hat{C}$ is downward biased. Lien (2012a) shows that the downward bias result can be extended to the case when a hedger is endowed with another type of strictly increasing concave utility function.

Because the certainty equivalent is not a strictly monotonically decreasing function of the portfolio variance (except when $E(\Delta f) = 0$ or when λ is infinitely large), the solution to variance minimization is not the same as the solution to certainty equivalent maximization. Therefore, OLS hedge ratio is not necessarily favored by the utility-based performance measure.

6. CONCLUSIONS

This paper analyzes the properties of Ederington hedging effectiveness (EHE) in general and within different statistical framework. The most popular EHE is measured by the percentage reduction in the unconditional return variance of the hedged portfolio relative to the unconditional return variance of the unhedged portfolio. Because of this emphasis on "unconditional" statistics, the OLS hedge strategy (which does not take into account any other information except current spot and futures returns) is most likely to outperform the optimal conditional hedge strategy.

This superiority of the OLS hedge ratio is challenged by the concern of the appropriateness to evaluate conditional hedging strategies by EHE. Nonetheless, the regime switching (RS) hedge ratio seems to be an exception. Under specific assumptions, the RS-OLS hedge ratio will outperform the conventional OLS hedge ratio.

Utility-based hedging effectiveness is another popular measure examined in the literature. The sample estimator of this effectiveness is, similar to the sample EHE estimator, is biased. On the other hand, the measure does not necessarily favor the OLS hedge ratio except when the futures price is a martingale or when the hedger is extremely risk averse.

Recently there have been several alternative effectiveness measures related to tail risk such as lower partial moment, value at risk, conditional value at risk. Similar problems prevail. That is, a hedge strategy may be chosen to minimize the conditional value at risk. However, when in the evaluation stage, it is the unconditional value at risk that counts. We do not address these issues in the current paper. It will be left for future research.

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