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Random Effects, Fixed Effcts and Hausman's Test for the **Generalized Spatial Panel Data Regression Model**

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Abstract

This paper suggests RE-GLS and FE-GLS estimators for the generalized spatial panel model of Baltagi, Egger and Pfaffermayr (2012) using the Generalized Moments method suggested by Kapoor, Kelejian, and Prucha (2007). We derive the asymptotic distributions of these estimators and suggest a Hausman test a la Mutl and Pfaffermayr (2011) based on the difference between them. Monte Carlo experiments are performed to investigate the performance of these estimators as well as the corresponding Hausman test.

Key Words: *Panel Data; Fixed Effects; Random Effects; Spatial Model; Hausman Test.*

JEL classification: C12; C13; C23

1 Introduction

Anselin (1988) and Kapoor, Kelejian, and Prucha (2007) considered two different variants of a random effects panel data model with spatially correlated errors. The first paper estimated it with maximum likelihood methods and the second estimated it with a generalized moments (GM) method that is computationally simpler. Baltagi, Egger and Pfaffermayr (2012) generalized this random effects spatial model to encompass both cases and derived LM and LR tests to distinguish between these models. The generalized model allows the individual effects and the remainder errors to have different spatial autoregressive parameters. Using maximum likelihood methods, Baltagi, Egger and Pfaffermayr (2008) examined the consequences of model misspecification in this context using Monte Carlo simulations. These papers assume that the underlying spatial panel model is *random effects* (RE). Spatial panel data model with *fixed-effects* (FE) have been considered by Baltagi and Li (2006), Mutl and Pfaffermayr (2011), and Lee and Yu (2010) to mention a few. In fact, Baltagi and Li (2006) obtained the maximum likelihood estimator of a first order spatial

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autoregressive model with fixed effects and used it to forecast the consumption of liquor across a panel of US states, while Lee and Yu (2010) established the asymptotic properties of a quasi-maximum likelihood estimator for the spatial panel data model with fixed-effects. However, as pointed out by Kapoor, Kelejian, and Prucha (2007), hereafter denoted by (KKP), the ML estimation of models variant to the one considered in Cliff and Ord (1973, 1981) entail substantial computational problems if the number of cross sectional units is large. To circumvent these computation problems, Mutl and Pfaffermayr (2011) suggest a Within-GLS estimator based on a generalized moments (GM) estimator a la Kapoor, Kelejian, and Prucha (2007) but applied to a spatial autoregressive panel data model. Mutl and Pfaffermayr (2011) also propose a Hausman test based on the difference between the fixed and random effects specification of this model. This paper applies the FE-GLS estimator of Mutl and Pfaffermayr (2011) to the generalized error component model considered by Baltagi, Egger and Pfaffermayr (2012). We also suggest a RE-GLS estimator using GM estimation of this generalized error component model, and apply a Hausman test based on the difference between the fixed and random effects specification of this model. Small sample properties of these estimators as well as the size of the proposed Hausman test are studied using Monte Carlo experiments. We show that a misspecified GM estimator can cause substantial loss in MSE and wrong size for the corresponding Hausman test.

The rest of the paper is organized as follows. Section 2 introduces the RE-GLS and FE-GLS estimators for the generalized spatial error component model proposed by Baltagi, Egger and Pfaffermayr (2012). Generalized moments (GM) estimators a la Kapoor, Kelejian and Prucha (2007) are proposed for this model and their asymptotic distribution is obtained. Following Mutl and Pfaffermayr (2011), a Hausman test is proposed based on the difference between the FE-GLS and feasible RE-GLS estimators of this spatial panel model. Simulation results are reported in section 3, while section 4 concludes the paper. All proofs are relegated to the Appendix.

2 The Model

Baltagi, Egger and Pfaffermayr (2012) considered a generalized spatial error components model. In each time period $t = 1, ..., T$, the data are generated according to the following model:

$$y_N(t) = X_N(t)\beta + u_N(t) \tag{1}$$

$$u_N(t) = u_{1N} + u_{2N}(t), \tag{2}$$

$$u_{1N} = \rho_1 W_N u_{1N} + \mu_N, \tag{3}$$

$$u_{2N}(t) = \rho_2 W_N u_{2N}(t) + \nu_N(t), \tag{4}$$

where $y_N(t)$ denotes the $N \times 1$ vector of observation on the dependent variable in period t. $X_N(t)$ denotes the $N \times K$ matrix of observations on exogenous regressors in period t, which may contain the constant term, β is the corresponding $K \times 1$ vector of regression parameters, and $u_N(t)$ denotes the $N \times 1$ vector of disturbance terms. The disturbance term follows an error component model which involves the sum of two disturbances. The $N \times 1$ vector of random variables u_{1N} captures the time-invariant unit-specific effects and therefore has no time subscript. The $N \times 1$ vector of the remainder disturbances $u_{2N}(t)$ varies with time. Both u_{1N} and $u_{2N}(t)$ are spatially correlated with the same spatial weights matrix W_N, but with different spatial autocorrelation parameters ρ_1 and ρ_2, respectively. W_N is an $N \times N$ weighting matrix of known constants which does not involve t.

Stacking the cross-sections over time yields

$$y_N = X_N\beta + u_N \tag{5}$$

$$u_N = Z_\mu u_{1N} + u_{2N}, \tag{6}$$

$$u_{1N} = \rho_1 W_N u_{1N} + \mu_N, \tag{7}$$

$$u_{2N} = \rho_2 (I_T \otimes W_N) u_{2N} + \nu_N, \tag{8}$$

where $y_N = [y_N'(1), \ldots, y_N'(T)]'$, $X_N = [X_N'(1), \ldots, X_N'(T)]'$, $u_N = [u_N'(1), \ldots, u_N'(T)]'$, $u_{2N} = [u_{2N}'(1), \ldots, u_{2N}'(T)]'$ and $v_N = [v_N'(1), \ldots, v_N'(T)]'$. The unit-specific errors u_{1N} are repeated in all time periods using the $NT \times N$ selector matrix $Z_\mu = \iota_T \otimes I_N$, where ι_T is a vector of ones of dimension T and I_N is an identity matrix of dimension N. Let $\{\mu_{i,N}\}$ and $\{\nu_{it,N}\}$ denote the elements of the $N \times 1$ vector of individual effects μ_N and the $n \times 1$ vector of remainder disturbances ν_N. Following Kapoor, Kelejian and Prucha (2007), we employ the following assumptions:

Assumption 1 *Let T be a fixed positive integer. (a) For all $1 \leq t \leq T$ and $1 \leq i \leq N$, $N \geq 1$ the error components $\nu_{it,N}$ are identically distributed with zero mean and variance σ_ν^2, $0 < \sigma_\nu^2 < b_v < \infty$, and finite* *fourth moments. In addition for each $N \geq 1$ and $1 \leq t \leq T$, $1 \leq i \leq N$ the error components $\nu_{it,N}$ are independently distributed. (b) For all $1 \leq i \leq N$, $N \geq 1$ the unit specific error components $\mu_{i,N}$ are independently distributed with zero mean and variance σ_μ^2, $0 < \sigma_\mu^2 < b_\mu < \infty$, and finite fourth moments. In addition for each $N \geq 1$ and $1 \leq i \leq N$ the unit specific error components $\mu_{i,N}$ are independently distributed. (c) The process $\{\mu_{i,N}\}$ and $\{\nu_{it,N}\}$ are independent.*

Assumption 2 *(a) All diagonal elements of W_N are zero. (b) $|\rho_1| < 1$ and $|\rho_2| < 1$. (c) The matrix $I_N - \rho_1 W_N$ and $I_N - \rho_2 W_N$ are nonsingular.*

As pointed out in Baltagi, Egger and Pfaffermayr (2012), this model nests the various spatial panels existing in the literature. When $\rho_1 = \rho_2$, it reduces to the model in Kapoor, Kelejian and Prucha (2007); when $\rho_1 = 0$, it reduces to the Anselin (1988) model also described in Baltagi, Song and Koh (2003) and Anselin, Le Gallo and Jayet (2008). When $\rho_1 = \rho_2 = 0$, it reduces to the familiar random effects (RE) panel data model; see Baltagi (2008).

2.1 Spatial GLS Estimator

Let $\Omega_u = var(u_N)$. The true GLS estimator of β is given by

$$\hat{\beta}_{GLS} = \left(X_N' \Omega_u^{-1} X_N\right)^{-1} X_N' \Omega_u^{-1} y_N \tag{9}$$

with variance $var\left(\hat{\beta}_{GLS}\right) = \left(X_N' \Omega_u^{-1} X_N\right)^{-1}$. As shown in Baltagi, Egger and Pfaffermayr (2012),

$$\Omega_u^{-1} = \bar{J}_T \otimes \left[T\sigma_\mu^2 (A'A)^{-1} + \sigma_\nu^2 (B'B)^{-1}\right]^{-1} + \sigma_\nu^{-2}\left[E_T \otimes (B'B)\right], \tag{10}$$

where $A = I_N - \rho_1 W_N$ and $B = I_N - \rho_2 W_N$. $E_T = I_T - \bar{J}_T$, $\bar{J}_T = J_T/T$ and J_T is a matrix of ones of dimension T.

Define $Q = I_{NT} - P$, wherer I_{NT} is an identity matrix of dimension NT and $P = \bar{J}_T \otimes I_N$. Kapoor, Kelejian and Prucha (2007) considers the special case where $\rho_1 = \rho_2$ and propose GM estimators of ρ_2 and σ_ν^2 based on the following three moment conditions:

$$\frac{1}{N(T-1)} E\left[\nu_N' Q \nu_N\right] = \sigma_\nu^2, \tag{11}$$

$$\frac{1}{N(T-1)} E\left[\bar{\nu}_N' Q \bar{\nu}_N\right] = \frac{1}{N}\sigma_\nu^2 tr\left(W_N' W_N\right), \tag{12}$$

$$\frac{1}{N(T-1)} E\left[\bar{\nu}_N' Q \nu_N\right] = 0, \tag{13}$$

where $\bar{\nu}_N = (I_T \otimes W_N)\nu_N$. Define $\bar{u}_{2N} = (I_T \otimes W_N) u_{2N}$ and $\bar{\bar{u}}_{2N} = (I_T \otimes W_N)\bar{u}_{2N}$. Substitute $\nu_N = u_{2N} - \rho_2 \bar{u}_{2N}$ and $\bar{\nu}_N = \bar{u}_{2N} - \rho_2 \bar{\bar{u}}_{2N}$ into the equation system above, we get

$$\frac{1}{N(T-1)}E\left[u_{2N}'Qu_{2N}\right]-\frac{2}{N(T-1)}\rho_2E\left[\bar{u}_{2N}'Qu_{2N}\right]+\frac{1}{N(T-1)}\rho_2^2E\left[\bar{u}_{2N}'Q\bar{u}_{2N}\right] = \sigma_\nu^2, \tag{14}$$

$$\frac{1}{N(T-1)}E\left[\bar{u}_{2N}'Q\bar{u}_{2N}\right]-\frac{2}{N(T-1)}\rho_2E\left[\bar{\bar{u}}_{2N}'Q\bar{u}_{2N}\right]+\frac{1}{N(T-1)}\rho_2^2E\left[\bar{\bar{u}}_{2N}'Q\bar{\bar{u}}_{2N}\right] = \frac{1}{N}\sigma_\nu^2 tr\left(W_N'W_N\right), \tag{15}$$

$$\frac{1}{N(T-1)}E\left[\bar{u}_{2N}'Qu_{2N}\right]-\frac{1}{N(T-1)}\rho_2E\left[\bar{\bar{u}}_{2N}'Qu_{2N}+\bar{u}_{2N}'Q\bar{u}_{2N}\right]+\frac{1}{N(T-1)}\rho_2^2E\left[\bar{\bar{u}}_{2N}'Q\bar{u}_{2N}\right] = 0. \tag{16}$$

Notice that $Q(\iota_T\otimes u_{1N})=0$. Therefore, for the general model in Equation (5)-(8), we have $u_N'Qu_N=u_{2N}'Qu_{2N}, \bar{u}_N'Q\bar{u}_N=\bar{u}_{2N}'Q\bar{u}_{2N}$ and $\bar{u}_N'Qu_N=\bar{u}_{2N}'Qu_{2N}$. This system can be expressed as

$$\Gamma_N^0\left[\rho_2,\rho_2^2,\sigma_\nu^2\right]'-\gamma_N^0=0, \tag{17}$$

where $\Gamma_N^0=\begin{pmatrix}\frac{2}{N(T-1)}E\left[\bar{u}_N'Qu_N\right] & -\frac{1}{N(T-1)}E\left[\bar{u}_N'Q\bar{u}_N\right] & 1\\ \frac{2}{N(T-1)}E\left[\bar{\bar{u}}_N'Q\bar{u}_N\right] & -\frac{1}{N(T-1)}E\left[\bar{\bar{u}}_N'Q\bar{\bar{u}}_N\right] & \frac{1}{N}tr\left(W_N'W_N\right)\\ \frac{1}{N(T-1)}E\left[\bar{\bar{u}}_N'Qu_N+\bar{u}_N'Q\bar{u}_N\right] & -\frac{1}{N(T-1)}E\left[\bar{\bar{u}}_N'Q\bar{u}_N\right] & 0\end{pmatrix}$ and $\gamma_N^0=\begin{pmatrix}\frac{1}{N(T-1)}E\left[u_N'Qu_N\right]\\ \frac{1}{N(T-1)}E\left[\bar{u}_N'Q\bar{u}_N\right]\\ \frac{1}{N(T-1)}E\left[\bar{u}_N'Qu_N\right]\end{pmatrix}$.

Let $\widehat{u}_N$ denote the OLS residuals from the (5), and let G_N and g_N^0 be the sample analogues of Γ_N^0 and γ_N^0 substituting $\widehat{u}_N$ for u_N.We can get a GM estimator by solving

$$\left(\tilde{\rho}_2,\tilde{\sigma}_\nu^2\right)=\arg\min\left\{\xi_N^0\left(\underline{\rho_2},\underline{\sigma_\nu^2}\right)'\xi_N^0\left(\underline{\rho_2},\underline{\sigma_\nu^2}\right),\underline{\rho_2}\in[-a_0,a_0],\underline{\sigma_\nu^2}\in[0,b_0]\right\}, \tag{18}$$

where $\xi_N^0\left(\rho_2,\sigma_\nu^2\right)=G_N^0\left[\rho_2,\rho_2^2,\sigma_\nu^2\right]'-g_N^1$, $a_0\geq 1$ and $b_0\geq b_v$.

Assumption 3 *The elements of X_N are bounded uniformly in absolute value. Furthermore, the limit $\Psi_0=\lim_{N\to\infty}\frac{1}{NT}X_N'\left\{\bar{J}_T\otimes\left[T\sigma_\mu^2\left(A'A\right)^{-1}+\sigma_\nu^2\left(B'B\right)^{-1}\right]^{-1}\right\}X_N$ and $\Psi_1=\lim_{N\to\infty}\frac{1}{NT}X_N'\left[E_T\otimes\left(B'B\right)\right]X_N$ are finite and nonsingular.*

Assumption 4 *The row and column sums of W_N, $\left(I_N-\rho_1W_N\right)^{-1}$ and $\left(I_N-\rho_2W_N\right)^{-1}$ are bounded uniformly in absolute values for all $|\rho_1|<1$ and $|\rho_2|<1$.*

Assumption 5 *The smallest eigenvalues of $\Gamma_N'\Gamma_N$ are bounded away from zero.*

Kapoor, Kelejian and Prucha (2007) showed that $\tilde{\rho}$ and $\tilde{\sigma}_\nu^2$ are consistent. It is worth pointing out that the condition $Q(\iota_T\otimes u_1)=0$ holds for the general model given in equation (5), and not only for the special case in Kapoor, Kelejian and Prucha (2007). Therefore, for all values of ρ_1 and σ_μ^2 in the parameter space, the GM estimator of ρ_2 and σ_ν^2 suggested by Kapoor, Kelejian and Prucha (2007) will always be consistent. By Theorem 1 of Kapoor, Kelejian and Prucha (2007), we have $\left(\tilde{\rho}_2,\tilde{\sigma}_\nu^2\right)\xrightarrow{p}\left(\rho_2,\sigma_\nu^2\right)$ under assumptions 1-5 as $N\to\infty$. Similarly, we introduce the following GM estimators of ρ_1 and σ_μ^2:

Define $\bar{\mu} = W_N \mu$. We have the following three moment conditions:

$$\frac{1}{N} E\left[\mu_N' \mu_N\right] = \sigma_\mu^2, \tag{19}$$
$$\frac{1}{N} E\left[\bar{\mu}_N' \bar{\mu}_N\right] = \frac{1}{N}\sigma_\mu^2 tr\left(W_N' W_N\right), \tag{20}$$
$$\frac{1}{N} E\left[\bar{\mu}_N' \mu_N\right] = 0. \tag{21}$$

Similarly define $\bar{u}_{1N} = W_N u_{1N}$ and $\bar{\bar{u}}_{1N} = W_N \bar{u}_{1N}$. Substitute $\mu_N = u_{1N} - \rho_1 \bar{u}_{1N}$ and $\bar{\mu}_N = \bar{u}_{1N} - \rho_1 \bar{\bar{u}}_{1N}$ into the equation system above, we get

$$\frac{1}{N} E\left[u_{1N}' u_{1N}\right] - \frac{2}{N}\rho_1 E\left[\bar{u}_{1N}' u_{1N}\right] + \frac{1}{N}\rho_1^2 E\left[\bar{u}_{1N}' \bar{u}_{1N}\right] = \sigma_\mu^2, \tag{22}$$
$$\frac{1}{N} E\left[\bar{u}_{1N}' \bar{u}_{1N}\right] - \frac{2}{N}\rho_1 E\left[\bar{\bar{u}}_{1N}' \bar{u}_{1N}\right] + \frac{1}{N}\rho_1^2 E\left[\bar{\bar{u}}_{1N}' \bar{\bar{u}}_{1N}\right] = \frac{1}{N}\sigma_\mu^2 tr\left(W_N' W_N\right), \tag{23}$$
$$\frac{1}{N} E\left[\bar{u}_{1N}' u_{1N}\right] - \frac{1}{N}\rho_1 E\left[\bar{\bar{u}}_{1N}' u_{1N} + \bar{u}_{1N}' \bar{u}_{1N}\right] + \frac{1}{N}\rho_1^2 E\left[\bar{\bar{u}}_{1N}' \bar{u}_{1N}\right] = 0. \tag{24}$$

This system can be expressed as

$$\Gamma_N^1 \left[\rho_1, \rho_1^2, \sigma_\mu^2\right]' - \gamma_N^1 = 0, \tag{25}$$

where $\Gamma_N^1 = \begin{pmatrix} \frac{2}{N} E\left[\bar{u}_{1N}' u_{1N}\right] & -\frac{1}{N} E\left[\bar{u}_{1N}' \bar{u}_{1N}\right] & 1 \\ \frac{2}{N} E\left[\bar{\bar{u}}_{1N}' \bar{u}_{1N}\right] & -\frac{1}{N} E\left[\bar{\bar{u}}_{1N}' \bar{\bar{u}}_{1N}\right] & \frac{1}{N} tr\left(W_N' W_N\right) \\ \frac{1}{N} E\left[\bar{\bar{u}}_{1N}' u_{1N} + \bar{u}_{1N}' \bar{u}_{1N}\right] & -\frac{1}{N} E\left[\bar{\bar{u}}_{1N}' \bar{u}_{1N}\right] & 0 \end{pmatrix}$ and $\gamma_N^1 = \begin{pmatrix} \frac{1}{N} E\left[u_{1N}' u_{1N}\right] \\ \frac{1}{N} E\left[\bar{u}_{1N}' \bar{u}_{1N}\right] \\ \frac{1}{N} E\left[\bar{u}_{1N}' u_{1N}\right] \end{pmatrix}$.

Define $S = P - \frac{1}{T-1} Q = S_T \otimes I_N$, where $S_T = \bar{J}_T - \frac{1}{T-1} E_T$. Also

$$\begin{aligned} \varphi_{kl,N} &= \frac{1}{NT} u_N' \left(I_T \otimes W_N^k\right)' S \left(I_T \otimes W_N^l\right) u_N \\ &= \frac{1}{NT} u_N' \left(I_T \otimes W_N^k\right)' \left(S_T \otimes I_N\right) \left(I_T \otimes W_N^l\right) u_N \\ &= \frac{1}{NT} u_N' \left(S_T \otimes W_N^{k\prime} W_N^l\right) u_N \end{aligned}$$

for $k, l = 0, 1, 2$. Hence

$$\begin{aligned} E\left(\varphi_{kl,N}\right) &= \frac{1}{NT} E\left[u_N' \left(S_T \otimes W_N^{k\prime} W_N^l\right) u_N\right] \\ &= \frac{1}{NT} E\left[\left(Z_\mu u_{1N} + u_{2N}\right)' \left(S_T \otimes W_N^{k\prime} W_N^l\right) \left(Z_\mu u_{1N} + u_{2N}\right)\right] \\ &= \frac{1}{NT} E\left[\left(Z_\mu u_{1N}\right)' \left(S_T \otimes W_N^{k\prime} W_N^l\right) \left(Z_\mu u_{1N}\right)\right] + \frac{1}{NT} E\left[u_{2N}' \left(S_T \otimes W_N^{k\prime} W_N^l\right) u_{2N}\right] \\ &\quad + \frac{2}{NT} E\left[u_{2N}' \left(S_T \otimes W_N^{k\prime} W_N^l\right) Z_\mu u_{1N}\right] \\ &\equiv I + II + III. \end{aligned}$$

Notice that

$$I = \frac{1}{NT}E\left[(Z_\mu u_{1N})'\left(S_T \otimes W_N^{k\prime}W_N^l\right)(Z_\mu u_{1N})\right] = \frac{1}{NT}E\left[u_{1N}'\left(\iota_T' S_T \iota_T \otimes W_N^{k\prime}W_N^l\right)u_{1N}\right] = \frac{1}{N}E\left(u_{1N}' W_N^{k\prime}W_N^l u_{1N}\right)$$

since $\iota_T' S_T \iota_T = \iota_T'\left(\bar{J}_T - \frac{1}{T-1}E_T\right)\iota_T = \iota_T'\bar{J}_T\iota_T - \frac{1}{T-1}\iota_T' E_T \iota_T = T$ using $\bar{J}_T\iota_T = \iota_T$ and $E_T\iota_T = 0$;

$$\begin{aligned} II &= \frac{1}{NT}E\left[u_{2N}'\left(S_T \otimes W_N^{k\prime}W_N^l\right)u_{2N}\right] = \frac{1}{NT}E\left[\nu_N'\left(I_T \otimes B^{-1}\right)'\left(S_T \otimes W_N^{k\prime}W_N^l\right)\left(I_T \otimes B^{-1}\right)\nu_N\right] \\ &= \frac{1}{NT}E\left[\nu_N'\left(S_T \otimes B^{-1\prime}W_N^{k\prime}W_N^l B^{-1}\right)\nu_N\right] \\ &= \frac{1}{NT}\sigma_v^2 tr\left(S_T\right) tr\left(B^{-1\prime}W_N^{k\prime}W_N^l B^{-1}\right) \\ &= 0 \end{aligned}$$

since $tr\left(S_T\right) = tr\left(\bar{J}_T - \frac{1}{T-1}E_T\right) = 0$; and

$$III = \frac{2}{NT}E\left[u_{2N}'\left(S_T \otimes W_N^{k\prime}W_N^l\right)Z_\mu u_{1N}\right] = 0$$

since u_{1N} and u_{2N} are independent by Assumption 1. Hence, one gets $E\left(\varphi_{kl,N}\right) = \frac{1}{N}E\left(u_{1N}'W_N^{k\prime}W_N^l u_{1N}\right)$; $\Gamma_N^1 = \begin{pmatrix} \frac{2}{NT}E\left[\bar{u}_N' S u_N\right] & -\frac{1}{NT}E\left[\bar{u}_N' S \bar{u}_N\right] & 1 \\ \frac{2}{NT}E\left[\bar{\bar{u}}_N' S \bar{u}_N\right] & -\frac{1}{NT}E\left[\bar{\bar{u}}_N' S \bar{\bar{u}}_N\right] & \frac{1}{N}tr\left(W_N' W_N\right) \\ \frac{1}{NT}E\left[\bar{\bar{u}}_N' S u_N + \bar{u}_N' S \bar{u}_N\right] & -\frac{1}{NT}E\left[\bar{\bar{u}}_N' S \bar{u}_N\right] & 0 \end{pmatrix}$; and $\gamma_N^1 = \begin{pmatrix} \frac{1}{NT}E\left[u_N' S u_N\right] \\ \frac{1}{NT}E\left[\bar{u}_N' S \bar{u}_N\right] \\ \frac{1}{NT}E\left[\bar{u}_N' S u_N\right] \end{pmatrix}$. The sample analogues to Γ_N^1 and γ_N^1 are $G_N^1 = \begin{pmatrix} \frac{2}{NT}\tilde{\bar{u}}_N' S \tilde{u}_N & -\frac{1}{NT}\tilde{\bar{u}}_N' S \tilde{\bar{u}}_N & 1 \\ \frac{2}{NT}\tilde{\bar{\bar{u}}}_N' S \tilde{\bar{u}}_N & -\frac{1}{NT}\tilde{\bar{\bar{u}}}_N' S \tilde{\bar{\bar{u}}}_N & \frac{1}{N}tr\left(W_N' W_N\right) \\ \frac{1}{NT}\left(\tilde{\bar{\bar{u}}}_N' S \tilde{u}_N + \tilde{\bar{u}}_N' S \tilde{\bar{u}}_N\right) & -\frac{1}{NT}\tilde{\bar{\bar{u}}}_N' S \tilde{\bar{u}}_N & 0 \end{pmatrix}$ and $g_N^1 = \begin{pmatrix} \frac{1}{NT}\tilde{u}_N' S \tilde{u}_N \\ \frac{1}{NT}\tilde{\bar{u}}_N' S \tilde{\bar{u}}_N \\ \frac{1}{NT}\tilde{\bar{u}}_N' S \tilde{u}_N \end{pmatrix}$, respectively. Hence, a GM estimator can be obtained from

$$\left(\tilde{\rho}_1, \tilde{\sigma}_\mu^2\right) = \arg\min\left\{\xi_N^1\left(\underline{\rho_1}, \underline{\sigma_\mu^2}\right)'\xi_N^1\left(\underline{\rho_1}, \underline{\sigma_\mu^2}\right), \underline{\rho_1} \in \left[-a_1, a_1\right], \underline{\sigma_\mu^2} \in \left[0, b_1\right]\right\}, \tag{26}$$

where $\xi_N^1\left(\rho_1, \sigma_\mu^2\right) = G_N^1\left[\rho_1, \rho_1^2, \sigma_\mu^2\right]' - g_N^1$, $a_1 \geq 1$ and $b_1 \geq b_\mu$.

Theorem 1 *Under Assumptions 1 -5, we have* $\left(\tilde{\rho}_1, \tilde{\sigma}_\mu^2\right) \xrightarrow{p} \left(\rho_1, \sigma_\mu^2\right)$ *as* $N \to \infty$.

With the GM estimators of $\tilde{\rho}_1$, $\tilde{\rho}_2$, $\tilde{\sigma}_\mu^2$ and $\tilde{\sigma}_\nu^2$, the corresponding feasible GLS estimator $\hat{\beta}_{FGLS}$ is given by

$$\hat{\beta}_{FGLS} = \left(X_N'\tilde{\Omega}_u^{-1}X_N\right)^{-1}X_N'\tilde{\Omega}_u^{-1}y_N, \tag{27}$$

where

$$\tilde{\Omega}_u^{-1} = \bar{J}_T \otimes \left[T\tilde{\sigma}_\mu^2 \left(\tilde{A}'\tilde{A} \right)^{-1} + \tilde{\sigma}_\nu^2 \left(\tilde{B}'\tilde{B} \right)^{-1} \right]^{-1} + \tilde{\sigma}_\nu^{-2} \left[E_T \otimes \left(\tilde{B}'\tilde{B} \right) \right]. \tag{28}$$

The theorem below establishes consistency and asymptotic normality of the feasible GLS estimators. The proof of the theorem is given in the appendix.

Theorem 2 *Under Assumptions 1 -5, we have* $\sqrt{NT}\left(\hat{\beta}_{FGLS} - \beta\right) \xrightarrow{d} N\left(0, \Psi^{-1}\right)$ *as* $N \to \infty$*, where* $\Psi = \Psi_0 + \sigma_\nu^{-2}\Psi_1$.

2.1.1 A special case–the Anselin model

Under the Anselin model, we have $\rho_1 = 0$ and hence $A = I_N$. Equation (10) reduces to

$$\Omega_u^{-1} = \bar{J}_T \otimes \left[T\sigma_\mu^2 I_N + \sigma_\nu^2 \left(B'B \right)^{-1} \right]^{-1} + \sigma_\nu^{-2} \left[E_T \otimes \left(B'B \right) \right]. \tag{29}$$

We can estimate σ_μ^2 from the first Equation in (19) as

$$\hat{\sigma}_\mu^2 = \frac{1}{NT}\tilde{u}_N' R \tilde{u}_N = \frac{1}{NT}\tilde{u}_N' P \tilde{u}_N - \frac{1}{NT\left(T-1\right)}\tilde{u}_N' Q \tilde{u}_N.$$

Notice that it is the same estimator of σ_μ^2 for the random effect model with $\rho_1 = \rho_2 = 0$. From the proof of Theorem 1, we know that under Assumptions 1-5, $\tilde{\sigma}_\mu^2 \xrightarrow{p} \sigma_\mu^2$ as $N \to \infty$. With these GM estimators of $\tilde{\rho}_2$, $\tilde{\sigma}_\mu^2$ and $\tilde{\sigma}_\nu^2$, the corresponding feasible GLS estimator $\hat{\beta}_{FGLS}$ is given by

$$\hat{\beta}_{FGLS} = \left(X_N' \tilde{\Omega}_u^{-1} X_N \right)^{-1} X_N' \tilde{\Omega}_u^{-1} y_N, \tag{30}$$

where

$$\tilde{\Omega}_u^{-1} = \bar{J}_T \otimes \left[T\tilde{\sigma}_\mu^2 + \tilde{\sigma}_\nu^2 \left(\tilde{B}'\tilde{B} \right)^{-1} \right]^{-1} + \tilde{\sigma}_\nu^{-2} \left[E_T \otimes \left(\tilde{B}'\tilde{B} \right) \right]. \tag{31}$$

Theorem 2 reduces to the following proposition.

Proposition 1 *Under Assumptions 1 -5,when* $\rho_1 = 0$, $\hat{\beta}_{FGLS}$ *has the same asymptotic distribution as in Theorem 2, with* Ψ_0 *reducing to* $\lim\limits_{N\to\infty} \frac{1}{NT} X_N' \left\{ \bar{J}_T \otimes \left[T\sigma_\mu^2 I_N + \sigma_\nu^2 \left(B'B \right)^{-1} \right]^{-1} \right\} X_N$.

2.2 Spatial Within-GLS Estimator

Let $\{u_{1i,N}\}$ and $\{X_{it,N}\}$ denote the elements of the $N \times 1$ vector of u_{1N} and the $NT \times K$ vector of X_N. A critical assumption for the consistency of the GLS estimator is that $E\left(u_{1i,N} | X_{it,N}\right) = 0$. If the unobserved individual invariant effects are correlated with X_{it}, then $E\left(u_{1i} | X_{it}\right) \neq 0$ and GLS is inconsistent. As pointed out in Lee and Yu (2010), with the fixed effects specification, the panel models in Baltagi, Egger and Pfaffermayr (2012), Kapoor, Kelejian and Prucha (2007) and Anselin (1988) have the same representation. More specifically, premultiplying equation (5) by the fixed effects (or within) transformation $Q = E_T \otimes I_N$, one obtains

$$Qy_N = QX_N\beta + Qu_{2N}, \tag{32}$$

since $Q(\iota_T \otimes u_{1N}) = 0$, see Baltagi (2008). The Within estimator $\hat{\beta}_{within} = (X_N'QX_N)^{-1} X_N'Qy_N$ wipes out the individual effects and does not require the estimation of ρ_1 or σ_μ^2. However, this estimator ignores the spatial autocorrelation in the error. To gain efficiency, one can apply the Cochrane-Orcutt type transformation on the within transformed model in Equation (32) to obtain the Within-GLS estimator as suggested in Mutl and Pfaffermayr (2011). More specifically, we premultiply equation (32) by $I_T \otimes B$, to get

$$(E_T \otimes B) y_N = (E_T \otimes B) X_N\beta + Q\nu_N. \tag{33}$$

This uses the fact that $(I_T \otimes B) Q = (E_T \otimes B) = Q(I_T \otimes B)$ and $(I_T \otimes B) Qu_{2N} = Q(I_T \otimes B) u_{2N} = Q\nu_N$. Hence, the Within-GLS estimator of β is given by

$$\begin{aligned} \hat{\beta}_{within-GLS} &= [X_N'(E_T \otimes B')(E_T \otimes B) X_N]^{-1} X_N'(E_T \otimes B')(E_T \otimes B) y_N \\ &= \{X_N'[E_T \otimes (B'B)] X_N\}^{-1} X_N'[E_T \otimes (B'B)] y_N, \end{aligned} \tag{34}$$

with variance $var\left(\hat{\beta}_{Within-GLS}\right) = \sigma_\nu^2 \{X_N'[E_T \otimes (B'B)] X_N\}^{-1}$. If $\rho_2 = 0$, then $B = I_N$ and the Within-GLS estimator in Equation (34) reduces to the within estimator $(X_N'QX_N)^{-1} X_N'Qy_N$. Using the GM estimators of $\tilde{\rho}_2$ and $\tilde{\sigma}_\nu^2$ from Equation (18), the corresponding feasible Within-GLS estimator $\hat{\beta}_{Within-FGLS}$ is obtained by replacing B by its estimator $\tilde{B} = I_N - \tilde{\rho}_2 W_N$, i.e.,

$$\hat{\beta}_{Within-FGLS} = \left\{X_N'\left[E_T \otimes \left(\tilde{B}'\tilde{B}\right)\right] X_N\right\}^{-1} X_N'\left[E_T \otimes \left(\tilde{B}'\tilde{B}\right)\right] y_N. \tag{35}$$

This estimator can be computed conveniently as the Within estimator after premultiplying the model in equation (5) by $I_T \otimes \tilde{B}$. The theorem below establishes consistency and asymptotic normality of the feasible Within-GLS estimators. The proof of the theorem is given in the appendix.

Theorem 3 *Under Assumptions 1 -5, we have* $\sqrt{NT}\left(\hat{\beta}_{Within-FGLS} - \beta\right) \xrightarrow{d} N\left(0, \sigma_\nu^2 \Psi_1^{-1}\right)$ *as* $N \to \infty$.

One of the advantages of the spatial within-GLS estimator of β is that it does not depend on σ_μ^2 and ρ_1. Hence, the Within-GLS estimator is robust to different values of σ_μ^2 and ρ_1. Another advantage of the Within-GLS estimator is that it is still consistent when $E(u_{1i}|x_{it}) \neq 0$, while the RE-GLS estimator is not.

2.3 Hausman's Test

One can perform Hausman's (1978) specification test for endogeneity on this generalized spatial panel data regression model. The null hypothesis is $H_0 : E(u_{1i,N}|X_{it,N}) = 0$. Under the null hypothesis H_0, $\hat{\beta}_{GLS}$ given in (9) is the efficient estimator, while under the alternative $H_1 : E(u_{1i,N}|X_{it,N}) \neq 0$, $\hat{\beta}_{GLS}$ is inconsistent . In contrast, $\hat{\beta}_{Within-GLS}$ is consistent under the null and alternative. Let $q = \hat{\beta}_{Within-GLS} - \hat{\beta}_{GLS}$ and note that

$$
\begin{aligned}
cov\left(\hat{\beta}_{Within-GLS}, \hat{\beta}_{GLS}\right) &= E\left[\left(\hat{\beta}_{Within-GLS} - \beta\right)\left(\hat{\beta}_{GLS} - \beta\right)'\right] \\
&= E\left[\left\{X_N'\left[E_T \otimes (B'B)\right]X_N\right\}^{-1} X_N'\left[E_T \otimes (B'B)\right]u_N u_N' \Omega_u^{-1} X_N \left(X_N'\Omega_u^{-1}X_N\right)^{-1}\right] \\
&= \left\{X_N'\left[E_T \otimes (B'B)\right]X_N\right\}^{-1} X_N'\left[E_T \otimes (B'B)\right]\Omega_u \Omega_u^{-1} X_N \left(X_N'\Omega_u^{-1}X_N\right)^{-1} \\
&= \left(X_N'\Omega_u^{-1}X_N\right)^{-1} = var\left(\hat{\beta}_{GLS}\right).
\end{aligned}
$$

Hence

$$
\begin{aligned}
var(q) &= var\left(\hat{\beta}_{Within-GLS} - \hat{\beta}_{GLS}\right) \\
&= var\left(\hat{\beta}_{Within-GLS}\right) + var\left(\hat{\beta}_{GLS}\right) - 2cov\left(\hat{\beta}_{Within-GLS}, \hat{\beta}_{GLS}\right) \\
&= var\left(\hat{\beta}_{Within-GLS}\right) + var\left(\hat{\beta}_{GLS}\right) - 2var\left(\hat{\beta}_{GLS}\right) \\
&= var\left(\hat{\beta}_{Within-GLS}\right) - var\left(\hat{\beta}_{GLS}\right).
\end{aligned}
$$

Under the null hypothesis H_0, the Hausman test $m = q'\left[var(q)\right]^{-1} q$ has a limiting χ^2 distribution with degrees of freedom equal to the rank of $var(q)$. In practice, estimates of both ρ_1 and ρ_2 are needed to calculate $var\left(\hat{\beta}_{GLS}\right)$. Under the Kapoor, Kelejian and Prucha (2007) random effects spatial model, $\rho_1 = \rho_2$ and under the Anselin (1988) random effects spatial model, $\rho_1 = 0$. One could perform a Hausman test based on these random effects spatial specifications versus the within-GLS estimator proposed in this paper. In fact, Mutl and Pfaffermayr (2011) suggested a Hausman test assuming $\rho_1 = \rho_2$. Its sensitivity under model misspecification (say $\rho_1 \neq \rho_2$) is checked in the following section via Monte Carlo experiments.

3 Extensions to MRSAR Model

This section considers the MRSAR model which replace Equation (5) by

$$
y_N = \lambda M_N y_N + X_N \beta + u_N, \tag{36}
$$

where M_n is an $N \times N$ spatial weight matrix for Y_N. M_N and W_N may or may not be the same. This generalizes the model in Equation (5) with SAR error by incorporating a spatial lag. As shown in Kelejian and Prucha (1998), the spatial lag $M_N y_N$ is correlated with the vector of disturbances u_N. Therefore, the Ordinary Least Squares estimator will be inconsistent. Define $Z_N = (M_N y_N, X_N)$ and $\delta = (\lambda, \beta')'$. The MRSAR model can be rewritten as

$$y_N = Z_N \delta + u_N \tag{37}$$

In the cross-section spatial autoregressive model, Kelejian and Prucha (1998) suggested instruments like $H_N = (X_N, M_N X_N, M_N^2 X_N)$. Applying the instruments to this $\Omega_u^{-1/2}$ transformed panel autoregressive spatial model, we get the random effects spatial two-stage least square estimator (RE-S2SLS) of δ given by

$$\widehat{\delta}_{RE-2SLS} = \left[Z_N' \Omega_u^{-1} H_N \left(H_N' \Omega_u^{-1} H_N\right)^{-1} H_N' \Omega_u^{-1} Z_N\right]^{-1} Z_N' \Omega_u^{-1} H_N \left(H_N' \Omega_u^{-1} H_N\right)^{-1} H_N' \Omega_u^{-1} y_N \tag{38}$$

Estimates of ρ_1, ρ_2, σ_μ^2 and σ_ν^2 can be obtained from Equation (18) and (26) using $\widehat{u}_N = y_N - Z_N \widehat{\delta}_{2SLS}$, where $\widehat{\delta}_{2SLS} = \left[Z_N' H_N \left(H_N' H_N\right)^{-1} H_N' Z_N\right]^{-1} Z_N' H_N \left(H_N' H_N\right)^{-1} H_N' y_N$. With the GM estimators of $\tilde{\rho}_1$, $\tilde{\rho}_2$, $\tilde{\sigma}_\mu^2$ and $\tilde{\sigma}_\nu^2$, the corresponding feasible GLS estimator $\widehat{\delta}_{RE-F2SLS}$ is given by

$$\widehat{\delta}_{RE-F2SLS} = \left[Z_N' \tilde{\Omega}_u^{-1} H_N \left(H_N' \tilde{\Omega}_u^{-1} H_N\right)^{-1} H_N' \tilde{\Omega}_u^{-1} Z_N\right]^{-1} Z_N' \tilde{\Omega}_u^{-1} H_N \left(H_N' \tilde{\Omega}_u^{-1} H_N\right)^{-1} H_N' \tilde{\Omega}_u^{-1} y_N \tag{39}$$

where $\tilde{\Omega}_u^{-1}$ is defined in Equation (31).

Assumption 6 *The elements of X are bounded uniformly in absolute value. Furthermore, the limit* $\Sigma_0 = \lim_{N\to\infty} \frac{1}{NT} H_N' \left\{ \bar{J}_T \otimes \left[T\sigma_\mu^2 (A'A)^{-1} + \sigma_\nu^2 (B'B)^{-1}\right]^{-1} \right\} H_N$, $\Sigma_1 = \lim_{N\to\infty} \frac{1}{NT} H_N' \left[E_T \otimes (B'B)\right] H_N$, $\Gamma_0 = \lim_{N\to\infty} \frac{1}{NT} H_N' \left\{ \bar{J}_T \otimes \left[T\sigma_\mu^2 (A'A)^{-1} + \sigma_\nu^2 (B'B)^{-1}\right]^{-1} \right\} Z_N$ *and* $\Gamma_1 = \lim_{N\to\infty} \frac{1}{NT} H_N' \left[E_T \otimes (B'B)\right] Z_N$ *are finite and nonsingular.*

The theorem below establishes consistency and asymptotic normality of the feasible GLS estimators. The proof of the theorem is given in the appendix.

Theorem 4 *Under Assumptions 1, 2,4, 5 and 6, we have* $\sqrt{NT}\left(\widehat{\delta}_{RE-F2SLS} - \delta\right) \xrightarrow{d} N\left(0, \Gamma' \Sigma^{-1} \Gamma\right)$ *as* $N \to \infty$, *where* $\Sigma = \Sigma_0 + \sigma_\nu^{-2} \Sigma_1$ *and* $\Gamma = \Gamma_0 + \sigma_\nu^{-2} \Gamma_1$.

Premultiplying equation (36) by the within transformation $Q = E_T \otimes I_N$, one obtains

$$Q y_N = Q Z_N \delta + Q u_{2N}. \tag{40}$$

A further Cochrane-Orcutt type transformation $I_T \otimes B$ yields:

$$(E_T \otimes B)\, y_N = (E_T \otimes B)\, Z_N \delta + Q \nu_N. \tag{41}$$

Applying the instruments $(E_T \otimes B) H_N$, we get the fixed effects spatial two-stage least square estimator (FE-S2SLS) of δ given by

$$\begin{aligned}\widehat{\delta}_{FE-2SLS} &= \left\{ Z_N' \left[E_T \otimes (B'B)\right] H_N \left(H_N' \left[E_T \otimes (B'B)\right] H_N\right)^{-1} H_N' \left[E_T \otimes (B'B)\right] Z_N \right\}^{-1} \\ & \quad Z_N' \left[E_T \otimes (B'B)\right] H_N \left(H_N' \left[E_T \otimes (B'B)\right] H_N\right)^{-1} H_N' \left[E_T \otimes (B'B)\right] y_N \end{aligned} \tag{42}$$

With the GM estimators of $\tilde{\rho}_2$ and $\tilde{\sigma}_\nu^2$, the corresponding feasible GLS estimator $\widehat{\delta}_{FE-F2SLS}$ is given by

$$\begin{aligned}\widehat{\delta}_{FE-F2SLS} &= \left\{ Z_N' \left[E_T \otimes \left(\tilde{B}'\tilde{B}\right)\right] H_N \left(H_N' \left[E_T \otimes \left(\tilde{B}'\tilde{B}\right)\right] H_N\right)^{-1} H_N' \left[E_T \otimes \left(\tilde{B}'\tilde{B}\right)\right] Z_N \right\}^{-1} \\ & \quad Z_N' \left[E_T \otimes \left(\tilde{B}'\tilde{B}\right)\right] H_N \left(H_N' \left[E_T \otimes \left(\tilde{B}'\tilde{B}\right)\right] H_N\right)^{-1} H_N' \left[E_T \otimes \left(\tilde{B}'\tilde{B}\right)\right] y_N \end{aligned} \tag{43}$$

The theorem below establishes consistency and asymptotic normality of the feasible FE-2SLS estimators. The proof of the theorem is given in the appendix.

Theorem 5 *Under Assumptions 1, 2,4, 5 and 6, we have* $\sqrt{NT}\left(\widehat{\delta}_{RE-F2SLS} - \delta\right) \xrightarrow{d} N\left(0, \sigma_\nu^2 \Gamma_1' \Sigma_1^{-1} \Gamma_1\right)$ *as* $N \to \infty$.

A Hausman test can be similiarly derived by replacing $q = \widehat{\delta}_{FE-2SLS} - \widehat{\delta}_{RE-2SLS}$ in section 2.3. Under the null hypothesis $H_0 : E\left(u_{1i,N}|X_{it,N}\right) = 0$, the Hausman test $m = q'\left[var\left(q\right)\right]^{-1} q$ has a limiting χ^2 distribution with degrees of freedom equal to the rank of $var(q)$.

4 Monte Carlo Simulation

This section performs some Monte Carlo experiments to study the finite sample performance of the proposed within-GLS estimator and the corresponding Hausman test. Following Baltagi, Egger and Pfaffermayr (2012), we consider the following model

$$y_{it} = \alpha + \beta x_{it} + u_{it}, \quad i = 1, \ldots, N \text{ and } t = 1, \ldots, T, \tag{44}$$

where $\alpha = 5$ and $\beta = 0.5$. x_{it} is generated by $x_{it} = \zeta_i + z_{it}$ with $\zeta_i \overset{iid}{\sim} U\left[-7.5, 7.5\right]$ and $z_{it} \overset{iid}{\sim} U\left[-5, 5\right]$. The individual specific effects are drawn from a normal distribution so that $\mu_i \overset{iid}{\sim} N\left(0, 20\theta\right)$. For the remainder error, we assume $\nu_{it} \overset{iid}{\sim} N\left(0, 20\left(1-\theta\right)\right)$ with $0 < \theta < 1$. $\theta = \frac{\sigma_\mu^2}{\sigma_\mu^2 + \sigma_\nu^2}$ is the proportion of the total variance due to the heterogeneity of the individual-specific effects. This implies that $\sigma_\mu^2 + \sigma_\nu^2 = 20$. We construct the spatial weights matrix such that its i-th row has non-zero elements in positions $i+1$ and $i-1$. Therefore, the i-th element of u is directly related to the one immediately before it and the one immediately after it.

This matrix is defined in a circular world so that the non-zero elements in rows 1 and N are, respectively, in positions $(2, N)$ and $(1, N-1)$. This matrix is row normalized so that all of its non-zero elements are equal to 1/2. As in Kelejian and Prucha (1999), this weighting matrix is referred as "1 ahead and 1 behind". ρ_1 and ρ_2 vary over the set $\{-0.8, -0.5, -0.2, 0, 0.2, 0.5, 0.8\}$. We consider a panel with $N = 100$ regions and $T = 5$ time periods. We also considered the case of $N = 49$ regions and $T = 10$. For each experiment, we perform 10,000 replications. For each replication we estimate the model using: (i) Fixed-Effects ignoring spatial correlation; (ii) Random Effects ignoring spatial correlation; (iii) Within-GLS for fixed effects with spatial correlation; (iv) KKP random effects with spatial correlation; (v) Anselin random effects with spatial correlation; (vi) General random effects with spatial correlation; and (vii) True GLS.

Table 1 reports the relative root mean squared error (RMSE) of each estimator of β with respect to true GLS for various values of ρ_1 and ρ_2 fixing $\sigma_\mu^2 = 10$ and $\sigma_\nu^2 = 10$, i.e., $\theta = 0.5$; and $N = 100$ and $T = 5$. Several conclusions emerge from this Table. Not surprisingly, true GLS is the most efficient estimator in terms of root mean squared error. When the true model is spatial RE, KKP or Anselin, the 'correct' feasible GLS estimator performs best and is the closest in RMSE to the true GLS. Within-GLS estimator performs much better than standard FE which ignores the spatial correlation. For example, for $\rho_1 = \rho_2 = 0.8$, their relative MSE with respect to true GLS is 2.356 and 1.232, respectively. Both estimators perform much worse than any feasible spatial RE-GLS estimator. There is also much gain from performing RE-GLS allowing for spatial correlation than ignoring it. For $\rho_1 = \rho_2 = 0.8$, the relative MSE of RE ignoring spatial correlation is 1.843 compared to 1.003 for KKP. For $\rho_1 = 0.8$ and $\rho_2 = 0.5$, the relative MSE of RE is 1.231 compared to 1.004 for the General spatial RE estimator of Baltagi, Egger and Pfaffermayr (2012). The gain in efficiency from using the correct feasible GLS for our experiments is not that large, for $\rho_1 = 0.8$ and $\rho_2 = -0.2$, the relative MSE of KKP is 1.171, compared to 1.129 for Anselin and 1.011 for General spatial RE estimator. Tables 2 and 3 repeat these experiments only varying the proportion of heterogeneity among the individuals from $\theta = 0.5$ in Table 1 to $\theta = 0.25$ in Table 2 and $\theta = 0.75$ in Table 3. By and large, the results are the same, but the magnitudes of the relative MSE are different. Estimators that handle heterogeneity only perform better as the degree of heterogeneity increases, and worse when the degree of heterogeneity decreases. We ran more experiments for $N = 49$ and $T = 10$, the results are reported in Tables 4, 5 and 6, for $\theta = 0.5$, 0.25 and 0.75, respectively. The flavor is the same, but the magnitudes of the relative MSE are different.

Table 7 reports the empirical size (at the 5% level) of the spatial Hausman test for various values of ρ_1 and ρ_2 fixing $\sigma_\mu^2 = 10$ and $\sigma_\nu^2 = 10$, i.e., $\theta = 0.5$; and $N = 100$ and $T = 5$ corresponding to the results reported in Table 1. This is based on the contrast of the KKP random effects GM estimator and the Within-GLS in the first column, and the contrast of the Anselin random effects GM estimator and the Within-GLS in the second column and the contrast of the General spatial RE estimator and the Within-GLS in the third column. We can see that the spatial Hausman test based on KKP is over-sized if the true model is an Anselin random effects model with $\rho_1 = 0$ and $\rho_2 = -0.8$ (or 0.8). In both cases, the test is oversized yielding a probability of type I error of 0.094 (or 0.090) when it should be 0.05. This oversizing of the test gets worse $\rho_1 = -0.8$ and $\rho_2 = 0.8$. The Hausman test based on KKP yields a type I error of 0.121. Also, when $\rho_1 = 0.8$ and $\rho_2 = -0.8$. In this case, The Hausman test based on KKP yields a type I error of 0.211. In contrast, the spatial Hausman test based on the Anselin random effects GM estimator is under-sized if the true model is KKP with $\rho_1 = \rho_2 = -0.8$ (or 0.8). In both cases, the test is undersized yielding a probability of type I error of 0.024 (or 0.023) when it should be 0.05. However, this undersizing does not get worse, and the Hausman test based on Anselin performs well when the true model is a general spatial RE estimator with size varying between 0.037 and 0.074. The spatial Hausman test based on the general spatial RE estimator performs well with size between 0.047 and 0.097. Tables 8 and 9 repeat the spatial Hausman test results only varying the proportion of heterogeneity among the individuals from $\theta = 0.5$ in Table 7 to $\theta = 0.25$ in Table 8 and $\theta = 0.75$ in Table 9. By and large, the results are the same, but the size magnitudes are different. Tables 10, 11 and 12 report the empirical size (at the 5% level) of the spatial Hausman test for $N = 49$ and $T = 10$, corresponding to the results reported in Tables 4, 5 and 6. For this relatively *smaller* N, *larger* T case, the KKP over-sizing of the Hausman test can be as large as 0.307 in Table 10, 0.278 in Table 11 and 0.317 in Table 12, all occurring when $\rho_1 = 0.8$ and $\rho_2 = -0.8$. For these values of ρ_1 and ρ_2, the corresponding over-sizing of the Hausman test based on the Anselin RE model is 0.117 in Table 10, 0.127 in Table 11, and 0.112 in Table 12, respectively. The corresponding over-sizing of the Hausman test based on the general spatial RE model is 0.088 in Table 10, 0.146 in Table 11, and 0.07 in Table 12, respectively.

5 Conclusion

This paper suggests simple RE-GLS and FE-GLS estimators for the generalized spatial error component model considered by Baltagi, Egger and Pfaffermayr (2012). These estimators apply the usual fixed effects transformation and the GM method of KKP and Mutl and Pfaffermayr (2011) and are easy to compute. We derive the asymptotic distribution of these estimators and investigate their performance using Monte Carlo experiments. Our results show that this Within-GLS estimator performs much better than standard FE which ignores the spatial correlation. There is also much gain from performing RE-GLS allowing for spatial correlation than the standard RE estimator which ignores the spatial correlation. Not surprisingly, the 'correct' feasible GLS estimator performs best and is the closest in RMSE to the true GLS. We also investigate the performance of the spatial Hausman test based on the contrast involving this Within-GLS estimator and the KKP, Anselin and General variants of the random effects spatial model. We show that this Hausman test can be misleading under misspecification.

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Appendix

A Proof of Theorem 1

Proof. First, let us show $\Gamma_N^1 = O(1)$, $\gamma_N^1 = O(1)$ and

$$G_N^{1*} - \Gamma_N^1 \xrightarrow{p} 0 \text{ and } g_N^{1*} - \gamma_N^1 \xrightarrow{p} 0 \text{ as } N \to \infty. \tag{45}$$

Let $\xi_N = \left(\xi_{1,N}, \ldots, \xi_{(T+1)N,N}\right)' = (\mu_N', \ldots, v_N')'$ so that $u_N = Z_\mu u_{1N} + u_{2N} = \left(\iota_T \otimes A^{-1}\right)\mu_N + \left(I_T \otimes B^{-1}\right)\nu_N = \left[\left(\iota_T \otimes A^{-1}\right), \left(I_T \otimes B^{-1}\right)\right]\xi_N$ and

$$\begin{aligned}
\varphi_{kl,N} &= \frac{1}{NT} u_N' \left(S_T \otimes W_N^{k\prime} W_N^l\right) u_N \\
&= \frac{1}{NT} \xi_N' \left[\left(\iota_T \otimes A^{-1}\right), \left(I_T \otimes B^{-1}\right)\right]' \left(S_T \otimes W_N^{k\prime} W_N^l\right) \left[\left(\iota_T \otimes A^{-1}\right), \left(I_T \otimes B^{-1}\right)\right] \xi_N \\
&= \frac{1}{NT} \xi_N' C_N \xi_N,
\end{aligned}$$

where $C_N = \begin{pmatrix} T & \iota_T' \\ \iota_T & S_T \end{pmatrix} \otimes \begin{pmatrix} A^{-1\prime} W_N^{k\prime} W_N^l A^{-1} & A^{-1\prime} W_N^{k\prime} W_N^l B^{-1} \\ B^{-1\prime} W_N^{k\prime} W_N^l A^{-1} & B^{-1\prime} W_N^{k\prime} W_N^l B^{-1} \end{pmatrix}$ using $\iota_T' S_T \iota_T = T$ and $S_T \iota_T = \iota_T$. Note the first matrix of the Kronecker product in C_N does not depend on N. The row and column sums of the second matrix of the Kronecker product in C_N are bounded uniformly in absolute value by Remark A2(b) in Kapoor, Kelejian and Prucha (2007). By the proof of Lemma A1 in Kapoor, Kelejian and Prucha (2007), we have $E\left(\varphi_{kl,N}\right) = O(1)$ and $\varphi_{kl,N} - E\left(\varphi_{kl,N}\right) \xrightarrow{p} 0$. Notice that $\varphi_{kl,N}$ are elements of G_N^{1*} and g_N^{1*}. $E\left(\varphi_{kl,N}\right)$ are elements of Γ_N^1 and γ_N^1, Equation (45) is proved.

Second, let us show

$$G_N^1 - G_N^{1*} \xrightarrow{p} 0 \text{ and } g_N^1 - g_N^{1*} \xrightarrow{p} 0 \text{ as } N \to \infty, \tag{46}$$

provided $\tilde{\beta}_N \xrightarrow{p} \beta$ as $N \to \infty$. Note that the elements of G_N^{1*} and g_N^{1*} are $\varphi_{kl,N} = \frac{1}{NT} u_N' \left(S_T \otimes W_N^{k\prime} W_N^l\right) u_N$. Since the row and column sums of the elements of W_N are uniformly bounded in absolute value by Assumption 4, it follows that the row and columns sums of the matrices $S_T \otimes W_N^{k\prime} W_N^l$ also have that property. Define $\tilde{\varphi}_{kl,N} = \frac{1}{NT} \tilde{u}_N' \left(S_T \otimes W_N^{k\prime} W_N^l\right) \tilde{u}_N$, which are the elements of G_N^1 and g_N^1. By the proof of Lemma A3 in Kapoor, Kelejian and Prucha (2007), we have $\tilde{\varphi}_{kl,N} - \varphi_{kl,N} \xrightarrow{p} 0$ as $N \to \infty$. This completes the proof of Equation (46).

Third, Let $\theta = \left(\rho_1, \sigma_\mu^2\right)$ and $\underline{\theta} = \left(\underline{\rho}_1, \underline{\sigma}_\mu^2\right)$. The objective function of the nonlinear least squares estimator and its corresponding nonstochastic counterpart are given by

$$\begin{aligned}
R_N^1(\underline{\theta}) &= \left[G_N^1 \left[\rho_1, \rho_1^2, \sigma_\mu^2\right]' - g_N^1\right]' \left[G_N^1 \left[\rho_1, \rho_1^2, \sigma_\mu^2\right]' - g_N^1\right], \\
\bar{R}_N^1(\underline{\theta}) &= \left[\Gamma_N^1 \left[\rho_1, \rho_1^2, \sigma_\mu^2\right]' - \gamma_N^1\right]' \left[\Gamma_N^1 \left[\rho_1, \rho_1^2, \sigma_\mu^2\right]' - \gamma_N^1\right],
\end{aligned}$$

respectively. Under Assumption 5, Equation (45) and (46), from the proof of Theorem 1 in Kapoor, Kelejian and Prucha (2007), we have

$$\sup_{\underline{\rho_1}\in[-a_1,a_1],\underline{\sigma}_\mu^2\in[0,b_1]} \left|R_N^1\left(\underline{\theta}\right)-\bar{R}_N^1\left(\underline{\theta}\right)\right| \xrightarrow{p} 0$$

as $N\to\infty$. The consistency of $\tilde{\rho}_1$ and $\tilde{\sigma}_\mu^2$ follows directly from Lemma 3.1 in Pötscher and Prucha (1997). ∎

B Proof of Theorem 2

Proof. First, using the central limit theorem and the law of large numbers, we have

$$\sqrt{NT}\left(\hat{\beta}_{GLS}-\beta\right)=\left\{\frac{1}{NT}X_N'\Omega_u^{-1}X_N\right\}^{-1}\frac{1}{\sqrt{NT}}X_N'\Omega_u^{-1}u_N \xrightarrow{d} N\left(0,\left(\Psi_0+\sigma_\nu^{-2}\Psi_1\right)^{-1}\right),$$

as $N\to\infty$ since

$$\begin{aligned}
&\frac{1}{NT}X_N'\Omega_u^{-1}X_N\\
=\;&\frac{1}{NT}X_N'\left\{\bar{J}_T\otimes\left[T\sigma_\mu^2\left(A'A\right)^{-1}+\sigma_\nu^2\left(B'B\right)^{-1}\right]^{-1}\right\}X_N+\sigma_\nu^{-2}\frac{1}{NT}X_N'\left[E_T\otimes\left(B'B\right)\right]X_N\\
&\xrightarrow{p}\Psi_0+\sigma_\nu^{-2}\Psi_1
\end{aligned}$$

and

$$\frac{1}{\sqrt{NT}}X_N'\Omega_u^{-1}u_N \xrightarrow{d} N\left(0,\lim_{N\to\infty}\frac{1}{NT}X_N'\Omega_u^{-1}X_N\right)=N\left(0,\Psi_0+\sigma_\nu^{-2}\Psi_1\right)$$

using Assumption 3.

Second, we show that with consistent estimators $\tilde{\rho}_1$, $\tilde{\rho}_2$, $\tilde{\sigma}_\mu^2$ and $\tilde{\sigma}_\nu^2$, by Lemma 4 of Baltagi, Egger and Pfaffermayr (2012), we have

$$\frac{1}{NT}X_N'\tilde{\Omega}_u^{-1}X_N-\frac{1}{NT}X_N'\Omega_u^{-1}X_N \xrightarrow{p} 0$$

and

$$\frac{1}{\sqrt{NT}}X_N'\tilde{\Omega}_u^{-1}u_N-\frac{1}{\sqrt{NT}}X_N'\Omega_u^{-1}u_N \xrightarrow{p} 0.$$

Therefore, we have $\sqrt{NT}\left(\hat{\beta}_{FGLS}-\hat{\beta}_{GLS}\right)\xrightarrow{p}0$ as $N\to\infty$. This proves the Theorem. ∎

C Proof of Theorem 3

Proof. First, using the central limit theorem and the law of large numbers, we have

$$\sqrt{NT}\left(\hat{\beta}_{Within-GLS}-\beta\right)=\left\{\frac{1}{NT}X_N'\left[E_T\otimes\left(B'B\right)\right]X_N\right\}^{-1}\frac{1}{\sqrt{NT}}X_N'\left[E_T\otimes B'\right]\nu_N \xrightarrow{d} N\left(0,\sigma_\nu^2\Psi_1^{-1}\right),$$

as $N \to \infty$ since

$$\frac{1}{NT} X_N' \left[E_T \otimes (B'B)\right] X_N \xrightarrow{p} \Psi_1$$

and

$$\frac{1}{\sqrt{NT}} X_N' \left[E_T \otimes B'\right] \nu_N \xrightarrow{d} N\left(0, \sigma_\nu^2 \Psi_1\right)$$

using Assumption 3.

Second, we show that with consistent estimators $\tilde{\rho}_2$ and $\tilde{\sigma}_\nu^2$, we have $\sqrt{NT}\left(\hat{\beta}_{Within-FGLS} - \hat{\beta}_{Within-GLS}\right) \xrightarrow{p} 0$ as $N \to \infty$. To prove this part it suffices to show that

$$\Delta_1 = \frac{1}{NT} X_N' \left[E_T \otimes \left(\tilde{B}'\tilde{B}\right)\right] X_N - \frac{1}{NT} X_N' \left[E_T \otimes (B'B)\right] X_N \xrightarrow{p} 0 \tag{47}$$

and

$$\Delta_2 = \frac{1}{\sqrt{NT}} X_N' \left[E_T \otimes \tilde{B}'\right] \nu_N - \frac{1}{\sqrt{NT}} X_N' \left[E_T \otimes B'\right] \nu_N \xrightarrow{p} 0. \tag{48}$$

We first demonstrate (47). It is readily seen that

$$\begin{aligned}
\tilde{B}'\tilde{B} - B'B &= (I_N - \tilde{\rho}_2 W_N)' (I_N - \tilde{\rho}_2 W_N) - (I_N - \rho_2 W_N)' (I_N - \rho_2 W_N) \\
&= \left[I_N - \tilde{\rho}_2 (W_N' + W_N) + \tilde{\rho}_2^2 W_N' W_N\right] - \left[I_N - \rho_2 (W_N' + W_N) + \rho_2^2 W_N' W_N\right] \\
&= -(\tilde{\rho}_2 - \rho_2)(W_N' + W_N) + \left(\tilde{\rho}_2^2 - \rho_2^2\right) W_N' W_N,
\end{aligned}$$

hence $\Delta_1 = -(\tilde{\rho}_2 - \rho_2) \frac{1}{NT} X_N' \left[E_T \otimes (W_N' + W_N)\right] X_N + \left(\tilde{\rho}_2^2 - \rho_2^2\right) \frac{1}{NT} X_N' \left[E_T \otimes (W_N' W_N)\right] X_N$. In light of Assumption 4, by Remark A2 of Kapoor, Kelejian and Prucha (2007), the elements of the matrices $\frac{1}{NT} X_N' \left[E_T \otimes (W_N' + W_N)\right] X_N$ and $\frac{1}{NT} X_N' \left[E_T \otimes (W_N' W_N)\right] X_N$ are uniformly bounded in absolute value. Since $\tilde{\rho}_2$ is a consistent estimator of ρ_2, it follows that $\Delta_1 \xrightarrow{p} 0$ as $N \to \infty$. We next demonstrate (48).

$$\tilde{B}' - B' = (I_N - \tilde{\rho}_2 W_N)' - (I_N - \rho_2 W_N)' = -(\tilde{\rho}_2 - \rho_2) W_N'.$$

Hence, $\Delta_2 = -(\tilde{\rho}_2 - \rho_2) \frac{1}{\sqrt{NT}} X_N' \left[E_T \otimes W_N'\right] \nu_N$. Since X_N and W_N are nonstochastic matrices, the expected value of the vector $\frac{1}{\sqrt{NT}} X_N' \left[E_T \otimes W_N'\right] \nu_N$ is a vector of zeros and its variance covariance matrix is $\sigma_\nu^2 \frac{1}{NT} X_N' \left[E_T \otimes (W_N' W_N)\right] X_N$. Hence, $\frac{1}{NT} X_N' \left[E_T \otimes (W_N' W_N)\right] X_N$ is uniformly bounded in absolute value. Therefore $\frac{1}{\sqrt{NT}} X_N' \left[E_T \otimes W_N'\right] \nu_N = O_p(1)$, and $\Delta_2 \xrightarrow{p} 0$ as $N \to \infty$, since $\tilde{\rho}_2$ is a consistent estimator of ρ_2. This proves the Theorem. ■

D Proof of Theorem 4

Proof. First, using the central limit theorem and the law of large numbers, we have

$$\begin{aligned}\sqrt{NT}\left(\widehat{\delta}_{RE-2SLS}-\delta\right) &= \left[\frac{Z_N'\Omega_u^{-1}H_N}{NT}\left(\frac{H_N'\Omega_u^{-1}H_N}{NT}\right)^{-1}\frac{H_N'\Omega_u^{-1}Z_N}{NT}\right]^{-1}\frac{Z_N'\Omega_u^{-1}H_N}{NT}\left(\frac{H_N'\Omega_u^{-1}H_N}{NT}\right)^{-1}\frac{H_N'\Omega_u^{-1}u_N}{\sqrt{NT}} \\ &\xrightarrow{d} N\left(0,\left(\Gamma'\Sigma^{-1}\Gamma\right)^{-1}\right),\end{aligned}$$

as $N\rightarrow\infty$ since

$$\begin{aligned}\frac{1}{NT}H_N'\Omega_u^{-1}H_N &= \frac{1}{NT}H_N'\left\{\bar{J}_T\otimes\left[T\sigma_\mu^2\left(A'A\right)^{-1}+\sigma_\nu^2\left(B'B\right)^{-1}\right]^{-1}\right\}H_N+\sigma_\nu^{-2}\frac{1}{NT}H_N'\left[E_T\otimes\left(B'B\right)\right]H_N \\ &\xrightarrow{p}\Sigma\equiv\Sigma_0+\sigma_\nu^{-2}\Sigma_1\end{aligned}$$

$$\begin{aligned}\frac{1}{NT}H_N'\Omega_u^{-1}Z_N &= \frac{1}{NT}H_N'\left\{\bar{J}_T\otimes\left[T\sigma_\mu^2\left(A'A\right)^{-1}+\sigma_\nu^2\left(B'B\right)^{-1}\right]^{-1}\right\}Z_N+\sigma_\nu^{-2}\frac{1}{NT}H_N'\left[E_T\otimes\left(B'B\right)\right]Z_N \\ &\xrightarrow{p}\Gamma\equiv\Gamma_0+\sigma_\nu^{-2}\Gamma_1\end{aligned}$$

and

$$\frac{1}{\sqrt{NT}}H_N'\Omega_u^{-1}u_N\xrightarrow{d}N\left(0,\lim_{N\rightarrow\infty}\frac{1}{NT}H_N'\Omega_u^{-1}H_N\right)=N\left(0,\Sigma\right)$$

using Assumption 6.

Second, it is easy to show that $\widehat{\delta}_{2SLS}\xrightarrow{p}\delta$ as $N\rightarrow\infty$. From Kapoor et al. (2007) and the proof of Theorem 1, we know the GM estimators of $\tilde{\rho}_1$, $\tilde{\rho}_2$, $\tilde{\sigma}_\mu^2$ and $\tilde{\sigma}_\nu^2$ are consistent. Similiar to Lemma 4 of Baltagi, Egger and Pfaffermayr (2012), we have

$$\frac{1}{NT}H_N'\tilde{\Omega}_u^{-1}H_N-\frac{1}{NT}H_N'\Omega_u^{-1}H_N\xrightarrow{p}0$$

$$\frac{1}{NT}H_N'\tilde{\Omega}_u^{-1}Z_N-\frac{1}{NT}H_N'\Omega_u^{-1}Z_N\xrightarrow{p}0$$

and

$$\frac{1}{\sqrt{NT}}H_N'\tilde{\Omega}_u^{-1}u_N-\frac{1}{\sqrt{NT}}H_N'\Omega_u^{-1}u_N\xrightarrow{p}0.$$

Therefore, we have $\sqrt{NT}\left(\widehat{\delta}_{RE-F2SLS}-\widehat{\delta}_{RE-2SLS}\right)\xrightarrow{p}0$ as $N\rightarrow\infty$. This proves the Theorem. ∎

E Proof of Theorem 5

Proof. First, using the central limit theorem and the law of large numbers, we have

$$\begin{aligned}\sqrt{NT}\left(\widehat{\delta}_{FE-2SLS}-\delta\right) &= \left\{\frac{Z_N'\left[E_T\otimes(B'B)\right]H_N}{NT}\left(\frac{H_N'\left[E_T\otimes(B'B)\right]H_N}{NT}\right)^{-1}\frac{H_N'\left[E_T\otimes(B'B)\right]Z_N}{NT}\right\}^{-1} \\ &\quad \frac{Z_N'\left[E_T\otimes(B'B)\right]H_N}{NT}\left(\frac{H_N'\left[E_T\otimes(B'B)\right]H_N}{NT}\right)^{-1}\frac{H_N'\left[E_T\otimes B'\right]v_N}{\sqrt{NT}} \\ &\xrightarrow{d} N\left(0,\sigma_\nu^2\left(\Gamma_1'\Sigma_1^{-1}\Gamma_1\right)^{-1}\right),\end{aligned}$$

as $N\to\infty$ since

$$\begin{aligned}\frac{1}{NT}H_N'\left[E_T\otimes(B'B)\right]H_N &\xrightarrow{p}\Sigma_1 \\ \frac{1}{NT}H_N'\left[E_T\otimes(B'B)\right]Z_N &\xrightarrow{p}\Gamma_1\end{aligned}$$

and

$$\frac{1}{\sqrt{NT}}H_N'\left[E_T\otimes B'\right]v_N\xrightarrow{d}N\left(0,\lim_{N\to\infty}\frac{1}{NT}H_N'\left[E_T\otimes(B'B)\right]H_N\right)=N\left(0,\sigma_\nu^2\Sigma_1\right)$$

using Assumption 6.

Second, similiar to the proof of Theorem 3, one can show that

$$\begin{aligned}\frac{1}{NT}H_N'\left[E_T\otimes\left(\tilde{B}'\tilde{B}\right)\right]H_N-\frac{1}{NT}H_N'\left[E_T\otimes(B'B)\right]H_N &\xrightarrow{p}0 \\ \frac{1}{NT}H_N'\left[E_T\otimes\left(\tilde{B}'\tilde{B}\right)\right]Z_N-\frac{1}{NT}H_N'\left[E_T\otimes(B'B)\right]Z_N &\xrightarrow{p}0\end{aligned}$$

and

$$\frac{1}{\sqrt{NT}}H_N'\left[E_T\otimes\tilde{B}'\right]v_N-\frac{1}{\sqrt{NT}}H_N'\left[E_T\otimes B'\right]v_N\xrightarrow{p}0.$$

Therefore, we have $\sqrt{NT}\left(\widehat{\delta}_{FE-F2SLS}-\widehat{\delta}_{FE-2SLS}\right)\xrightarrow{p}0$ as $N\to\infty$. This proves the Theorem. ■

Table 1: Relative Efficiencies of Spatial Panel Data Estimators (N = 100, T= 5)

	ρ_1	ρ_2	FE	RE	Within-GLS	KKP	Anselin	General
RE	0	0	1.226	1.006	1.232	1.005	1.004	1.015
KKP	-0.8	-0.8	2.179	1.768	1.190	1.000	1.080	1.007
	-0.5	-0.5	1.434	1.179	1.202	1.001	1.033	1.006
	-0.2	-0.2	1.214	1.023	1.190	1.008	1.009	1.012
	0.2	0.2	1.242	1.031	1.194	1.008	1.020	1.014
	0.5	0.5	1.448	1.199	1.213	0.999	1.034	1.009
	0.8	0.8	2.356	1.843	1.232	1.003	1.091	1.003
Anselin	0	-0.8	2.108	1.489	1.149	1.065	1.012	1.013
	0	-0.5	1.425	1.133	1.194	1.035	1.005	1.010
	0	-0.2	1.217	1.021	1.193	1.017	1.007	1.016
	0	0.2	1.230	1.021	1.183	1.003	1.011	1.014
	0	0.5	1.412	1.130	1.183	1.023	1.001	1.004
	0	0.8	2.266	1.489	1.184	1.039	1.005	1.012
General	-0.8	-0.5	1.456	1.273	1.220	1.024	1.109	1.015
	-0.8	-0.2	1.222	1.105	1.197	1.061	1.090	1.007
	-0.8	0	1.207	1.082	1.209	1.084	1.086	1.002
	-0.8	0.2	1.212	1.095	1.167	1.104	1.074	1.014
	-0.8	0.5	1.387	1.233	1.159	1.136	1.074	1.014
	-0.8	0.8	2.187	1.709	1.143	1.131	1.072	1.063
	-0.5	-0.8	2.156	1.584	1.176	1.014	1.032	1.007
	-0.5	-0.2	1.214	1.050	1.190	1.011	1.033	1.015
	-0.5	0	1.216	1.042	1.220	1.045	1.042	1.010
	-0.5	0.2	1.211	1.029	1.165	1.042	1.023	1.013
	-0.5	0.5	1.398	1.161	1.170	1.085	1.023	1.013
	-0.5	0.8	2.226	1.543	1.162	1.094	1.031	1.027
	-0.2	-0.8	2.127	1.510	1.160	1.038	1.014	1.010
	-0.2	-0.5	1.432	1.148	1.200	1.020	1.014	1.013
	-0.2	0	1.223	1.010	1.228	1.010	1.011	1.016
	-0.2	0.2	1.227	1.020	1.181	1.018	1.012	1.014
	-0.2	0.5	1.408	1.136	1.179	1.050	1.008	1.005
	-0.2	0.8	2.254	1.508	1.177	1.056	1.012	1.021
	0.2	-0.8	2.087	1.481	1.137	1.084	1.011	1.015
	0.2	-0.5	1.426	1.134	1.195	1.054	1.004	1.015
	0.2	-0.2	1.214	1.026	1.190	1.030	1.009	1.011
	0.2	0	1.233	1.012	1.239	1.014	1.013	1.013
	0.2	0.5	1.426	1.148	1.194	1.011	1.013	1.002
	0.2	0.8	2.279	1.508	1.191	1.023	1.006	1.006
	0.5	-0.8	2.052	1.502	1.118	1.119	1.021	1.018
	0.5	-0.5	1.413	1.155	1.184	1.099	1.025	1.013
	0.5	-0.2	1.228	1.073	1.204	1.087	1.057	1.014
	0.5	0	1.245	1.055	1.251	1.056	1.059	1.012
	0.5	0.2	1.257	1.064	1.208	1.031	1.050	1.003
	0.5	0.8	2.304	1.595	1.205	0.999	1.026	1.000
	0.8	-0.8	2.036	1.653	1.113	1.151	1.063	1.038
	0.8	-0.5	1.418	1.244	1.189	1.164	1.085	1.011
	0.8	-0.2	1.238	1.152	1.214	1.171	1.129	1.011
	0.8	0	1.258	1.130	1.263	1.141	1.143	1.008
	0.8	0.2	1.280	1.171	1.232	1.099	1.150	1.008
	0.8	0.5	1.469	1.308	1.231	1.031	1.106	1.004

Notes: (a) Relative mean square error with respect to the true GLS. (b) 10,000 replications. (c) $\sigma_\mu^2 = 10$ and $\sigma_v^2 = 10$, i.e., $\theta = 0.5$.

Table 2: Relative Efficiencies of Spatial Panel Data Estimators (N = 100, T= 5)

	ρ_1	ρ_2	FE	RE	Within-GLS	KKP	Anselin	General
RE	0	0	1.468	1.041	1.474	1.001	1.001	1.010
KKP	-0.8	-0.8	2.567	1.811	1.402	0.999	1.124	1.005
	-0.5	-0.5	1.683	1.200	1.411	1.007	1.022	1.012
	-0.2	-0.2	1.439	1.046	1.409	1.009	1.014	1.017
	0.2	0.2	1.474	1.063	1.419	1.002	1.014	1.008
	0.5	0.5	1.730	1.246	1.448	1.007	1.040	1.002
	0.8	0.8	2.867	1.966	1.500	1.013	1.129	1.018
Anselin	0	-0.8	2.398	1.503	1.308	1.042	1.032	1.017
	0	-0.5	1.656	1.134	1.389	1.032	1.006	1.012
	0	-0.2	1.432	1.035	1.402	1.002	1.007	1.011
	0	0.2	1.461	1.053	1.406	1.003	1.003	1.019
	0	0.5	1.673	1.175	1.400	1.013	1.003	1.006
	0	0.8	2.654	1.598	1.386	1.042	1.041	1.020
General	-0.8	-0.5	1.694	1.315	1.421	1.017	1.113	1.005
	-0.8	-0.2	1.434	1.160	1.404	1.078	1.130	1.013
	-0.8	0	1.418	1.140	1.422	1.126	1.124	1.008
	-0.8	0.2	1.399	1.127	1.346	1.143	1.097	1.009
	-0.8	0.5	1.573	1.243	1.317	1.194	1.101	1.043
	-0.8	0.8	2.433	1.657	1.270	1.200	1.087	1.182
	-0.5	-0.8	2.513	1.634	1.372	1.008	1.052	1.010
	-0.5	-0.2	1.435	1.072	1.405	1.017	1.040	1.013
	-0.5	0	1.435	1.062	1.440	1.027	1.026	1.011
	-0.5	0.2	1.417	1.045	1.364	1.049	1.030	1.011
	-0.5	0.5	1.606	1.159	1.344	1.088	1.024	1.005
	-0.5	0.8	2.515	1.563	1.313	1.127	1.068	1.053
	-0.2	-0.8	2.448	1.547	1.336	1.018	1.037	1.015
	-0.2	-0.5	1.663	1.152	1.396	1.012	1.001	1.009
	-0.2	0	1.453	1.037	1.460	1.003	1.000	1.009
	-0.2	0.2	1.445	1.039	1.391	1.015	1.010	1.018
	-0.2	0.5	1.646	1.157	1.378	1.033	1.008	1.002
	-0.2	0.8	2.601	1.569	1.358	1.064	1.048	1.032
	0.2	-0.8	2.363	1.486	1.289	1.077	1.041	1.023
	0.2	-0.5	1.658	1.140	1.390	1.074	1.029	1.030
	0.2	-0.2	1.422	1.035	1.393	1.016	1.002	1.006
	0.2	0	1.474	1.049	1.480	1.013	1.012	1.010
	0.2	0.5	1.700	1.199	1.423	1.011	1.012	1.004
	0.2	0.8	2.702	1.628	1.411	1.028	1.036	1.010
	0.5	-0.8	2.307	1.489	1.258	1.150	1.064	1.047
	0.5	-0.5	1.633	1.153	1.369	1.142	1.058	1.032
	0.5	-0.2	1.416	1.064	1.386	1.072	1.040	1.006
	0.5	0	1.481	1.090	1.487	1.060	1.054	1.013
	0.5	0.2	1.486	1.105	1.430	1.025	1.047	1.008
	0.5	0.8	2.791	1.732	1.458	1.011	1.047	1.013
	0.8	-0.8	2.268	1.598	1.238	1.285	1.128	1.173
	0.8	-0.5	1.612	1.242	1.350	1.274	1.132	1.042
	0.8	-0.2	1.403	1.156	1.374	1.213	1.148	1.007
	0.8	0	1.473	1.176	1.479	1.169	1.170	1.013
	0.8	0.2	1.502	1.229	1.444	1.107	1.166	1.009
	0.8	0.5	1.744	1.368	1.461	1.021	1.119	0.996

Notes: (a) Relative mean square error with respect to the true GLS. (b) 10,000 replications. (c) $\sigma_\mu^2 = 5$ and $\sigma_v^2 = 15$, i.e., $\theta = 0.25$.

Table 3: Relative Efficiencies of Spatial Panel Data Estimators (N = 100, T= 5)

	ρ_1	ρ_2	FE	RE	Within-GLS	KKP	Anselin	General
RE	0	0	1.098	1.006	1.101	1.018	1.023	1.023
KKP	-0.8	-0.8	1.984	1.831	1.084	1.005	1.044	1.010
	-0.5	-0.5	1.299	1.195	1.089	1.005	1.034	1.006
	-0.2	-0.2	1.094	1.018	1.072	1.003	1.000	1.012
	0.2	0.2	1.107	1.023	1.065	1.002	1.000	1.003
	0.5	0.5	1.286	1.201	1.076	1.004	1.017	1.007
	0.8	0.8	2.066	1.881	1.083	1.004	1.043	1.008
Anselin	0	-0.8	1.937	1.629	1.056	1.032	1.002	1.005
	0	-0.5	1.294	1.178	1.086	1.019	1.004	0.999
	0	-0.2	1.094	1.016	1.072	1.013	1.000	1.006
	0	0.2	1.103	1.014	1.061	1.006	0.994	1.005
	0	0.5	1.279	1.166	1.071	1.024	1.012	1.016
	0	0.8	2.029	1.636	1.060	1.020	1.005	1.006
General	-0.8	-0.5	1.306	1.235	1.096	1.022	1.059	1.009
	-0.8	-0.2	1.099	1.058	1.075	1.032	1.043	1.008
	-0.8	0	1.088	1.032	1.090	1.042	1.042	1.006
	-0.8	0.2	1.104	1.064	1.064	1.056	1.042	1.001
	-0.8	0.5	1.280	1.225	1.067	1.066	1.050	1.010
	-0.8	0.8	2.018	1.814	1.055	1.059	1.033	1.014
	-0.5	-0.8	1.961	1.724	1.070	1.008	1.016	1.002
	-0.5	-0.2	1.092	1.026	1.070	1.002	1.014	1.006
	-0.5	0	1.089	1.014	1.090	1.024	1.023	1.010
	-0.5	0.2	1.099	1.026	1.057	1.020	1.010	1.002
	-0.5	0.5	1.274	1.186	1.066	1.056	1.027	1.005
	-0.5	0.8	2.013	1.692	1.051	1.047	1.017	1.007
	-0.2	-0.8	1.943	1.644	1.059	1.020	1.004	1.009
	-0.2	-0.5	1.298	1.180	1.089	1.010	1.016	1.010
	-0.2	0	1.094	1.005	1.097	1.027	1.023	1.021
	-0.2	0.2	1.101	1.018	1.060	1.009	1.003	1.002
	-0.2	0.5	1.279	1.173	1.070	1.041	1.011	1.015
	-0.2	0.8	2.017	1.641	1.054	1.025	1.004	1.004
	0.2	-0.8	1.931	1.633	1.053	1.048	1.000	1.008
	0.2	-0.5	1.294	1.172	1.086	1.029	1.003	1.009
	0.2	-0.2	1.097	1.024	1.076	1.021	1.008	1.002
	0.2	0	1.106	1.014	1.109	1.023	1.020	1.028
	0.2	0.5	1.283	1.176	1.074	1.017	1.013	1.011
	0.2	0.8	2.036	1.643	1.064	1.011	1.006	1.009
	0.5	-0.8	1.931	1.671	1.055	1.064	1.012	1.007
	0.5	-0.5	1.290	1.182	1.082	1.048	1.010	1.006
	0.5	-0.2	1.104	1.045	1.083	1.047	1.025	1.009
	0.5	0	1.107	1.025	1.111	1.033	1.034	1.019
	0.5	0.2	1.110	1.035	1.068	1.005	1.026	1.002
	0.5	0.8	2.039	1.709	1.066	0.995	1.006	0.999
	0.8	-0.8	1.938	1.779	1.060	1.086	1.038	1.011
	0.8	-0.5	1.293	1.214	1.082	1.070	1.034	1.007
	0.8	-0.2	1.109	1.073	1.090	1.082	1.060	1.006
	0.8	0	1.115	1.070	1.117	1.080	1.081	1.018
	0.8	0.2	1.118	1.076	1.077	1.048	1.063	1.002
	0.8	0.5	1.295	1.249	1.081	1.009	1.047	1.007

Notes: (a) Relative mean square error with respect to the true GLS. (b) 10,000 replications. (c) $\sigma_\mu^2 = 15$ and $\sigma_v^2 = 5$, i.e., $\theta = 0.75$.

Table 4: Relative Efficiencies of Spatial Panel Data Estimators (N = 49, T= 10)

	ρ_1	ρ_2	FE	RE	Within-GLS	KKP	Anselin	General
RE	0	0	1.091	0.995	1.098	0.997	1.003	1.008
KKP	-0.8	-0.8	1.968	1.765	1.133	1.014	1.071	1.012
	-0.5	-0.5	1.301	1.180	1.112	1.007	1.032	1.009
	-0.2	-0.2	1.173	1.034	1.117	0.995	1.002	1.001
	0.2	0.2	1.123	1.021	1.101	1.005	1.003	1.015
	0.5	0.5	1.314	1.189	1.096	0.996	1.025	1.001
	0.8	0.8	2.099	1.908	1.113	0.998	1.055	1.002
Anselin	0	-0.8	1.913	1.540	1.101	1.058	1.012	1.015
	0	-0.5	1.280	1.148	1.093	1.025	1.006	1.013
	0	-0.2	1.168	1.019	1.112	0.999	0.995	1.001
	0	0.2	1.122	1.027	1.100	1.012	1.006	1.011
	0	0.5	1.303	1.127	1.086	1.011	1.001	0.996
	0	0.8	2.059	1.638	1.091	1.044	1.007	1.019
General	-0.8	-0.5	1.323	1.244	1.132	1.029	1.075	1.013
	-0.8	-0.2	1.194	1.127	1.138	1.059	1.084	0.999
	-0.8	0	1.121	1.074	1.130	1.077	1.078	1.019
	-0.8	0.2	1.131	1.080	1.107	1.101	1.074	1.008
	-0.8	0.5	1.331	1.229	1.108	1.099	1.062	1.017
	-0.8	0.8	2.047	1.817	1.084	1.109	1.064	1.061
	-0.5	-0.8	1.946	1.626	1.121	1.022	1.035	1.016
	-0.5	-0.2	1.185	1.067	1.129	1.012	1.032	1.003
	-0.5	0	1.108	1.028	1.115	1.031	1.029	1.016
	-0.5	0.2	1.132	1.051	1.109	1.060	1.033	1.014
	-0.5	0.5	1.308	1.154	1.090	1.064	1.016	1.003
	-0.5	0.8	2.044	1.698	1.082	1.095	1.025	1.028
	-0.2	-0.8	1.923	1.568	1.107	1.046	1.016	1.012
	-0.2	-0.5	1.284	1.146	1.097	1.004	1.011	1.015
	-0.2	0	1.098	1.003	1.105	1.006	1.004	1.010
	-0.2	0.2	1.122	1.027	1.100	1.022	1.009	1.008
	-0.2	0.5	1.299	1.132	1.082	1.026	0.996	0.991
	-0.2	0.8	2.052	1.664	1.088	1.067	1.011	1.019
	0.2	-0.8	1.907	1.555	1.097	1.072	1.016	1.020
	0.2	-0.5	1.271	1.142	1.085	1.046	1.007	1.014
	0.2	-0.2	1.171	1.026	1.115	1.020	1.003	1.011
	0.2	0	1.087	1.000	1.094	1.007	1.008	1.004
	0.2	0.5	1.306	1.144	1.089	0.998	1.005	0.995
	0.2	0.8	2.076	1.690	1.100	1.039	1.008	1.009
	0.5	-0.8	1.881	1.592	1.082	1.087	1.032	1.021
	0.5	-0.5	1.266	1.172	1.081	1.080	1.029	1.009
	0.5	-0.2	1.171	1.047	1.114	1.047	1.027	1.012
	0.5	0	1.088	1.021	1.096	1.032	1.028	0.996
	0.5	0.2	1.122	1.054	1.100	1.014	1.027	1.016
	0.5	0.8	2.081	1.721	1.103	1.014	1.016	1.004
	0.8	-0.8	1.860	1.721	1.070	1.091	1.038	1.031
	0.8	-0.5	1.264	1.222	1.081	1.104	1.061	1.004
	0.8	-0.2	1.175	1.103	1.119	1.116	1.084	1.006
	0.8	0	1.102	1.074	1.109	1.078	1.078	1.008
	0.8	0.2	1.131	1.091	1.108	1.046	1.074	1.017
	0.8	0.5	1.329	1.267	1.108	1.020	1.064	1.004

Notes: (a) Relative mean square error with respect to the true GLS. (b) 10,000 replications. (c) $\sigma_\mu^2 = 10$ and $\sigma_v^2 = 10$, i.e., $\theta = 0.5$.

Table 5: Relative Efficiencies of Spatial Panel Data Estimators (N = 49, T= 10)

	ρ_1	ρ_2	FE	RE	Within-GLS	KKP	Anselin	General
RE	0	0	1.239	1.008	1.248	1.007	1.005	1.013
KKP	-0.8	-0.8	2.239	1.774	1.288	1.000	1.119	1.014
	-0.5	-0.5	1.482	1.190	1.267	1.010	1.054	1.019
	-0.2	-0.2	1.344	1.041	1.280	1.009	1.010	1.006
	0.2	0.2	1.265	1.029	1.240	1.017	1.025	1.025
	0.5	0.5	1.508	1.199	1.254	1.004	1.043	1.017
	0.8	0.8	2.382	1.895	1.262	1.005	1.091	1.010
Anselin	0	-0.8	2.105	1.470	1.211	1.051	1.022	1.029
	0	-0.5	1.427	1.123	1.218	1.035	1.008	1.013
	0	-0.2	1.334	1.031	1.270	1.011	1.008	1.010
	0	0.2	1.257	1.019	1.232	1.017	1.015	1.025
	0	0.5	1.477	1.130	1.230	1.032	1.009	1.012
	0	0.8	2.285	1.585	1.209	1.066	1.042	1.028
General	-0.8	-0.5	1.505	1.309	1.287	1.036	1.130	1.009
	-0.8	-0.2	1.389	1.212	1.324	1.112	1.164	1.025
	-0.8	0	1.257	1.123	1.267	1.129	1.127	1.008
	-0.8	0.2	1.257	1.124	1.231	1.167	1.119	1.024
	-0.8	0.5	1.463	1.252	1.216	1.195	1.103	1.028
	-0.8	0.8	2.213	1.742	1.172	1.203	1.110	1.299
	-0.5	-0.8	2.174	1.564	1.251	0.996	1.030	1.009
	-0.5	-0.2	1.363	1.089	1.299	1.031	1.047	1.018
	-0.5	0	1.251	1.049	1.262	1.043	1.046	1.010
	-0.5	0.2	1.249	1.048	1.223	1.059	1.041	1.018
	-0.5	0.5	1.457	1.148	1.212	1.117	1.029	1.013
	-0.5	0.8	2.226	1.592	1.179	1.148	1.061	1.061
	-0.2	-0.8	2.141	1.501	1.232	1.027	1.029	1.020
	-0.2	-0.5	1.449	1.134	1.239	1.018	1.013	1.020
	-0.2	0	1.242	1.011	1.252	1.009	1.011	1.013
	-0.2	0.2	1.257	1.020	1.232	1.027	1.023	1.030
	-0.2	0.5	1.463	1.121	1.217	1.054	1.002	1.006
	-0.2	0.8	2.247	1.564	1.189	1.093	1.045	1.033
	0.2	-0.8	2.073	1.457	1.193	1.083	1.033	1.038
	0.2	-0.5	1.416	1.123	1.209	1.069	1.012	1.023
	0.2	-0.2	1.331	1.034	1.267	1.028	1.017	1.017
	0.2	0	1.246	1.025	1.256	1.019	1.024	1.018
	0.2	0.5	1.494	1.144	1.244	1.018	1.018	1.016
	0.2	0.8	2.306	1.602	1.221	1.041	1.032	1.026
	0.5	-0.8	2.031	1.471	1.169	1.148	1.053	1.064
	0.5	-0.5	1.398	1.142	1.193	1.144	1.041	1.027
	0.5	-0.2	1.322	1.064	1.257	1.083	1.049	1.015
	0.5	0	1.241	1.052	1.250	1.055	1.058	1.016
	0.5	0.2	1.271	1.069	1.246	1.035	1.053	1.024
	0.5	0.8	2.354	1.693	1.247	1.024	1.041	1.027
	0.8	-0.8	1.998	1.632	1.150	1.212	1.101	1.164
	0.8	-0.5	1.395	1.238	1.191	1.240	1.137	1.040
	0.8	-0.2	1.303	1.138	1.239	1.187	1.131	1.001
	0.8	0	1.246	1.132	1.256	1.143	1.150	1.016
	0.8	0.2	1.282	1.160	1.255	1.096	1.143	1.022
	0.8	0.5	1.517	1.315	1.264	1.029	1.130	1.011

Notes: (a) Relative mean square error with respect to the true GLS. (b) 10,000 replications. (c) $\sigma_\mu^2 = 5$ and $\sigma_v^2 = 15$, i.e., $\theta = 0.25$.

Table 6: Relative Efficiencies of Spatial Panel Data Estimators (N = 49, T= 10)

	ρ_1	ρ_2	FE	RE	Within-GLS	KKP	Anselin	General
RE	0	0	1.037	1.005	1.043	1.013	1.012	1.015
KKP	-0.8	-0.8	1.826	1.747	1.051	1.005	1.036	1.008
	-0.5	-0.5	1.206	1.153	1.033	1.003	1.008	1.002
	-0.2	-0.2	1.109	1.033	1.056	1.005	1.009	1.012
	0.2	0.2	1.050	1.017	1.028	1.004	1.000	1.010
	0.5	0.5	1.242	1.194	1.038	1.007	1.016	1.009
	0.8	0.8	1.963	1.890	1.041	0.999	1.034	1.008
Anselin	0	-0.8	1.800	1.651	1.036	1.019	1.007	1.000
	0	-0.5	1.204	1.153	1.031	1.017	1.005	1.008
	0	-0.2	1.101	1.024	1.049	1.003	1.000	1.007
	0	0.2	1.053	1.023	1.031	1.007	1.003	1.009
	0	0.5	1.244	1.171	1.039	1.011	1.006	1.012
	0	0.8	1.957	1.769	1.037	1.033	1.001	1.012
General	-0.8	-0.5	1.215	1.185	1.041	1.010	1.023	1.004
	-0.8	-0.2	1.112	1.079	1.064	1.036	1.047	1.010
	-0.8	0	1.043	1.032	1.048	1.035	1.034	1.004
	-0.8	0.2	1.066	1.056	1.044	1.051	1.040	1.011
	-0.8	0.5	1.247	1.209	1.039	1.043	1.034	1.001
	-0.8	0.8	1.957	1.881	1.038	1.061	1.035	1.015
	-0.5	-0.8	1.826	1.697	1.050	1.015	1.014	1.008
	-0.5	-0.2	1.110	1.053	1.056	1.008	1.021	1.012
	-0.5	0	1.033	1.007	1.041	1.010	1.011	1.006
	-0.5	0.2	1.059	1.034	1.037	1.029	1.018	1.006
	-0.5	0.5	1.239	1.174	1.033	1.022	1.003	1.000
	-0.5	0.8	1.957	1.797	1.039	1.056	1.019	1.017
	-0.2	-0.8	1.810	1.657	1.041	1.019	1.006	1.002
	-0.2	-0.5	1.205	1.148	1.031	1.011	1.004	1.006
	-0.2	0	1.037	1.006	1.044	1.011	1.012	1.015
	-0.2	0.2	1.051	1.021	1.030	1.011	1.002	1.007
	-0.2	0.5	1.243	1.163	1.038	1.018	1.008	1.009
	-0.2	0.8	1.961	1.773	1.039	1.043	1.006	1.016
	0.2	-0.8	1.792	1.641	1.031	1.020	0.999	1.003
	0.2	-0.5	1.209	1.167	1.036	1.033	1.011	1.009
	0.2	-0.2	1.101	1.029	1.049	1.010	1.000	1.008
	0.2	0	1.037	1.009	1.044	1.017	1.014	1.017
	0.2	0.5	1.244	1.181	1.039	1.012	1.005	1.013
	0.2	0.8	1.959	1.775	1.038	1.020	1.005	1.012
	0.5	-0.8	1.794	1.678	1.034	1.033	1.007	1.006
	0.5	-0.5	1.205	1.172	1.033	1.039	1.018	1.006
	0.5	-0.2	1.099	1.046	1.049	1.022	1.013	1.009
	0.5	0	1.034	1.012	1.041	1.018	1.018	1.011
	0.5	0.2	1.051	1.021	1.029	1.002	1.010	1.010
	0.5	0.8	1.958	1.819	1.037	1.006	1.014	1.004
	0.8	-0.8	1.790	1.732	1.033	1.037	1.010	1.003
	0.8	-0.5	1.210	1.195	1.038	1.054	1.034	1.012
	0.8	-0.2	1.103	1.071	1.051	1.057	1.040	1.010
	0.8	0	1.038	1.031	1.042	1.038	1.040	1.009
	0.8	0.2	1.049	1.033	1.027	1.016	1.026	1.005
	0.8	0.5	1.242	1.211	1.037	1.006	1.030	1.010

Notes: (a) Relative mean square error with respect to the true GLS. (b) 10,000 replications. (c) $\sigma_\mu^2 = 15$ and $\sigma_v^2 = 5$, i.e., $\theta = 0.75$.

Table 7: Size of the Spatial Hausman Test (N = 100, T= 5)

	ρ_1	ρ_2	KKP	Anselin	General
RE	0	0	0.048	0.048	0.050
KKP	-0.8	-0.8	0.054	0.024	0.056
	-0.5	-0.5	0.054	0.048	0.056
	-0.2	-0.2	0.051	0.052	0.053
	0.2	0.2	0.048	0.047	0.049
	0.5	0.5	0.050	0.041	0.050
	0.8	0.8	0.050	0.023	0.050
Anselin	0	-0.8	0.094	0.057	0.064
	0	-0.5	0.069	0.055	0.059
	0	-0.2	0.054	0.052	0.054
	0	0.2	0.053	0.048	0.051
	0	0.5	0.067	0.051	0.053
	0	0.8	0.090	0.056	0.061
General	-0.8	-0.5	0.037	0.037	0.055
	-0.8	-0.2	0.033	0.042	0.050
	-0.8	0	0.037	0.037	0.050
	-0.8	0.2	0.057	0.044	0.052
	-0.8	0.5	0.087	0.047	0.054
	-0.8	0.8	0.121	0.058	0.097
	-0.5	-0.8	0.070	0.041	0.059
	-0.5	-0.2	0.048	0.051	0.051
	-0.5	0	0.048	0.048	0.050
	-0.5	0.2	0.061	0.050	0.051
	-0.5	0.5	0.085	0.057	0.053
	-0.5	0.8	0.117	0.067	0.072
	-0.2	-0.8	0.083	0.052	0.062
	-0.2	-0.5	0.063	0.053	0.059
	-0.2	0	0.048	0.048	0.050
	-0.2	0.2	0.055	0.049	0.051
	-0.2	0.5	0.074	0.053	0.052
	-0.2	0.8	0.100	0.061	0.065
	0.2	-0.8	0.107	0.062	0.067
	0.2	-0.5	0.077	0.057	0.058
	0.2	-0.2	0.055	0.051	0.054
	0.2	0	0.047	0.046	0.049
	0.2	0.5	0.060	0.047	0.052
	0.2	0.8	0.082	0.050	0.057
	0.5	-0.8	0.143	0.069	0.068
	0.5	-0.5	0.100	0.059	0.059
	0.5	-0.2	0.063	0.053	0.054
	0.5	0	0.049	0.048	0.049
	0.5	0.2	0.042	0.044	0.050
	0.5	0.8	0.068	0.037	0.052
	0.8	-0.8	0.211	0.074	0.083
	0.8	-0.5	0.148	0.066	0.059
	0.8	-0.2	0.084	0.058	0.053
	0.8	0	0.054	0.053	0.047
	0.8	0.2	0.034	0.047	0.048
	0.8	0.5	0.035	0.038	0.049

Notes: (a) 10,000 replications. (b) $\sigma_\mu^2 = 10$ and $\sigma_v^2 = 10$, i.e., $\theta = 0.5$.

Table 8: Size of the Spatial Hausman Test (N = 100, T= 5)

	ρ_1	ρ_2	KKP	Anselin	General
RE	0	0	0.047	0.047	0.048
KKP	-0.8	-0.8	0.052	0.025	0.051
	-0.5	-0.5	0.053	0.046	0.054
	-0.2	-0.2	0.050	0.051	0.050
	0.2	0.2	0.048	0.046	0.049
	0.5	0.5	0.048	0.042	0.049
	0.8	0.8	0.049	0.022	0.048
Anselin	0	-0.8	0.078	0.067	0.062
	0	-0.5	0.060	0.052	0.054
	0	-0.2	0.051	0.051	0.050
	0	0.2	0.050	0.047	0.049
	0	0.5	0.059	0.051	0.053
	0	0.8	0.071	0.070	0.061
General	-0.8	-0.5	0.043	0.034	0.054
	-0.8	-0.2	0.040	0.042	0.050
	-0.8	0	0.040	0.040	0.051
	-0.8	0.2	0.053	0.045	0.053
	-0.8	0.5	0.075	0.052	0.066
	-0.8	0.8	0.107	0.067	0.132
	-0.5	-0.8	0.062	0.046	0.056
	-0.5	-0.2	0.049	0.049	0.050
	-0.5	0	0.047	0.047	0.050
	-0.5	0.2	0.055	0.049	0.051
	-0.5	0.5	0.071	0.059	0.056
	-0.5	0.8	0.094	0.078	0.074
	-0.2	-0.8	0.070	0.060	0.059
	-0.2	-0.5	0.058	0.051	0.056
	-0.2	0	0.047	0.047	0.049
	-0.2	0.2	0.051	0.049	0.050
	-0.2	0.5	0.063	0.053	0.053
	-0.2	0.8	0.080	0.076	0.065
	0.2	-0.8	0.087	0.069	0.065
	0.2	-0.5	0.064	0.058	0.056
	0.2	-0.2	0.053	0.051	0.052
	0.2	0	0.048	0.047	0.048
	0.2	0.5	0.055	0.048	0.051
	0.2	0.8	0.064	0.064	0.057
	0.5	-0.8	0.114	0.079	0.076
	0.5	-0.5	0.079	0.062	0.059
	0.5	-0.2	0.058	0.054	0.054
	0.5	0	0.048	0.047	0.048
	0.5	0.2	0.044	0.044	0.049
	0.5	0.8	0.057	0.049	0.051
	0.8	-0.8	0.182	0.086	0.122
	0.8	-0.5	0.122	0.070	0.064
	0.8	-0.2	0.075	0.060	0.051
	0.8	0	0.050	0.049	0.048
	0.8	0.2	0.037	0.041	0.049
	0.8	0.5	0.040	0.033	0.046

Notes: (a) 10,000 replications. (b) $\sigma_\mu^2 = 5$ and $\sigma_v^2 = 15$, i.e., $\theta = 0.25$.

Table 9: Size of the Spatial Hausman Test (N = 100, T= 5)

	ρ_1	ρ_2	KKP	Anselin	General
RE	0	0	0.050	0.049	0.055
KKP	-0.8	-0.8	0.055	0.031	0.057
	-0.5	-0.5	0.056	0.052	0.060
	-0.2	-0.2	0.055	0.055	0.058
	0.2	0.2	0.052	0.049	0.056
	0.5	0.5	0.050	0.044	0.050
	0.8	0.8	0.050	0.035	0.051
Anselin	0	-0.8	0.110	0.056	0.065
	0	-0.5	0.079	0.058	0.064
	0	-0.2	0.056	0.054	0.062
	0	0.2	0.057	0.052	0.057
	0	0.5	0.074	0.050	0.057
	0	0.8	0.106	0.054	0.063
General	-0.8	-0.5	0.034	0.041	0.056
	-0.8	-0.2	0.032	0.042	0.051
	-0.8	0	0.036	0.035	0.051
	-0.8	0.2	0.059	0.043	0.052
	-0.8	0.5	0.089	0.042	0.053
	-0.8	0.8	0.129	0.048	0.071
	-0.5	-0.8	0.079	0.047	0.059
	-0.5	-0.2	0.049	0.054	0.054
	-0.5	0	0.049	0.049	0.052
	-0.5	0.2	0.068	0.054	0.054
	-0.5	0.5	0.094	0.054	0.054
	-0.5	0.8	0.133	0.063	0.066
	-0.2	-0.8	0.096	0.051	0.064
	-0.2	-0.5	0.068	0.058	0.064
	-0.2	0	0.050	0.050	0.055
	-0.2	0.2	0.061	0.053	0.055
	-0.2	0.5	0.082	0.052	0.054
	-0.2	0.8	0.117	0.059	0.063
	0.2	-0.8	0.124	0.059	0.063
	0.2	-0.5	0.090	0.057	0.064
	0.2	-0.2	0.059	0.053	0.058
	0.2	0	0.050	0.049	0.054
	0.2	0.5	0.065	0.049	0.054
	0.2	0.8	0.094	0.050	0.060
	0.5	-0.8	0.160	0.063	0.063
	0.5	-0.5	0.112	0.060	0.059
	0.5	-0.2	0.068	0.052	0.056
	0.5	0	0.050	0.050	0.048
	0.5	0.2	0.043	0.047	0.053
	0.5	0.8	0.076	0.042	0.054
	0.8	-0.8	0.224	0.066	0.065
	0.8	-0.5	0.160	0.062	0.057
	0.8	-0.2	0.088	0.056	0.055
	0.8	0	0.055	0.055	0.046
	0.8	0.2	0.035	0.052	0.050
	0.8	0.5	0.029	0.046	0.049

Notes: (a) 10,000 replications. (b) $\sigma_\mu^2 = 15$ and $\sigma_v^2 = 5$, i.e., $\theta = 0.75$.

Table 10: Size of the Spatial Hausman Test (N = 49, T= 10)

	ρ_1	ρ_2	KKP	Anselin	General
RE	0	0	0.055	0.056	0.063
KKP	-0.8	-0.8	0.055	0.028	0.059
	-0.5	-0.5	0.059	0.048	0.065
	-0.2	-0.2	0.057	0.052	0.062
	0.2	0.2	0.055	0.058	0.062
	0.5	0.5	0.061	0.061	0.062
	0.8	0.8	0.053	0.048	0.051
Anselin	0	-0.8	0.129	0.062	0.076
	0	-0.5	0.092	0.059	0.069
	0	-0.2	0.063	0.056	0.062
	0	0.2	0.058	0.056	0.062
	0	0.5	0.081	0.061	0.069
	0	0.8	0.108	0.065	0.073
General	-0.8	-0.5	0.031	0.043	0.059
	-0.8	-0.2	0.034	0.048	0.055
	-0.8	0	0.047	0.047	0.056
	-0.8	0.2	0.081	0.051	0.058
	-0.8	0.5	0.138	0.055	0.064
	-0.8	0.8	0.196	0.067	0.114
	-0.5	-0.8	0.084	0.039	0.065
	-0.5	-0.2	0.047	0.052	0.062
	-0.5	0	0.051	0.051	0.062
	-0.5	0.2	0.069	0.056	0.060
	-0.5	0.5	0.114	0.063	0.067
	-0.5	0.8	0.166	0.071	0.088
	-0.2	-0.8	0.110	0.053	0.071
	-0.2	-0.5	0.078	0.053	0.068
	-0.2	0	0.052	0.052	0.064
	-0.2	0.2	0.061	0.055	0.062
	-0.2	0.5	0.093	0.062	0.069
	-0.2	0.8	0.128	0.068	0.079
	0.2	-0.8	0.152	0.070	0.078
	0.2	-0.5	0.109	0.066	0.068
	0.2	-0.2	0.073	0.059	0.063
	0.2	0	0.060	0.059	0.062
	0.2	0.5	0.073	0.060	0.067
	0.2	0.8	0.092	0.060	0.070
	0.5	-0.8	0.200	0.086	0.082
	0.5	-0.5	0.145	0.078	0.065
	0.5	-0.2	0.093	0.070	0.062
	0.5	0	0.068	0.066	0.057
	0.5	0.2	0.055	0.064	0.058
	0.5	0.8	0.074	0.053	0.061
	0.8	-0.8	0.307	0.117	0.088
	0.8	-0.5	0.234	0.107	0.059
	0.8	-0.2	0.146	0.097	0.055
	0.8	0	0.092	0.091	0.051
	0.8	0.2	0.059	0.088	0.051
	0.8	0.5	0.042	0.080	0.054

Notes: (a) 10,000 replications. (b) $\sigma_\mu^2 = 10$ and $\sigma_v^2 = 10$, i.e., $\theta = 0.5$.

Table 11: Size of the Spatial Hausman Test (N = 49, T= 10)

	ρ_1	ρ_2	KKP	Anselin	General
RE	0	0	0.053	0.053	0.059
KKP	-0.8	-0.8	0.055	0.020	0.056
	-0.5	-0.5	0.056	0.044	0.059
	-0.2	-0.2	0.052	0.052	0.056
	0.2	0.2	0.052	0.054	0.055
	0.5	0.5	0.056	0.051	0.058
	0.8	0.8	0.048	0.034	0.046
Anselin	0	-0.8	0.105	0.068	0.074
	0	-0.5	0.078	0.057	0.064
	0	-0.2	0.057	0.055	0.057
	0	0.2	0.053	0.053	0.057
	0	0.5	0.071	0.058	0.063
	0	0.8	0.087	0.075	0.072
General	-0.8	-0.5	0.039	0.036	0.057
	-0.8	-0.2	0.036	0.044	0.055
	-0.8	0	0.049	0.048	0.055
	-0.8	0.2	0.075	0.053	0.057
	-0.8	0.5	0.128	0.063	0.074
	-0.8	0.8	0.180	0.084	0.203
	-0.5	-0.8	0.073	0.041	0.062
	-0.5	-0.2	0.047	0.047	0.058
	-0.5	0	0.051	0.051	0.057
	-0.5	0.2	0.062	0.055	0.059
	-0.5	0.5	0.100	0.065	0.068
	-0.5	0.8	0.138	0.087	0.095
	-0.2	-0.8	0.090	0.059	0.070
	-0.2	-0.5	0.068	0.052	0.062
	-0.2	0	0.052	0.051	0.059
	-0.2	0.2	0.056	0.054	0.058
	-0.2	0.5	0.077	0.061	0.064
	-0.2	0.8	0.103	0.079	0.077
	0.2	-0.8	0.123	0.076	0.081
	0.2	-0.5	0.090	0.064	0.066
	0.2	-0.2	0.064	0.055	0.059
	0.2	0	0.056	0.056	0.059
	0.2	0.5	0.065	0.056	0.060
	0.2	0.8	0.077	0.067	0.065
	0.5	-0.8	0.168	0.096	0.095
	0.5	-0.5	0.121	0.077	0.068
	0.5	-0.2	0.077	0.064	0.058
	0.5	0	0.063	0.062	0.056
	0.5	0.2	0.052	0.058	0.052
	0.5	0.8	0.063	0.053	0.056
	0.8	-0.8	0.278	0.127	0.146
	0.8	-0.5	0.209	0.107	0.070
	0.8	-0.2	0.129	0.089	0.055
	0.8	0	0.084	0.083	0.049
	0.8	0.2	0.055	0.074	0.049
	0.8	0.5	0.045	0.060	0.054

Notes: (a) 10,000 replications. (b) $\sigma_\mu^2 = 5$ and $\sigma_v^2 = 15$, i.e., $\theta = 0.25$.

Table 12: Size of the Spatial Hausman Test (N = 49, T= 10)

	ρ_1	ρ_2	KKP	Anselin	General
RE	0	0	0.057	0.056	0.068
KKP	-0.8	-0.8	0.056	0.037	0.061
	-0.5	-0.5	0.059	0.052	0.069
	-0.2	-0.2	0.058	0.055	0.066
	0.2	0.2	0.060	0.061	0.071
	0.5	0.5	0.060	0.069	0.063
	0.8	0.8	0.052	0.072	0.053
Anselin	0	-0.8	0.142	0.062	0.076
	0	-0.5	0.100	0.060	0.074
	0	-0.2	0.065	0.059	0.069
	0	0.2	0.062	0.059	0.070
	0	0.5	0.085	0.059	0.070
	0	0.8	0.122	0.063	0.076
General	-0.8	-0.5	0.030	0.045	0.061
	-0.8	-0.2	0.036	0.049	0.056
	-0.8	0	0.048	0.046	0.057
	-0.8	0.2	0.085	0.050	0.056
	-0.8	0.5	0.142	0.051	0.062
	-0.8	0.8	0.203	0.059	0.079
	-0.5	-0.8	0.092	0.047	0.069
	-0.5	-0.2	0.048	0.052	0.062
	-0.5	0	0.050	0.050	0.063
	-0.5	0.2	0.075	0.056	0.063
	-0.5	0.5	0.124	0.058	0.068
	-0.5	0.8	0.176	0.064	0.080
	-0.2	-0.8	0.121	0.056	0.073
	-0.2	-0.5	0.085	0.057	0.071
	-0.2	0	0.054	0.053	0.067
	-0.2	0.2	0.065	0.058	0.068
	-0.2	0.5	0.099	0.057	0.071
	-0.2	0.8	0.143	0.065	0.077
	0.2	-0.8	0.168	0.069	0.076
	0.2	-0.5	0.118	0.065	0.069
	0.2	-0.2	0.074	0.062	0.070
	0.2	0	0.062	0.061	0.065
	0.2	0.5	0.074	0.061	0.069
	0.2	0.8	0.104	0.061	0.072
	0.5	-0.8	0.216	0.082	0.072
	0.5	-0.5	0.158	0.076	0.067
	0.5	-0.2	0.100	0.072	0.063
	0.5	0	0.071	0.071	0.059
	0.5	0.2	0.059	0.068	0.064
	0.5	0.8	0.082	0.060	0.063
	0.8	-0.8	0.317	0.112	0.070
	0.8	-0.5	0.242	0.108	0.057
	0.8	-0.2	0.151	0.101	0.056
	0.8	0	0.096	0.095	0.053
	0.8	0.2	0.059	0.096	0.056
	0.8	0.5	0.043	0.096	0.055

Notes: (a) 10,000 replications. (b) $\sigma^2_\mu = 15$ and $\sigma^2_v = 5$, i.e., $\theta = 0.75$.