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Abstract

The objective of this manuscript is to study the importance of banking competition in the formulation of monetary policy. While the Friedman rule can be optimal in the presence of perfect competition, it is never the case when the banking system is not competitive. Furthermore, when market power in the banking system is highly distortionary, it is optimal to impose a higher tax on money when the banking system is not competitive.

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1 Introduction

Should monetary policy be designed according to the competitive structure of the banking system? Providing an answer to this question is of significant importance given the important role that banks play in the economy and in the transmission of monetary policy.[1,2] Motivated by the wave of consolidations the financial sector has witnessed over the past few decades around the world, a number of recent studies attempted to address this issue. For example, Paal, Smith, and Wang (2005) study an overlapping generations production economy where banks provide risk sharing services to their depositors. An economy where banks are perfectly competitive is compared to an economy where the banking

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[1]A large amount of work has been devoted to study the linkages between developments in the banking sector and economic growth. From previous studies, I cite King and Levine (1993), Levine (1997), and Levine, Loayza, and Beck (2000), among others.

[2]The important role of banks in the transmission of monetary policy has been highlighted by numerous studies. Previous work includes, Bernanke and Blinder (1992), Bernanke and Gertler (1995), Kashyap and Stein (2000), Kishan and Opiela (2000), Nilsen (2002), and Gan (2007).

system is monopolistic. In their setting, banks are able to distort the deposit market, yet the credit or capital market is competitive. The authors demonstrate that the effect of banking competition on economic growth is ambiguous. Moreover, the Friedman Rule, where money and capital yield the same real return, can be optimal under both competitive structures.[3] The Friedman rule also guarantees that depositors receive complete insurance against idiosyncratic risk. More recent work by Matsuoka (2011) follows a similar approach to Paal et al. (2005) albeit in an endowment economy with an exogenous opportunity cost to holding money. The author demonstrates that the Friedman Rule is optimal when the banking sector is not competitive and zero inflation is optimal in a perfectly competitive setting.

While the results described above are important, it is hard to believe that capital markets cannot be distorted by lack of competition in the banking system, which in turn could have implications for monetary policy. In fact, recent empirical evidence points out to an asymmetric relationship between banking competition and credit market activity. For example, in a cross-country study, Beck et al. (2004) find that banking concentration reduces access to finance only in less developed countries.[4] Similar results are found in recent work by Deidda and Fattouh (2005) linking banking concentration and economic growth. More importantly, central banks around the world do not adopt the Friedman rule. For example, the European central bank has an explicit long-run inflation target of 2%. Obviously, previous experiences of the United States and Japan with the Friedman rule was not pleasant.

The objective of this manuscript is to highlight the importance of different distortions that could arise from lack of competition in the banking sector for the formulation of monetary policy. In order to do so, I examine a setting where banks can exert market power on both sides of their balance sheet. In particular, I study a two-period overlapping generations production model. The economy is populated by two types of agents, depositors and bankers. Following Townsend (1987) and Schreft and Smith (1997), depositors are born on one of two geographically separated, yet symmetric locations. Agents work when young and invest all their income in the economy's assets (money and physical capital). With some probability, depositors must relocate to the other location after they make their portfolio choice. Due to private information and limited communication relocated agents must liquidate their assets (physical capital) into cash to be able to consume. Depositors can choose between intermediating their savings and investing directly in asset markets. Bankers take deposits, insure their depositors against relocation shocks, and invest in the economy's assets to maximize profits. Finally, there is a government that targets the rate of money creation and rebates seigniorage revenue to young depositors.

In this setting, I study the optimal monetary policy under two competitive

[3] Ghossoub, Laosuthi, and Reed (2012) and Ghossoub (2012) examine the linkages between monetary policy and banking competition. However, these papers do not study optimal monetary policy.

[4] Berger et al. (2004) provides a nice overview of the literature on the effects of banking competition on various markets.

banking structures: perfect competition and monopoly.[5] Following previous studies such as Paal, Smith, and Wang (2005) and Matsuoka (2011) I use the expected utility of a particular generation of depositors as a proxy for welfare. In this manner, the monetary authority chooses the rate of money growth to maximize steady-state welfare.[6]

When the banking sector is fully concentrated, the bank has an incentive to restrict the amount of investment in order to raise the return from capital. However, the bank also has market power in the deposit market and is capable of extracting all the surplus from its depositors by offering deposit contracts such that agents are indifferent between using and not using the bank. Therefore, the relative return to depositors is much lower when the banking sector is not competitive. This translates into two outcomes. First, depositors receive better insurance against idiosyncractic liquidity risk when the banking sector is not competitive. Moreover, due to lower deposit rates, the marginal cost of capital investment is also lower when the banking system is fully concentrated. Consequently, as previously found (although for slightly different reasons) in Paal et al. (2005), a fully concentrated banking system could promote capital formation and improve welfare compared to a perfectly competitive banking system.

As in standard random relocation models, raising the rate of money growth involves a trade-off between less risk sharing (due to lower insurance to depositors) and higher capital formation (and higher deposits). If welfare is strictly decreasing with the rate of money creation, the Friedman rule is optimal when the banking system is competitive. At the Friedman rule, both money and capital yield the same real return, which also implies that depositors receive complete risk sharing against idiosyncratic liquidity shocks. However, when the banking system is not competitive, complete risk sharing is realized at a level of money creation much higher than the one prescribed by the Friedman rule. More importantly, the Friedman rule rate of money growth triggers a bank run on deposits when the banking system is not competitive. This in turn causes agents to opt out from the banking system in anticipation of the run. Therefore, the Friedman rule cannot support a banking equilibrium when the banking system is not competitive.[7]

This result differs significantly from previous work that emphasizes only on

[5] Earlier work by Boyd, De Nicolo, and Smith (2004) also compares two competitive structures to study the implications of banking competition for the probability of a banking crisis. The authors conclude that the link between banking crises and banking competition depends on the stance of monetary policy. For example, the probability of a banking crisis is lower under a competitive banking system when inflation is relatively low.

[6] Ghossoub (2012) studies a two sector model where banks can also have market power in capital markets. However, the author treats the outside option of depositors as exogenous, which renders the equilibrium welfare of depositors also exogenous. Consequently, welfare analysis and optimal monetary policy were not studied

[7] When the rate of money growth drops slightly below the rate that yields complete risk sharing, agents drop out from the banking system and exhibit a discrete drop in their welfare. If the rate of money grow is further decreases, their welfare could improve. However, at the Friedman rule money becomes too good of an asset and agents choose not to invest in physical capital. In a production economy, the only equilibrium available is the trivial equilibrium.

distortions in the deposit markets. In addition, the result is at stark difference from previous studies that study banking crises such as Champ et al. (1996), Antinolfi et al. (2001), and Smith (2002) in a competitive endowment economy. In these models, the Friedman rule completely eliminates a banking crisis (a situation where banks run out of cash reserves) as banks allocate all their deposits into cash balances when the value of money is too high. In this manuscript, market power distorts capital markets, which in turn distorts the ability of the bank to provide risk sharing services at low levels of inflation. Hence, deflationary episodes such as the ones experienced by the United Sates during the great depression can be attributed to banking crises in this model.

Furthermore, in a competitive environment, Bhattacharya, Haslag, and Martin (2009) attribute the sub-optimality of the Friedman rule to the presence of a Tobin Effect -the positive relationship between the rate of money growth and the capital stock (or economic growth in their model). This manuscript provides an alternative explanation based on the degree of competitiveness of banks in the credit or capital market. In particular, market power in the banking sector exacerbates information frictions, which automatically rules out the Friedman rule as the optimal policy. This result is independent of the welfare criteria adopted in this manuscript and the presence of a Tobin effect.

As I demonstrate in the text, inflation has adverse effects on welfare under both competitive structures when a fully concentrated banking system significantly distorts capital markets.[8] Given that the monopolist is holding too much cash relative to perfect competition, it is optimal to impose a higher inflation tax on the economy with a fully concentrated banking system to stimulate capital investment and welfare. In contrast, if the lack of competition stimulates capital formation, complete risk sharing is not optimal.[9] More specifically, the optimal inflation tax is much higher when the banking system is competitive.

The results in this manuscript suggest that central banks should take into account changes in the industrial organization of the banking sector while setting long-run inflation targets. However, policymakers should pay a closer attention to any distortionary effects that could arise under more concentration. In this manner, my work also sheds some light on the variations in the inflation rate across countries. In less developed countries where banking concentration is highly distortionary, the inflation tax should be higher.[10] In contrast, in economies where banking concentration might improve the level of economic activity such as the Japanese experience, monetary policy should be more restrictive.

[8] This occurs when the degree of liquidity risk is below some threshold level.

[9] The level of investment under a monopoly banking system exceeds that under perfect competition when depositors are highly exposed to liquidity risk. When agents are highly exposed to liquidity risk, the banking system is holding a large amount of cash to insure its depositors against idiosyncratic risk. Therefore, deviating from the Friedman rule (or complete risk sharing) to stimulate capital investment can be welfare improving.

[10] A number of studies such as Roubini and Sala-i-Martin (1985) and Bencivenga and Smith (1992) suggest that it is optimal to lower the value of money when tax evasion is high as in less developed countries. I provide an alternative explanation for variations in the stance of monetary policy across countries based on the degree of banking competition.

The paper is organized as follows. In Section 2, I provide a description of the model. Section 3, describes the outside option of potential depositors in absence of financial intermediation. Sections 5 studies an economy where the banking system is perfectly competitive, while in section 6 the banking system is fully concentrated. I offer concluding remarks in Section 6.Technical details are available in the Appendix.

2 Environment

Consider a two period overlapping generations economy divided into two geographically separated, yet symmetric locations. Time is discrete and is denoted by an index, $t = 1, 2, ..\infty$. At the beginning of each period, a continuum of young workers (potential depositors) with unit mass and $N \geq 1$ bankers are born.

Agents are assumed to value only their old age consumption, c_t, of the economy's single perishable good. Workers are assumed to be risk averse with logarithmic preferences, $u(c_t) = \ln c_t$.[11] Furthermore, bankers are assumed to be risk neutral. Workers are endowed with one unit of labor effort when young, which they supply inelastically, and are retired when old. By comparison, bankers do not receive any endowments.

Consumption goods are produced by a representative firm using capital and labor as inputs. The firm has access to a constant returns to scale technology of the Cobb-Douglas form, with $Y_t = AK_t^{\alpha} L_t^{1-\alpha}$, where Y_t, K_t, and L_t are period t aggregate output, capital stock, and labor, respectively. In addition, $\alpha \epsilon (0, 1)$ is capital's share of total output and A reflects total factor productivity. Equivalently, output per worker is expressed by $y_t = Ak_t^{\alpha}$, with $k_t = \frac{K_t}{L_t}$ is the capital labor ratio. Further, I assume that the capital stock depreciates completely in the production process.

The capital stock is produced in the following manner. One unit of goods allocated towards investment in physical capital in period t, becomes one unit of capital in period $t+1$. Let i_t, reflect the level of investment in capital goods per worker. As capital depreciates completely after production, the capital stock per worker in $t + 1$ is: $k_{t+1} = i_t$.

In addition to physical capital, there is a stock of money (fiat currency) that circulates in the economy. Denote the per worker nominal monetary base by M_t. Money is universally recognizable, durable and divisible object. At the initial date 0, the generation of old workers at each location is endowed with the aggregate capital, K_0 and money supply, M_0. Since the population of workers is equal to one, these variables also represent aggregate values. Assuming that the price level is common across locations, I refer to P_t as the number of units of currency per unit of goods at time t.

[11]Similar insights can be obtained if workers have general types of preferences. While technical work is not tractable when agents have CRRA preferences, numerical work with a coefficient of risk aversion below unity yield similar conclusions to the ones derived in this manuscript.

Following Schreft and Smith (1997, 1998), workers are randomly chosen to change location after they make their portfolio choice. As the number of workers is unity, the probability of relocation, π, also reflects the number of relocating agents (movers). Furthermore, there is limited communication between different locations and information is private. Therefore, agents cannot trade claims to assets they own on their original location. As in standard random relocation models, fiat money overcomes these trade frictions and is assumed to be the only asset that can be carried across islands. As a result, workers who learn they will be relocated will liquidate all their asset holdings into currency. As in Diamond and Dybvig (1983), financial intermediaries (bankers) play an important role insuring works against liquidity risk (random relocation shocks). In contrast to workers, bankers are not subject to relocation shocks.

The final agent in this economy is a government (or central bank) that adopts a constant money growth rule. The evolution of real money balances, m, between periods $t-1$ and t is expressed as:

$$m_t = \sigma \frac{P_{t-1}}{P_t} m_{t-1} \tag{1}$$

where $\sigma > 0$ is the gross rate of money creation (or destruction) chosen at the beginning of time and $\frac{P_{t-1}}{P_t}$ is the gross rate of return on money balances between period $t-1$ and t. The government rebates back seigniorage income to young workers. Denote the amount of transfers at the beginning of period t by τ_t, where

$$\tau_t = \frac{\sigma - 1}{\sigma} m_t \tag{2}$$

2.1 Trade

2.2 Factors Markets

In period t, a representative firm rents capital and hires workers in perfectly competitive factor markets at rates r_t and w_t, respectively. The inverse demands for labor and capital by a typical firm are expressed by

$$w_t = (1-\alpha) A k_t^{\alpha} \equiv w(k_t) \tag{3}$$

and

$$r_t = \alpha A k_t^{\alpha - 1} \tag{4}$$

3 Portfolio Choice Under Direct Investment

In absence of financial intermediation, I assume that agents have no means to share risk. At the beginning of period t, agents receive their labor income and government transfers, which are entirely saved. Before the shock is realized, agents allocate their savings towards real money balances, m_t and physical capital, k_{t+1}. Therefore, the following budget constraint must hold:

$$w_t + \tau_t = m_t + k_{t+1} \tag{5}$$

Moreover, if an agent relocates (is a mover), she loses all her capital investment and her consumption in $t+1$ comes from existing cash on hand, $m_t \frac{P_t}{P_{t+1}}$:

$$c^m_{t+1} = m_t \frac{P_t}{P_{t+1}} \tag{6}$$

By comparison, an agent that does not have to move will earn income from both assets, money and capital. Therefore, the consumption of a non-mover satisfies:

$$c^n_{t+1} = m_t \frac{P_t}{P_{t+1}} + r_{t+1} k_{t+1} \tag{7}$$

A typical agent maximizes her expected utility, $\underline{u}$ subject to the constraints above. Specifically, the problem of a worker is summarized as:

$$\underline{u} = \underset{c^m_{t+1}, c^n_{t+1}, m_t, k_{t+1}}{Max} \pi \ln c^m_{t+1} + (1-\pi) \ln c^n_{t+1}$$

subject to $(5) - (7)$.

Substituting the constraints into the objective function, the problem can be reduced into a choice of real money balances:

$$\underline{u} \equiv \underset{m_t}{Max} \pi \ln m_t \frac{P_t}{P_{t+1}} + (1-\pi) \ln \left(m_t \frac{P_t}{P_{t+1}} + r_{t+1} (w_t + \tau_t - m_t) \right) \tag{8}$$

The choice of money holding is such that:

$$\frac{\partial u}{\partial m_t} = \frac{\pi}{m_t} - (1-\pi) \frac{r_{t+1} - \frac{P_t}{P_{t+1}}}{\left(m_t \frac{P_t}{P_{t+1}} + r_{t+1} (w_t + \tau_t - m_t) \right)} \tag{9}$$

Given that money provides a liquidity advantage over physical capital, both assets are held in equilibrium if the expected return to capital exceeds that to money. That is, $(1-\pi) r_{t+1} \geq \frac{P_t}{P_{t+1}}$. Equivalently, define $I_t = r_{t+1} \frac{P_{t+1}}{P_t}$ to be the gross nominal return to capital between t and $t+1$. Money is ex-ante dominated in rate of return if: $I_t \geq \frac{1}{1-\pi} > 1$. It follows from (9) that the individual choice of money balances is such that:

$$m_t = \begin{cases} (w_t + \tau_t) & \text{if } I_t \leq \frac{1}{1-\pi} \\ \frac{\pi}{1-\frac{1}{I_t}} (w_t + \tau_t) & \text{if } I_t > \frac{1}{1-\pi} \end{cases} \tag{10}$$

Analogously, using (5) and the expression for money demand, (10), individual capital investment is such that:

$$k_{t+1} = \begin{cases} 0 & \text{if } I_t \leq \frac{1}{1-\pi} \\ \left(1 - \frac{\pi}{1-\frac{1}{I_t}}\right)(w_t + \tau_t) & \text{if } I_t > \frac{1}{1-\pi} \end{cases} \tag{11}$$

Finally, upon substituting (10), (11) into (8), the expected utility of a typical agent is:

$$\underline{u} = \begin{cases} \ln (w_t + \tau_t) \frac{P_t}{P_{t+1}} & \text{if } I_t \leq \frac{1}{1-\pi} \\ \ln \frac{(1-\pi)^{1-\pi}\pi^{\pi}}{(I_t - 1)^{\pi}} I_t (w_t + \tau_t) \frac{P_t}{P_{t+1}} & \text{if } I_t > \frac{1}{1-\pi} \end{cases} \tag{12}$$

4 Perfectly Competitive Bankers

To begin, I analyze an economy where the banking sector is perfectly competitive. In this setting, bankers engage in price competition in both capital and deposit markets. Given that banks provide identical financial services, perfect competition is realized when the number of banks exceeds unity. That is, $N > 1$.

At the beginning of period t, each banker announces deposit rates. In particular, because agents' types are publicly observable, banks are able to offer deposits contracts that are contingent on the realization of the shock. A bank promises a gross real return on deposits, r_t^m if a young individual is relocated and a gross real return r_t^n if not.

The bank allocates deposits between real money balances, m_t and capital investment, i_t. A typical bank's balance sheet is expressed by:

$$w_t + \tau_t = m_t + i_t \tag{13}$$

Because relocated agents need cash to transact, total payments made to movers, satisfy:

$$\pi r_t^m (w_t + \tau_t) = m_t \frac{P_t}{P_{t+1}} \tag{14}$$

As banks attract a large number of depositors, they are able to completely diversify idiosyncratic risk. Therefore, banks will not hold excess reserves. A bank's total payments to non-movers are therefore paid out of its revenue from renting capital to firms in $t+1$. The constraint on payments to non-movers is such that:

$$(1-\pi) r_t^n (w_t + \tau_t) = r_{t+1} k_{t+1} \tag{15}$$

Furthermore, the contract between the bank and its depositors has to be incentive compatible. Therefore, the following self-selection constraint has to hold:

$$\frac{r_t^n}{r_t^m} \geq 1 \tag{16}$$

Finally, in order to induce workers to participate in the banking sector, the expected utility of a typical agent when their savings are intermediated, u_t^{PC} must be at least as high as that under direct investment. That is, the following participation constraint must hold:

$$u_t^{PC} \geq \underline{u} \tag{17}$$

Due to perfect competition, banks make zero profits in equilibrium and make their portfolio choice to maximize the expected utility of their depositors. A typical bank's problem is summarized by

$$u_t^{PC} = \underset{r_t^m, r_t^n, m_t, k_{t+1}}{Max} \pi \ln r_t^m (w_t + \tau_t) + (1-\pi) \ln r_t^n (w_t + \tau_t) \tag{18}$$

subject to (13)-(17), and $i_t = k_{t+1}$.

The solution to the bank's problem generates the demand for real money balances:

$$m_t = \pi (w_t + \tau_t) \tag{19}$$

Alternatively,

$$\gamma_t^{PC} = \frac{m_t}{w_t + \tau_t} = \pi \tag{20}$$

where γ_t^{pc} is the reserves to deposits ratio under a perfectly competitive banking system.

Due to logarithmic preferences, banks allocate a constant fraction of their deposits into cash reserves. That is, the demand for cash reserves does not depend on the return to different assets. This occurs because the income and substitution effects from different rates of returns changes exactly offset each other.

Furthermore, using (13) and (19), the quantity of capital demanded by banks:

$$k_{t+1} = (1-\pi)(w(k_t) + \tau_t) \tag{21}$$

Finally, using (13) and (19) in (14) and (15), the relative return to depositors is:

$$\frac{r_t^n}{r_t^m} = I_t \tag{22}$$

where I_t is the gross nominal return to capital. In this manner, agents receive less insurance against liquidity risk when the cost of holding money increases.

4.1 General Equilibrium

I proceed to characterize the equilibrium for the economy with perfectly competitive banks. Equilibrium is characterized by a set of non-negative quantities,

(k_{t+1}, L_t, m_t) and prices, $\left(r_{t+1}, w_t, \frac{P_t}{P_{t+1}}\right)$ that clear output, capital, labor, and money markets.

In equilibrium labor receives its marginal product, (3), and the labor market clears, with $L_t = 1$. Substituting (2), (3) into (21) generates the equilibrium law of motion for capital:

$$k_{t+1} = \left(\frac{1}{1 + \frac{\pi}{\sigma(1-\pi)}} (1-\alpha) A \right) k_t^{\alpha} \tag{23}$$

Furthermore, from (1), (3), and (19), equilibrium in the money market requires that prices evolve such that:

$$\frac{P_{t+1}}{P_t} = \sigma \left(\frac{k_t}{k_{t+1}} \right)^{\alpha} \tag{24}$$

Equations (23) and (24) characterize the behavior of the economy at a given point in time.

I proceed to study the stationary behavior of the economy. To begin, imposing steady-state on (24), the long-run inflation rate is equal to the rate of money creation, $\frac{P_{t+1}}{P_t} = \sigma$. Furthermore, imposing steady-state on (23), the steady-state capital stock is given by

$$k^{PC} = \left(\frac{(1-\alpha) A}{1 + \frac{\pi}{1-\pi} \frac{1}{\sigma}} \right)^{\frac{1}{1-\alpha}} \tag{25}$$

where the superscript PC, designates the outcome under perfect competition. Incorporating the expression for transfers, (2) into (19), the steady-state amount of cash reserves held by the banking sector is:

$$m^{PC} = \frac{\pi}{1-\pi} \left(\frac{(1-\alpha) A}{1 + \frac{\pi}{1-\pi} \frac{1}{\sigma}} \right)^{\frac{1}{1-\alpha}} \tag{26}$$

Moreover, using (4) and (25) in the steady-state, the gross real return to capital and the nominal return to capital are respectively expressed by

$$r^{PC} = \frac{\alpha}{1-\alpha} \left(1 + \frac{1}{\sigma} \frac{\pi}{(1-\pi)} \right) \tag{27}$$

$$\left(\frac{r^n}{r^m} \right)^{PC} = I^{PC} = \frac{\alpha}{1-\alpha} \left(\sigma + \frac{\pi}{(1-\pi)} \right) \tag{28}$$

Proposition 1. *Suppose $\sigma \geq \underline{\sigma}_0$, where $\underline{\sigma}_0 = \frac{1-\alpha}{\alpha} - \frac{\pi}{1-\pi}$. Under this condition, a steady-state in an economy with a perfectly competitive banking sector exists and is unique.*

A steady-state in an economy with a perfectly competitive banking sector exists if money is dominated in rate of return and the self-selection condition is satisfied. From (22), the return to non-movers is at least as high to that of movers when money is dominated in rate of return. Moreover, using (27), the return to capital exceeds that to money if the inflation rate is sufficiently large. Finally, it is trivial to show that a perfectly competitive banking system generates a higher expected utility to depositors for all $I > 1$.

Furthermore, it is clear from (25) that a higher rate of money creation stimulates capital formation and income. When the banking sector is perfectly competitive, agents receive a higher amount of transfers from the government when the rate of money growth increase. This in turn raises the total amount of deposits available to be invested in the economy's assets. As banks supply more capital in the rental market, the real return to capital is lower under a higher rate of money creation. However, by (28) the relative return to capital is increasing with the rate of money growth. This in turn indicates that depositors receive a lower amount of insurance against idiosyncratic risk at higher inflation rates.

Welfare Analysis Under a Perfectly Competitive Banking System:

Upon substituting the equilibrium conditions, (25) – (28) into (18), the steady-state expected utility of a typical depositor is:

$$u^{PC} = \ln \frac{(1-\alpha)^{\pi + \frac{\alpha}{1-\alpha}} A^{\frac{1}{1-\alpha}}}{\left(\frac{1}{\pi} - \frac{\sigma - 1}{\sigma}\right)^{\pi}} \frac{\alpha^{1-\pi}}{\sigma^{\pi}} \left[\frac{1}{1 + \frac{\pi}{1-\pi}\frac{1}{\sigma}}\right]^{\frac{\alpha}{1-\alpha}} \tag{29}$$

We begin with the following observation:

Proposition 2.
i. Suppose $\alpha \geq \frac{1}{2}$. *Under this condition,* $\frac{du^{PC}}{d\sigma} < 0$ *for all* $\sigma \geq \underline{\sigma}$.
ii. Suppose $\alpha < \frac{1}{2}$.
a. If $\pi \leq \frac{1-2\alpha}{1-\alpha}$, $\frac{du^{PC}}{d\sigma} \leq 0$ *for all* $\sigma \geq \underline{\sigma}$.
b. If $\pi > \frac{1-2\alpha}{1-\alpha}$, $\frac{du^{PC}}{d\sigma} \geq (<)\, 0$ *for all* $\sigma \leq (>)\, \hat{\sigma}$, *where* $\hat{\sigma} = \frac{\alpha}{1-\alpha}\frac{1}{1-\pi} > \underline{\sigma}$.

The result in Proposition 2 indicates that a change in the rate of money creation has a non-monotonic effect on total welfare. In this setting, the welfare of depositors is affected by inflation through two primary channels. First, as discussed above, a higher rate of money creation promotes capital formation and increases total output. The higher amount of output raises the demand for labor exerting upward pressures on wages and welfare. However, the ability of banks to insure their depositors against random relocations shocks deteriorates when inflation is higher. Therefore, inflation adversely affects risk averse depositors' welfare through this channel.

In an economy where production is capital intensive, the return capital is highly sensitive to changes in the value of money. More importantly from (28), the marginal effects from a higher inflation rate on risk sharing are significant and dominate any gains that come about from higher wages. Therefore, inflation has adverse effects on total welfare and the Friedman rule where money and capital yield the same rate of return is optimal. When the banking sector is perfectly competitive, the Friedman rule also implies complete risk sharing.[12]

By comparison, if $\alpha < \frac{1}{2}$, the net impact of inflation on welfare depends on the extent of liquidity risk in the economy. In particular, when the level of liquidity risk in the economy is low, banks allocate a large fraction of their deposits towards capital investment. In this manner, slight deviations from the Friedman rule will bring little gain to depositors from capital formation. Therefore, welfare is strictly decreasing with the inflation rate when the degree of liquidity risk is low.

However, when agents are highly exposed to liquidity risk, banks hold a lot of cash reserves to meet the high anticipated demand for money. Consequently, little resource are devoted towards capital investment, exerting downward pressure on wage income and welfare. At the Friedman rule, agents are completely insured against idiosyncratic risk. However, their expected income is low when the probability of relocation is high. As a result, deviations from the Friedman rule can be welfare improving.

5 A Monopoly Banking Sector

I proceed to examine an economy where the banking sector is fully concentrated. That is, the population of bankers is equal to unity, $N = 1$. At the beginning of period t, the banker announces deposit rates, r_t^m and r_t^n. The bank exerts its market power by extracting all surplus from deposit markets. Hence, the participation constraint, (17) holds with equality.

Given that the banker only values old age consumption, all deposits are invested in asset markets. Therefore, the banker makes her portfolio and pricing decisions, $(m_t, k_{t+1}, r_t^m, r_t^n, r_{t+1})$ to maximize profits in $t+1$, Π_{t+1}

$$\Pi_{t+1} = \underset{m_t, k_{t+1}, r_t^m, r_t^n, r_{t+1}}{Max} r_{t+1}k_{t+1} + m_t \frac{P_t}{P_{t+1}} - \pi r_t^m (w_t + \tau_t) - (1-\pi) r_t^n (w_t + \tau_t) \tag{30}$$

subject to (13) and (14). Further, payments made to non-relocated agents are made out of the return from renting capital. The banker is willing to provide financial services only if she makes positive profits. Thus, the constraint on payments to non-movers is such that:

$$(1-\pi) r_t^n (w_t + \tau_t) < r_{t+1}k_{t+1} \tag{31}$$

Because the bank is the sole supplier of capital in the rental market, it faces a downward sloping demand for capital, (4). Consequently, it takes into account

[12] As I discuss in the following section, this is not the case when the banking system is not competitive.

that it must earn a lower return from capital under a higher level of investment.

In sum, the bank maximizes (30) subject to (4), (17), (13), (14), and (31). Substituting the binding constraints into the objective function, the problem is reduced into a choice of capital,

$$\Pi_{t+1} = \underset{k_{t+1}}{Max} \alpha A k_{t+1}^{\alpha} - \frac{(1-\pi)\,\pi^{\frac{\pi}{1-\pi}} e^{\frac{u}{1-\pi}} \left(\frac{P_{t+1}}{P_t}\right)^{\frac{\pi}{1-\pi}}}{(w_t + \tau_t - k_{t+1})^{\frac{\pi}{1-\pi}}} \tag{32}$$

The profit maximizing choice of capital is such that

$$\Pi_1 \equiv \frac{\partial \Pi_{t+1}}{\partial k_{t+1}} = \alpha^2 A k_{t+1}^{\alpha-1} - \frac{\pi}{1-\pi} \frac{\pi^{\frac{\pi}{1-\pi}} \left(\frac{P_{t+1}}{P_t}\right)^{\frac{\pi}{1-\pi}} (1-\pi)\, e^{\frac{u}{1-\pi}}}{(w_t + \tau_t - k_{t+1})^{\frac{\pi}{1-\pi}+1}} = 0 \tag{33}$$

Where the term, $\alpha^2 A k_{t+1}^{\alpha-1}$ reflects the marginal revenue from renting one unit of capital to firms and $\frac{\pi}{1-\pi} \frac{\pi^{\frac{\pi}{1-\pi}} \left(\frac{P_{t+1}}{P_t}\right)^{\frac{\pi}{1-\pi}} (1-\pi) e^{\frac{u}{1-\pi}}}{(w_t+\tau_t-k_{t+1})^{\frac{\pi}{1-\pi}+1}}$ is the marginal cost of a unit of capital. The marginal cost of capital to the bank is the additional return that must be paid to non-movers under a higher level of capital formation. Specifically, under a higher rate of capital formation, the bank must pay capital goods producers a higher price to stimulate production. The higher amount investment requires the bank to cut its money holdings and thus making lower payments to relocated agents. In order to induce agents to participate in financial markets, the bank must pay a higher return to agents (as a group) in the event they do not relocate.

Using (13) and (33), the equilibrium amount of cash holdings by the bank is:

$$m_t = \begin{cases} \frac{\pi}{\alpha^{1-\pi} I_t^{1-\pi}} (w_t + \tau_t) & \text{if } I_t \le \frac{1}{1-\pi} \\ \frac{(1-\pi)^{1-\pi} \pi^{1+\pi}}{\alpha^{1-\pi} \left(1 - \frac{1}{I_t}\right)^{\pi}} (w_t + \tau_t) & \text{if } I_t > \frac{1}{1-\pi} \end{cases} \tag{34}$$

Equivalently, the fraction of deposits allocated towards cash reserves is:

$$\gamma_t = \begin{cases} \frac{\pi}{\alpha^{1-\pi} I_t^{1-\pi}} & \text{if } I_t \le \frac{1}{1-\pi} \\ \frac{(1-\pi)^{1-\pi} \pi^{1+\pi}}{\alpha^{1-\pi} \left(1 - \frac{1}{I_t}\right)^{\pi}} & \text{if } I_t > \frac{1}{1-\pi} \end{cases} \tag{35}$$

In contrast to the perfectly competitive case, the bank allocates a smaller fraction of its deposits into cash reserves when the return to capital is higher. This happens because the bank generates more profits from capital investment.

Furthermore, using (17), (14), and (34), the relative return to depositors is:

$$\frac{r_t^n}{r_t^m} = \alpha I_t \tag{36}$$

Unlike a competitive banking system, complete risk sharing is not achieved when money and capital yield the same rate of return (the Friedman Rule).

This happens because the bank uses the marginal revenue from capital to make its investment decision. Interestingly, at the Friedman rule, $I_t = 1$ and $r_t^n < r_t^m$, which induces the moral hazard problem as non-movers will claim to be movers. Therefore, a fully concentrated banking system cannot operate when the nominal return to capital is zero. This imposes a lower bound on the nominal return to capital for the bank to operate. Specifically, $\frac{r_t^n}{r_t^m} \geq 1$ if $I_t \geq \underline{I} = \frac{1}{\alpha}$.

5.1 General Equilibrium

Upon substituting (2) into (34), the general equilibrium demand for money balances is:

$$m_t = \begin{cases} \dfrac{w_t}{\frac{(\alpha I_t)^{1-\pi}}{\pi} - \frac{\sigma-1}{\sigma}} & \text{if } I_t \leq \frac{1}{1-\pi} \\ \dfrac{w_t}{\frac{\alpha^{1-\pi}\left(1-\frac{1}{I_t}\right)^{\pi}}{(1-\pi)^{1-\pi}\pi^{1+\pi}} - \frac{\sigma-1}{\sigma}} & \text{if } I_t > \frac{1}{1-\pi} \end{cases} \tag{37}$$

Furthermore, using (2), (3), and (37) into the bank's balance sheet, (13), the supply of capital goods by the bank is such that:

$$k_{t+1} = \begin{cases} \left(1 - \frac{1}{\sigma}\dfrac{1}{\frac{(\alpha I_t)^{1-\pi}}{\pi} - \frac{\sigma-1}{\sigma}}\right)(1-\alpha) A k_t^{\alpha} & \text{if } I_t \leq \frac{1}{1-\pi} \\ \left(1 - \frac{1}{\sigma}\dfrac{1}{\frac{\alpha^{1-\pi}\left(1-\frac{1}{I_t}\right)^{\pi}}{(1-\pi)^{1-\pi}\pi^{1+\pi}} - \frac{\sigma-1}{\sigma}}\right)(1-\alpha) A k_t^{\alpha} & \text{if } I_t > \frac{1}{1-\pi} \end{cases} \tag{38}$$

whereas from (4) and the definition of the nominal return to capital, the demand for capital goods is expressed by:

$$I_t = \alpha A k_{t+1}^{\alpha-1} \frac{P_{t+1}}{P_t} \tag{39}$$

Additionally, by the substitution of $(3), (34)$, and (39) into (1), the money market clearing condition yields the evolution of the nominal return to capital:

$$I_{t+1} = \begin{cases} \frac{1}{\alpha}\left(\dfrac{k_{t+1} I_t\left(\frac{(\alpha I_t)^{1-\pi}}{\pi} - \frac{\sigma-1}{\sigma}\right)}{\sigma\alpha A k_t^{\alpha}} + \frac{\sigma-1}{\sigma}\right)^{\frac{1}{1-\pi}} \pi^{\frac{1}{1-\pi}} & \text{if } I_t \leq \frac{1}{1-\pi} \\ \left[1 - \left(\dfrac{I_t\left(\frac{\alpha^{1-\pi}\left(1-\frac{1}{I_t}\right)^{\pi}}{(1-\pi)^{1-\pi}\pi^{1+\pi}} - \frac{\sigma-1}{\sigma}\right)k_{t+1}}{\sigma\alpha A k_t^{\alpha}} + \frac{\sigma-1}{\sigma}\right)^{\frac{1}{\pi}} \dfrac{(1-\pi)^{\frac{1-\pi}{\pi}}\pi^{\frac{1+\pi}{\pi}}}{\alpha^{\frac{1-\pi}{\pi}}}\right]^{-1} & \text{if } I_t > \frac{1}{1-\pi} \end{cases} \tag{40}$$

The loci defined by (38) and (40) characterize the behavior of the economy under a monopolistic banking sector at each point in time.

As in the previous section, I focus on the steady-state behavior of the economy. By imposing steady-state on the system of equations (38)-(40) and using the expression for the demand for capital, (4), the stationary nominal return to capital that clears the capital market is the solution to:

$$\psi(I) = 1 \tag{41}$$

where

$$\psi(I) = \begin{cases} \frac{\alpha\sigma}{(1-\alpha)I} + \frac{1}{1+\left[\frac{(\alpha I)^{1-\pi}}{\pi}-1\right]\sigma} & \text{if } I \le \frac{1}{1-\pi} \\ \frac{\alpha\sigma}{(1-\alpha)I} + \frac{1}{1+\left(\frac{\alpha^{1-\pi}\left(1-\frac{1}{I}\right)^{\pi}}{(1-\pi)^{1-\pi}\pi^{1+\pi}}-1\right)\sigma} & \text{if } I > \frac{1}{1-\pi} \end{cases} \tag{42}$$

The function $\psi(I)$ represents the bank's total assets (net of transfers) as a fraction of total wages. More specifically, it indicates how the bank's net asset position changes with the nominal return to capital. The the term $\frac{\alpha\sigma}{(1-\alpha)I}$ is the capital-wage ratio, $\frac{k}{w}$, while the other term in the equation is the money net of transfers to wage ratio, $\frac{m-\tau}{w}$. The following Lemma characterizes the behavior of ψ.

Lemma 1. *$\psi'(I) < 0$ and $\lim_{I\to\infty} \psi \to \underline{\psi} = \frac{1}{\left(\frac{\alpha^{1-\pi}}{(1-\pi)^{1-\pi}\pi^{1+\pi}}-1\right)\sigma+1}$. Moreover, $\psi\left(\frac{1}{\alpha}\right) = \frac{\alpha^2\sigma}{(1-\alpha)} + \frac{1}{\left(\frac{\alpha^{1-\pi}(1-\alpha)^{\pi}}{(1-\pi)^{1-\pi}\pi^{1+\pi}}-1\right)\sigma+1}$ if $\pi < 1-\alpha$ and $\psi\left(\frac{1}{\alpha}\right) = \frac{\alpha^2\sigma}{(1-\alpha)} + \frac{1}{1+\left[\frac{1-\pi}{\pi}\right]\sigma}$ if $\pi > 1-\alpha$.*

The result in Lemma 1 indicates that the bank's asset holdings is strictly decreasing with the return to capital. Intuitively, consumer goods firms demand less capital under higher rental rates which induces the bank to hold less capital. Moreover, from the description of the money demand equation, the bank holds less cash reserves when capital yields a higher return. Both effects explain the behavior of $\psi(I)$.

Additionally, we distinguish between two cases. First, if $\pi < 1-\alpha$, the lower bound on the return to capital, $\underline{I}$ exceeds $\frac{1}{1-\pi}$. Therefore, in equilibrium, it must be that $I > \frac{1}{\alpha} > \frac{1}{1-\pi}$ and agents invest in both capital and money under financial autarky. This case is illustrated in Figure 1 below. By comparison, if $\pi > 1-\alpha \iff \frac{1}{\alpha} < \frac{1}{1-\pi}$, agents only devote resources towards cash balances when they self-insure against liquidity risk, this necessarily happens when $I < \frac{1}{1-\pi}$.

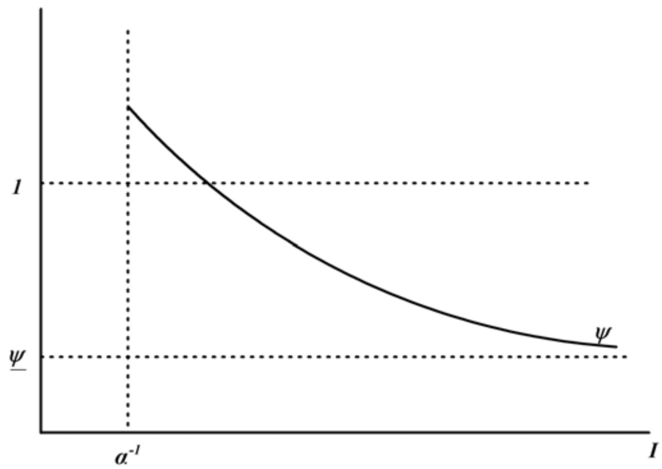

Figure 1. Equilibrium Under Monopolistic Banking, $\pi < 1 - \alpha$

I proceed to examine the existence and uniqueness of steady-state equilibrium in the following Proposition. All the proofs can be found in the appendix.

Proposition 3. *Suppose $\sigma > \max(\underline{\sigma}_1, \sigma_2)$. Under this condition, a steady-state where the banking system is fully concentrated exists and is unique.*

From our characterization of $\psi(I)$, the polynomial (41) has a unique non-trivial solution as illustrated in the Figure above. An equilibrium where the banking system is fully concentrated exists if the incentive compatibility constraint, (36) holds, money is dominated in rate of return, and the bank earns non-negative profits.

To begin from our analysis of the self-selection constraint, (36), the equilibrium return to non-movers is at least as high as that to movers if $I \geq \frac{1}{\alpha}$.This happens if the return to money is sufficiently low. Consequently, the rate of inflation must exceed a certain threshold, $\underline{\sigma}_1$. For all $\sigma \geq \underline{\sigma}_1$, workers intermediate their savings. Moreover given that $\alpha < 1$, this also implies that money is dominated in rate of return and the bank holds non-negative amounts of both money and capital. Obviously, for all $\sigma < \underline{\sigma}_1$ depositors will self-insure against relocation shocks and the bank will not operate.

Moreover, using the equilibrium conditions, (12), (13), and (33), the steady-state maximized profits of the bank are:

$$\Pi = \left[\frac{1}{(1-\alpha)} - \frac{(1-\pi)}{\pi}\left[I - \frac{\alpha\sigma}{(1-\alpha)}\right]\right]\alpha w \tag{43}$$

A quick look at (43) indicates that a necessary and sufficient condition for positive profits is:

$$I < \left[\frac{\pi}{1-\pi} + \alpha\sigma\right]\frac{1}{1-\alpha} = \bar{I}$$

which puts an upper (lower) bound on the nominal return to capital (capital formation). That is, we need the capital stock to be high enough. Notably, $I^* < \bar{I}$ if $\psi\left(\bar{I}\right) < 1$. As we discuss below, the bank earns higher profits at higher inflation rates. Therefore, equilibrium profits are realized when inflation is high enough, $\sigma > \sigma_2$.

Proposition 4. *$\frac{dk}{d\sigma} > 0$, $\frac{dI}{d\sigma} > 0$, $\frac{d\Pi}{d\sigma} > 0$, and $\frac{dr}{d\sigma} < 0$.*

In order to better understand how monetary policy affects the economy when the banking sector is fully concentrated, one may want to take a closer look at the profit maximizing choice of capital, (33). In contrast to a perfectly competitive banking system, monetary policy affects the economy through three primary channels. First, a higher rate of money growth reduces the value of money and the real value of payments made to depositors in the event they relocate. In order to prevent deposit withdrawals, the bank has to compensate its depositors in the good state (in the event they do not relocate). This in turn raises the marginal cost of capital investment and induces the bank to hold a more liquid portfolio. Second, when money loses some of its value, agents anticipate a lower utility under financial autarky, which reduces the marginal cost of capital investment as the bank has to pay its depositors less in the good state. Finally, as under a competitive banking system, depositors receive higher transfers under a higher rate of money growth, which raises depositors and stimulates investment in the economy's assets. Overall, an inflationary monetary policy promotes capital formation in the economy (Tobin effect). The higher amount of capital raises the bank's profits albeit it reduces the real return to capital.

I proceed to examine how different economic outcomes vary with the industrial organization of the banking system. Let the variables under a monopolistic banking system be indexed with the superscript "$Mono$".

Proposition 5.

i. Suppose $\pi < 1-\alpha$. Under this condition, $\sigma \leq (>)\, \sigma_3$, $I^{Mono} \geq (<)\, I^{PC}$ and $k^{Mono} \leq (>)\, k^{PC}$. However, $\left(\frac{r^n}{r^m}\right)^{Mono} < \left(\frac{r^n}{r^m}\right)^{PC}$ for all $\sigma > \max\left(\sigma_0, \sigma_2\right)$.

ii. Suppose $\pi > 1-\alpha$. Under this condition, $I^{Mono} < I^{PC}$, $k^{Mono} > k^{PC}$, and $\left(\frac{r^n}{r^m}\right)^{Mono} < \left(\frac{r^n}{r^m}\right)^{PC}$.

In contrast to the standard literature on Industrial organization, Proposition 5 suggests that market power in banking can promote capital formation and raise total output. As conventional wisdom might suggest, the bank has an incentive to restrict capital investment to raise the return from capital relative to perfect competition. However, its willingness and ability to do so from a general equilibrium perspective depend on other factors in the economy such as the degree of liquidity risk and the value of money, which affect the marginal cost of investing in capital markets.

In particular, if agents are highly exposed to liquidity risk, their need to intermediate savings is very high. More importantly, agents anticipate a low expected utility if they were to self-insure against idiosyncratic shocks. Market power in the deposit market enables the bank to make very low payments to its depositors, which reduces the marginal cost of investing in capital goods. This effect offsets the willingness of the bank to distort capital markets. Consequently, a fully concentrated banking system promotes capital formation compared to perfect competition when agents are highly exposed to liquidity risk. By diminishing returns, the nominal and real returns to capital are unambiguously lower when the banking system is not competitive.

By comparison, for a given stance of monetary policy, depositors must receive a higher expected utility when the degree of liquidity risk in the economy is low. The reservation utility is even higher when the value of money is high as depositors hold an abundant amount of cash under financial autarky. The high marginal cost of investment and lower marginal revenue (compared to perfect competition) drive the bank to significantly distort financial markets. Therefore, a fully concentrated banking system hinders capital formation when the degree of liquidity risk and inflation are both low. However, this is not the case when inflation is high enough. Sufficiently high inflation rates erode the value of money and the ability of depositors to self-insure against random relocation shocks. Therefore, the marginal cost of investment is also low. This in turn causes a fully concentrated banking system to promote capital formation vis-a-vis perfect competition.

Finally, as I demonstrate in the appendix, the monopoly bank always offers better insurance against liquidity shocks in comparison to perfectly competition. This result is driven by the profit maximizing motive of the bank as it has an incentive to make low payments to its depositors in the good state of nature (in the event they do not relocate) to make more profits. In order to retain their deposits at the bank, depositors are compensated in the event they relocate. Therefore, market power in the banking sector reduces the variability of depositors' rate of return.

I proceed to study the effects of inflation on steady-state welfare and the implications of banking competition for welfare. Following Paal and Smith (2000) and Matsuoka (2011), I use the expected utility of depositors as a proxy for welfare. As I demonstrate in the appendix, similar insights can be obtained if the bank's profits are rebated to young depositors.[13]

Welfare analysis under a fully concentrated banking system:

[13] Alternatively, one may assume that the banker has logarithmic preferences. Given that she only values old age consumption, the solution to the problem is intact. Therefore, total welfare would include the utility of the banker. Numerical work suggests that similar conclusions to the ones obtained in Proposition 6 can be drawn under this welfare criteria. Numerical results can be furnished upon request.

The steady-state expected utility of a generation of depositors under a monopolistic banking system can be obtained by substituting (2) and (37) into (12) to generate:

$$u^{Mono} = \begin{cases} \ln\left(\frac{w\frac{1}{\sigma}}{1-\frac{\pi}{\alpha^{1-\pi}I^{1-\pi}}\frac{\sigma-1}{\sigma}}\right) & \text{if } I \leq \frac{1}{1-\pi} \\ \ln\frac{\frac{\alpha^{1-\pi}}{\pi}I^{1-\pi}w\frac{1}{\sigma}}{\left(\frac{\alpha^{1-\pi}\left(1-\frac{1}{I}\right)^{\pi}}{(1-\pi)^{1-\pi}\pi^{1+\pi}}-\frac{\sigma-1}{\sigma}\right)} & \text{if } I > \frac{1}{1-\pi} \end{cases} \tag{44}$$

We proceed with the following observation:

Proposition 6. *Suppose $\pi < 1-\alpha$. Under this condition, $\frac{du^{Mono}}{d\sigma} \geq (<)\, 0$ if $\sigma \leq (>)\, \hat{\sigma}$, where $\hat{\sigma}$ is defined in the appendix.*[14]

The intuition behind this result is straight forward. As under a perfectly competitive banking system, there is a trade-off between less risk sharing and higher income that comes about under a higher inflation rate. Due to the presence of a Tobin effect, depositors' income from wages is low when inflation levels are low. However, the bank is providing a significant amount of insurance against liquidity risk when the value of money is high. Consequently, taxing money holdings when inflation is initially low can be welfare improving. As I demonstrate in the appendix, $\hat{\sigma} < \underline{\sigma}_1$ when the probability of relocation is significantly low.[15] Therefore, welfare is strictly decreasing with inflation when π is sufficiently small and complete risk sharing is locally optimal. That is if we restrict the parameter space such that an economy with a fully concentrated banking system exists, the optimal rate of money growth is such that: $\sigma^*_{Mono} = \underline{\sigma}_1 > \sigma_0$. However, as we point out below, the globally optimal rate of money growth is the one that induces agents to self-insure against risk. By comparison, numerical work indicates that $\hat{\sigma} > \underline{\sigma}_1$ when agents are highly exposed to liquidity risk. Consequently, incomplete risk sharing is optimal. This result suggests that the ability of the banking system to operate and function efficiently depends on the degree of liquidity risk in the economy.

In order to highlight this issue, I construct a numerical example using the following parameters: $A = 1$, $\alpha = 0.35$. Next, I examine how the inflation - welfare relationship depends on the probability of relocation by setting, $\pi = .25$ and $\pi = .95$, which I respectively illustrate in Figures 1-2 and 3 below.

Unlike a perfectly competitive banking system, financial intermediation does not always dominate self-insurance on welfare grounds when the banking system is not competitive. More importantly, the Friedman rule cannot sustain a banking equilibrium when the banking system is not competitive. For instance, when the degree of liquidity risk in the economy is relatively low as in Figure 1

[14] Although the analytical work in Proposition 6 focuses on cases where $\pi < 1-\alpha$, similar insights are obtained for $\pi > 1-\alpha$. We elaborate on this issue below.

[15] In the appendix, I provide a necessary and sufficient condition for $\hat{\sigma}$ to exceed $\underline{\sigma}_1$. However, I omit discussing the condition in the text as it is too complicated for insights to be drawn from.

below, welfare is strictly decreasing with the inflation rate when savings are intermediated. Although reducing the rate of money creation below $\underline{\sigma}_1$(or σ^*_{Mono} in the figure) causes a discrete drop in depositors' welfare due to agents opting out from the banking system, agents can do much better on their own when the value of money is sufficiently high. In deed, financial autarky is optimal in this case when the banking sector is not competitive. The optimal rate of money creation, $\sigma^*_{autarky}$ is also far above the Friedman rule level. From (11), agents will not invest in capital at the Friedman rule. From a general equilibrium perspective, this leads to the trivial equilibrium, where agents receive no consumption, which is obviously not an optimal resource allocation.

In Figure 2, we illustrate total welfare under both competitive structures. As observed in the Figure, perfect competition dominates both financial autarky and a fully concentrated banking system on welfare grounds when π is low. As indicated in the discussion of Proposition 2, the Friedman rule is the optimal policy in an economy with a perfectly competitive banking system when the degree of liquidity risk in the economy is below some threshold level. The Friedman rule also implies complete risk sharing under perfect competition. Notably, $\underline{\sigma}_1 > \sigma_0$. Therefore, if a banking economy under both competitive structures exists, the optimal rate of money growth is much higher when the banking system is fully concentrated.

By comparison, when the probability of relocation is sufficiently high, the welfare function is bell-shaped under both competitive structures. Moreover, welfare is always higher when the banking system is not competitive compared to financial autarky. As observed in Figure 3, a fully concentrated banking system yields a higher total welfare compared to perfect competition when inflation is below some threshold level. This occurs for two reasons. First, as discussed above, the level of output and wages are much higher under a monopoly banking system when agents are highly exposed to liquidity risk. Furthermore, depositors' anticipate a higher utility from self-insuring when the value of money is high. These two reasons combined enable a fully concentrated banking system to improve welfare at low inflation rates.

Our numerical result also indicates that optimal monetary policy is more restrictive when the banking system is not competitive. This occurs because the monopoly bank is holding less cash reserves (more capital) relative to a competitive banking when the probability of relocation is high enough. Therefore, it is optimal to impose a higher tax on money when the banking system is more competitive to stimulate capital investment.

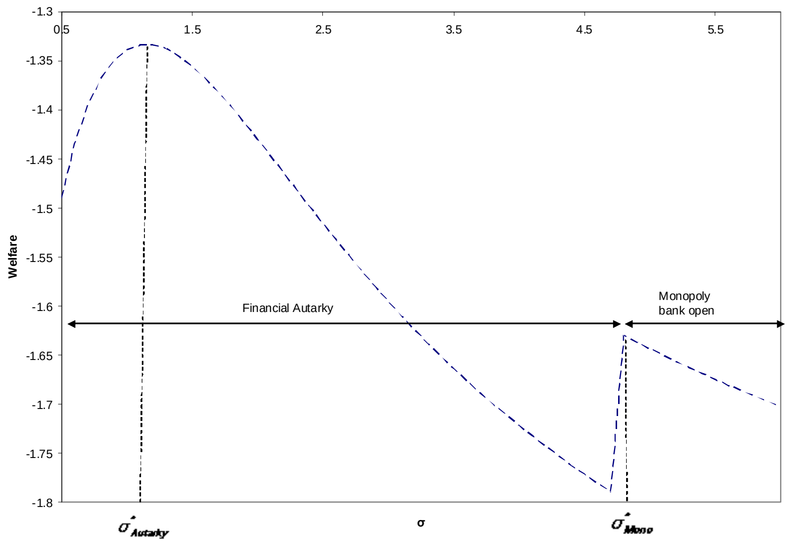

Figure 1. Welfare Under Monopoly, low π

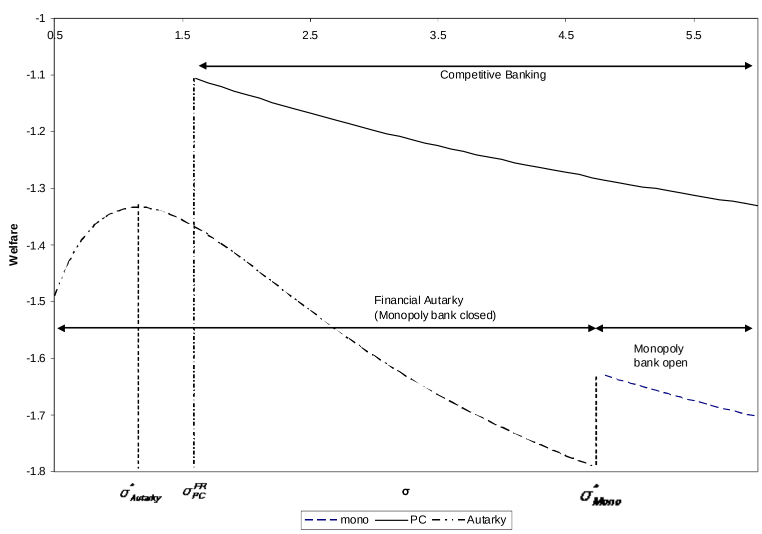

Figure 2. Welfare and Banking Competition, low π

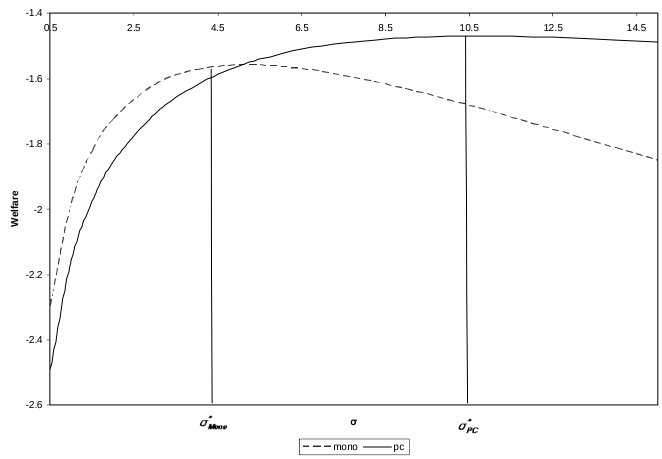

Figure 3. Welfare and Banking Competition, high π

6 Conclusion

Does the formulation of monetary policy depend on the industrial organization of the banking system? The answer to this question is simply, yes. In a setting where banks insure their depositors against idiosyncratic liquidity shocks, I study optimal monetary policy under two competitive banking structures: perfect competition and monopolistic. Unlike previous work by Matsuoka (2011), market power can be exerted in both deposit and capital markets. Interestingly, market power in capital markets exacerbates information frictions, which renders the Friedman rule sub-optimal when the banking system is fully concentrated. Furthermore, I demonstrate that it is optimal to impose a higher inflation tax when the banking system is not competitive if lack of competition is highly distortionary. In contrast, if a monopolistic banking system stimulates capital formation, monetary policy should be more expansionary under perfect competition.

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7 Technical Appendix

1. **Proof of Proposition 3**. From the discussion in the text, existence of an economy with a fully concentrated banking system requires that the incentive compatibility constraint holds, (16), which takes place if $I \geq \frac{1}{\alpha}$. Additionally, as discussed in the text, the bank earns non-negative profits if $I < \bar{I}$.

If $\pi < 1 - \alpha$, then $\frac{1}{\alpha} > \frac{1}{1-\pi}$. Therefore by Lemma 1, $\psi\left(\frac{1}{\alpha}\right) \equiv \frac{\alpha^2\sigma}{(1-\alpha)} + \frac{1}{\left(\frac{\alpha^{1-\pi}(1-\alpha)^{\pi}}{(1-\pi)^{1-\pi}\pi^{1+\pi}} - 1\right)\sigma + 1}$. In this manner, $I \geq \frac{1}{\alpha}$ if $\psi\left(\frac{1}{\alpha}\right) \geq 1$. With some simplifying algebra, this condition reduces to:

$$\sigma \geq \left(\frac{(1-\alpha)}{\alpha^2} - \frac{1}{\left(\frac{\alpha^{1-\pi}(1-\alpha)^{\pi}}{(1-\pi)^{1-\pi}\pi^{1+\pi}} - 1\right)}\right) = \underline{\sigma}_1$$

where $\underline{\sigma}_1 : \psi\left(\frac{1}{\alpha}\right) = 1$.

Finally, steady-state profits are positive if $\psi\left(\bar{I}\right) < 1$, where $\bar{I} = \left[\frac{\pi}{1-\pi} + \alpha\sigma\right]\frac{1}{1-\alpha}$ as defined in the text. Upon substituting $\bar{I}$ into $\psi(I)$, profits are positive if:

$$\frac{1-\alpha}{\frac{\pi}{1-\pi} + \alpha\sigma} + \left[1 + \alpha\frac{1-\pi}{\pi}\right]^{\frac{1}{\pi}} \frac{(1-\pi)^{\frac{1-\pi}{\pi}}\pi^{\frac{1+\pi}{\pi}}}{\alpha^{\frac{1-\pi}{\pi}}} < 1$$

Equivalently, we need:

$$\sigma > \left[\frac{1-\alpha}{1 - \left[1 + \alpha\frac{1-\pi}{\pi}\right]^{\frac{1}{\pi}} \frac{(1-\pi)^{\frac{1-\pi}{\pi}}\pi^{\frac{1+\pi}{\pi}}}{\alpha^{\frac{1-\pi}{\pi}}}} - \frac{\pi}{1-\pi}\right]\frac{1}{\alpha} = \sigma_2$$

In this manner, a steady-state exists and is unique when $\sigma > \max\left(\underline{\sigma}_1, \sigma_2\right)$. Given that ψ is monotonically decreasing in I, this condition also implies that $\bar{I} > \frac{1}{\alpha}$, which is necessary for existence.

Next, suppose $\pi > 1 - \alpha \iff \frac{1}{\alpha} < \frac{1}{1-\pi}$. Under this parameter space, $\psi\left(\frac{1}{\alpha}\right) = \frac{\alpha^2\sigma}{(1-\alpha)} + \frac{1}{1+\left[\frac{1-\pi}{\pi}\right]\sigma}$ by Lemma 1. Therefore, the incentive compatibility constraint holds if:

$$\sigma \geq \left[\frac{(1-\alpha)}{\alpha^2} - \frac{\pi}{1-\pi}\right] = \underline{\sigma}_3$$

Using the expression for $\bar{I}$, $\bar{I} > \frac{1}{\alpha}$if

$$\sigma > \left[\frac{1-\alpha}{\alpha} - \frac{\pi}{1-\pi}\right]\frac{1}{\alpha} > \underline{\sigma}_3$$

Finally, the condition for positive profits is identical to the case where $\pi < 1-\alpha$. Therefore, when $\sigma > \max\left(\underline{\sigma}_1, \sigma_2\right)$, a steady-state in an economy where the banking system is fully concentrated always exists and is unique. This completes the proof of Proposition 3.

2. **Proof of Proposition 4**. We begin by showing that $\frac{dI}{d\sigma} > 0$. First, suppose $\pi < 1-\alpha$, where $I > \frac{1}{\alpha} > \frac{1}{1-\pi}$ in equilibrium. Differentiating (41) with respect to σ we get:

$$\frac{dI}{d\sigma} = \frac{\frac{\alpha}{(1-\alpha)}}{\left[\frac{\pi}{I-1}\left(\frac{1-\alpha}{\alpha}I - \sigma\right) + 1\right]} > 0$$

where using Cobb-Douglas, the bank's balance sheet condition, (13), and the fact that $\tau = \frac{\sigma-1}{\sigma}m$, we have: $\Omega = \frac{k}{w(k)} = \frac{\alpha\sigma}{(1-\alpha)I} = 1 - \frac{1}{\sigma}m < 1$. Therefore, $\frac{1-\alpha}{\alpha}I - \sigma > 0$.

Next, differentiate $\frac{\alpha\sigma}{(1-\alpha)I}$ with respect to σ, to get:

$$\frac{d\Omega}{d\sigma} = \frac{\alpha}{(1-\alpha)}\left(\frac{1}{I} - \frac{\sigma}{I^2}\frac{dI}{d\sigma}\right)$$

where $\frac{d\Omega}{d\sigma} > 0$ if: $\frac{\sigma}{I}\frac{dI}{d\sigma} < 1$. Using the expression for $\frac{dI}{d\sigma}$ derived above and some algebra, $\frac{\sigma}{I}\frac{dI}{d\sigma} < 1$ if:

$$0 < \left(\frac{\pi}{I-1}\frac{1-\alpha}{\alpha}I + 1\right)\left(1 - \frac{\sigma}{I}\frac{\alpha}{(1-\alpha)}\right)$$

which always holds. Therefore, $\frac{d\Omega}{d\sigma} > 0$, which also implies that $\frac{dk}{d\sigma} > 0$ give that $\Omega = \frac{k}{w(k)} = \frac{k^{1-\alpha}}{(1-\alpha)A}$ is increasing in k. By diminishing returns, this result also implies that $\frac{dr}{d\sigma} < 0$.

I proceed to study the effects of inflation on the bank's profits. To begin, I re-write the expression for the bank's profits, (43) :

$$\Pi = \left\{\frac{1}{(1-\alpha)} - \frac{(1-\pi)}{\pi}\left[I - \frac{\alpha\sigma}{(1-\alpha)}\right]\right\}\alpha w$$

Define the term in curly bracket by $\Gamma(\sigma) = \left\{\frac{1}{(1-\alpha)} - \frac{(1-\pi)}{\pi}\left[I - \frac{\alpha\sigma}{(1-\alpha)}\right]\right\}$. It is easily found that:

$$\frac{d\Pi}{d\sigma} = \frac{d\Gamma}{d\sigma}\alpha w + \alpha\Gamma\frac{dw}{d\sigma}$$

where $\frac{dw}{d\sigma} > 0$ by the Tobin effect. Therefore, a sufficient condition for profits to increase under a higher rate of money creation is that $\frac{d\Gamma}{d\sigma} > 0$, where $\frac{d\Gamma}{d\sigma} = \left[\frac{\alpha}{1-\alpha} - \frac{dI}{d\sigma}\right]\frac{(1-\pi)}{\pi}$ and $\frac{d\Gamma}{d\sigma} > 0$ if $\frac{dI}{d\sigma} < \frac{\alpha}{1-\alpha}$. Using the expression for $\frac{dI}{d\sigma}$ derived above, $\frac{dI}{d\sigma} < \frac{\alpha}{1-\alpha}$ simplifies to:

$$\frac{1-\alpha}{\alpha}I - \sigma > 0$$

which always holds as shown above. Consequently, $\frac{d\Pi}{d\sigma} > 0$. It is trivial to show that these results also hold under the case where $\pi > 1-\alpha$. This completes the proof of Proposition 4.

3. **Proof of Proposition 5**. I begin by comparing the outcome under both competitive structures when $\pi < 1-\alpha$. The nominal return to capital is lower under a monopoly banking sector if:

$$\psi\left(I^{PC}\right) < 1$$

Upon substituting for the expression of I^{PC}, (28) into (41) and some simplifying algebra, this condition can be written as:

$$\sigma > \frac{(1-\alpha)}{\alpha} \frac{1}{\left[1 - \frac{(1-\pi)^{\frac{1-\pi}{\pi}}\pi}{\alpha^{\frac{1-\pi}{\pi}}}\right]} - \frac{\pi}{1-\pi} = \sigma_3$$

By definition of I, $I = r\sigma$. Therefore, $I^{Mono} \geq (<)\, I^{PC}$ and $k^{Mono} \leq (>)\, k^{PC}$ if $\sigma \leq (>)\, \sigma_3$.

I proceed to compare the amount of insurance received by depositors under both competitive structures. As I demonstrate in the text, the relative return to depositors under a monopoly bank is, $\frac{r^n}{r^m} = \alpha I^{Mono}$, while under perfect competition, we have $\frac{r^n}{r^m} = I^{PC}$. Therefore, the monopolist provides better insurance if:

$$I^{Mono} < \frac{1}{\alpha} I^{PC}$$

It is sufficient to show that $\frac{1}{\alpha} I^{PC} > \bar{I}$, where $\bar{I} : \Pi = 0$. If this is the case, then $I^{Mono} < \bar{I} < \frac{1}{\alpha} I^{PC}$, which implies better insurance under the monopoly banking system. Using the expression for $\bar{I}$, this condition is written as:

$$\frac{1}{\alpha} I^{PC} > \left[\frac{\pi}{1-\pi} + \alpha\sigma\right] \frac{1}{1-\alpha}$$

Substituting for I^{PC} and simplifying yields:

$$1 > \alpha$$

which always holds for all $\pi > 0$. Consequently, the monopolist always provides better risk sharing than a perfectly competitive banking system. This completes the first part of Proposition 5.

Next, suppose $\pi > 1-\alpha$. Interest rates are lower under a monopoly banking sector if: $\psi\left(I^{PC}\right) < 1$. Equivalently:

$$\frac{\alpha\sigma}{(1-\alpha)\,\sigma \frac{\alpha}{(1-\alpha)} \left(1 + \frac{\pi}{1-\pi}\frac{1}{\sigma}\right)} + \frac{1}{1 + \left[\frac{(\alpha I)^{1-\pi} - \pi}{\pi}\right]\sigma} < 1$$

Simplifying, this condition becomes:

$$\sigma > \frac{1-\alpha}{\alpha}\frac{1}{\alpha} - \frac{\pi}{1-\pi}$$

which always holds when both economies exist. In particular, a necessary condition for an economy with a monopolistic banking system and an economy with a perfectly competitive banking system to exist is that: $I^{PC} \geq \frac{1}{\alpha}$. Using the expression for I^{PC}, this condition becomes:

$$\sigma \geq \left[\frac{1-\alpha}{\alpha} - \frac{\pi}{1-\pi}\right]\frac{1}{\alpha}$$

which obviously implies that $\sigma > \frac{1-\alpha}{\alpha}\frac{1}{\alpha} - \frac{\pi}{1-\pi}$ give that $\alpha < 1$. As a result, $k^{Mono} > k^{PC}$, $I^{Mono} < I^{PC}$, and $\left(\frac{r^n}{r^m}\right)^{Mono} < \left(\frac{r^n}{r^m}\right)^{PC}$ when $\pi > 1-\alpha$ and both economies exist. This completes the proof of Proposition 5.

4. **Proof of Proposition 6.** Suppose $\pi < 1-\alpha$. From (44), the welfare of depositors under a monopoly bank is such that:

$$u^{Mono} = \ln \frac{\frac{\alpha^{1-\pi}}{\pi} I^{1-\pi} w \frac{1}{\sigma}}{\left(\frac{\alpha^{1-\pi}\left(1-\frac{1}{I}\right)^{\pi}}{(1-\pi)^{1-\pi}\pi^{1+\pi}} - \frac{\sigma-1}{\sigma}\right)}$$

Using the expression for wages, (3) and the equilibrium condition, (41) we can re-write the expected utility of a depositors as:

$$u^{Mono} = \ln \frac{\alpha^{1-\pi}}{\pi}\left(1 - \frac{\alpha\sigma}{(1-\alpha)I}\right) I^{1-\pi}(1-\alpha)Ak^{\alpha} \tag{45}$$

Finally, from the definition of I and the expression for the rental rate, (4), we have: $k = \left(\frac{\sigma\alpha A}{I}\right)^{\frac{1}{1-\alpha}}$. Substitute into the expected utility of depositors with some algebra to get:

$$u^{Mono} = \ln (1-\alpha)(\alpha A)^{\frac{\alpha}{1-\alpha}} A\frac{\alpha^{1-\pi}}{\pi}\left(I - \frac{\alpha\sigma}{(1-\alpha)}\right)\frac{1}{I^{\pi+\frac{\alpha}{1-\alpha}}}\sigma^{\frac{\alpha}{1-\alpha}}$$

or equivalently:

$$u^{Mono} = \ln (1-\alpha)(\alpha A)^{\frac{\alpha}{1-\alpha}} A\frac{\alpha^{1-\pi}}{\pi} + \ln\left(I - \frac{\alpha\sigma}{(1-\alpha)}\right) - \left(\pi + \frac{\alpha}{1-\alpha}\right)\ln I + \frac{\alpha}{1-\alpha}\ln\sigma$$

Differentiating with respect to σ yields:

$$\frac{du^{IC}}{d\sigma} = \frac{\frac{dI}{d\sigma} - \frac{\alpha}{(1-\alpha)}}{\left(I - \frac{\alpha\sigma}{(1-\alpha)}\right)} - \frac{\left(\pi + \frac{\alpha}{1-\alpha}\right)}{I}\frac{dI}{d\sigma} + \frac{\alpha}{1-\alpha}\frac{1}{\sigma}$$

Using the expression for $\frac{dI}{d\sigma}$ derived in the proof of Proposition 4, $\frac{du^{IC}}{d\sigma} \geq 0$ if:

$$\left[(1-\alpha)\frac{I}{\sigma} - 1\right]\frac{1}{\alpha}\frac{\pi}{1-\frac{1}{I}} + \frac{I}{\sigma} \geq \pi + \frac{\alpha}{1-\alpha} \tag{46}$$

Clearly, the term on the LHS of (46) is strictly decreasing with inflation since $\frac{d\left(\frac{I}{\sigma}\right)}{d\sigma} < 0$ (by the Tobin effect) and $\frac{dI}{d\sigma} > 0$. Therefore, define $\hat{\sigma}$ such that the above holds with equality. For all $\sigma \leq (>)\, \hat{\sigma}$, $\frac{du^{IC}}{d\sigma} \geq (<)\, 0$. This completes the proof of Proposition 6.

4. **Profits rebated to young agents:** Suppose that the government taxes away all the accrued profits by the bank. The procedure is transferred to young agents in the form of lump sum transfers. Therefore, young agents born in period t receive $w_t + \tau_t$, where now τ_t is such that:

$$\tau_t = \frac{\sigma - 1}{\sigma} m_t + \Pi_t \tag{47}$$

Since the bank takes the level of deposits as given, the general solution to the monopoly problem is identical to the work above for a given τ_t. Focusing on steady-state equilibrium, the bank's balance sheet implies that:

$$k = w - \frac{1}{\sigma} m + \Pi \tag{48}$$

where

$$\Pi = \alpha A k^{\alpha} - \frac{(1-\pi)\, e^{\frac{\underline{u}}{1-\pi}}}{\left(\frac{1}{\pi} m \frac{1}{\sigma}\right)^{\frac{\pi}{1-\pi}}}$$

and the reservation utility is expressed by (12).

The bank's problem yields:

$$m = \frac{\pi \sigma e^{\underline{u}}}{\alpha^{1-\pi} I^{1-\pi}} \tag{49}$$

Moreover, using (49) into the profit function to get:

$$\Pi = \alpha A k^{\alpha} - \alpha \frac{(1-\pi)}{\pi \sigma} I m \tag{50}$$

From (12), (47), and (49) we get the equilibrium amount of money demand:

$$m = \begin{cases} \dfrac{A k^{\alpha}}{\left[\frac{\alpha^{1-\pi} I^{1-\pi}}{\pi} - \frac{\sigma - 1}{\sigma} + \alpha \frac{(1-\pi)}{\pi \sigma} I\right]} & \text{if } I \leq \frac{1}{1-\pi} \\ \dfrac{A k^{\alpha}}{\left[\frac{\alpha^{1-\pi}\left(1 - \frac{1}{I}\right)^{\pi}}{(1-\pi)^{1-\pi} \pi^{1+\pi}} - \frac{\sigma - 1}{\sigma} + \frac{(1-\pi) I \alpha}{\pi \sigma}\right]} & \text{if } I > \frac{1}{1-\pi} \end{cases} \tag{51}$$

and upon substituting (50) and (51) into (48), the equilibrium nominal return to capital is the solution to the following polynomial: $\Gamma(I) = 1$, where

$$\Gamma(I) = \begin{cases} \alpha \frac{\sigma}{I} + \dfrac{\left[1 + \frac{(1-\pi) I \alpha}{\pi}\right]}{\left[\left(\frac{\alpha^{1-\pi} I^{1-\pi}}{\pi} - \frac{\sigma - 1}{\sigma}\right) \sigma + \alpha \frac{(1-\pi)}{\pi} I\right]} & \text{if } I \leq \frac{1}{1-\pi} \\ \alpha \frac{\sigma}{I} + \frac{1}{\sigma} \dfrac{1 + \frac{(1-\pi) I \alpha}{\pi}}{\left[\frac{\alpha^{1-\pi}\left(1 - \frac{1}{I}\right)^{\pi}}{(1-\pi)^{1-\pi} \pi^{1+\pi}} - \frac{\sigma - 1}{\sigma} + \frac{(1-\pi) I \alpha}{\pi \sigma}\right]} & \text{if } I > \frac{1}{1-\pi} \end{cases}$$

Due to the high non-linearity of the system above, I resort to numerical simulation to draw some insights from the model. In the following examples, I highlight that the primary results when the bank's profits are sunk hold when profits are rebated to young agents. In table 1 below, I consider the case where agents are highly exposed to liquidity risk, $\pi > 1 - \alpha$. As can be observed from the Table, total welfare is higher under a fully concentrated banking system when inflation is below some threshold level (gross inflation rate around 30). Furthermore, complete risk sharing is not optimal under the parameters considered.

By comparison, perfect competition dominates a fully concentrated banking system on welfare grounds when the probability of relocation is relatively low. The case where $\pi < 1 - \alpha$ is considered in Table 2 below. Moreover, welfare is strictly decreasing with the inflation rate under both competitive structures. Therefore, complete risk sharing is optimal.

It is important to note one difference in the results compared to the previous section. In particular, for a large set of parameters, the capital stock is always higher under a fully concentrated banking system compared to perfect competition. This happens because of the additional inter-generational transfers made when profits are rebated to young agents. The additional rebates raise depositors' savings and deposits, which enables the bank to invest more in the economy's assets. All the other results are identical.

α	0.35
A	1
π	0.95

Monopoly Bank

τ	0.065	0.361	1.092	1.460	1.639	1.812	2.142	2.446	2.724	2.977	3.205	4.450	4.922
σ	1.000	2.000	4.000	5.000	5.500	6.000	7.000	8.000	9.000	10.000	11.000	20.000	30.000
I	4.989	5.222	5.711	5.967	6.098	6.230	6.499	6.774	7.054	7.340	7.630	10.395	13.667
k	0.017	0.045	0.115	0.151	0.170	0.188	0.223	0.257	0.289	0.320	0.349	0.544	0.667
m	0.204	0.535	1.281	1.645	1.819	1.987	2.303	2.593	2.856	3.093	3.306	4.431	4.819
$w+\tau$	0.220	0.581	1.396	1.796	1.988	2.174	2.526	2.850	3.145	3.413	3.655	4.975	5.486
w	0.155	0.220	0.305	0.336	0.349	0.362	0.384	0.404	0.421	0.436	0.450	0.525	0.564
r	4.989	2.611	1.428	1.193	1.109	1.038	0.928	0.847	0.784	0.734	0.694	0.520	0.456
c^m	0.214	0.282	0.337	0.346	0.348	0.349	0.346	0.341	0.334	0.326	0.316	0.233	0.169
c^n	0.374	0.515	0.674	0.723	0.743	0.760	0.788	0.809	0.825	0.836	0.845	0.848	0.809
u	-1.512	-1.236	-1.052	-1.024	-1.017	-1.015	-1.019	-1.032	-1.051	-1.075	-1.102	-1.391	-1.699
Π	0.065	0.093	0.130	0.145	0.151	0.157	0.168	0.177	0.186	0.193	0.200	0.240	0.263
r^n/r^m	1.746	1.828	1.999	2.089	2.134	2.181	2.275	2.371	2.469	2.569	2.670	3.638	4.784

Perfect Competition

τ	0.000	0.131	0.498	0.701	0.805	0.908	1.115	1.319	1.519	1.713	1.902	3.330	4.450
σ	1.000	2.000	4.000	5.000	5.500	6.000	7.000	8.000	9.000	10.000	11.000	20.000	30.000
I	10.769	11.308	12.385	12.923	13.192	13.462	14.000	14.538	15.077	15.615	16.154	21.000	26.385
k	0.005	0.014	0.035	0.046	0.052	0.057	0.068	0.079	0.090	0.100	0.110	0.184	0.242
m	0.098	0.263	0.664	0.877	0.983	1.090	1.301	1.507	1.708	1.903	2.092	3.505	4.604
$w+\tau$	0.103	0.277	0.699	0.923	1.035	1.147	1.369	1.587	1.798	2.004	2.202	3.690	4.846
w	0.103	0.145	0.201	0.221	0.231	0.239	0.254	0.268	0.280	0.291	0.300	0.360	0.396
r	10.769	5.654	3.096	2.585	2.399	2.244	2.000	1.817	1.675	1.562	1.469	1.050	0.879
c^m	0.103	0.138	0.175	0.185	0.188	0.191	0.196	0.198	0.200	0.200	0.200	0.184	0.162
c^n	1.106	1.565	2.164	2.385	2.483	2.574	2.738	2.883	3.013	3.129	3.234	4.262	4.262
u	-2.157	-1.856	-1.619	-1.562	-1.541	-1.524	-1.500	-1.484	-1.475	-1.470	-1.469	-1.659	-1.659
Π	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
r^n/r^m	10.769	11.308	12.385	12.923	13.192	13.462	14.000	14.538	15.077	15.615	16.154	26.385	26.385

Table 1. Example 1, High π

α	0.35						
A	1						
π	0.25						
		Monopoly Bank			Perfect Competition		
τ	0.410	0.424	0.435		0.132	0.137	0.141
σ	6.030	7.000	8.000		6.030	7.000	8.000
l	2.860	3.258	3.671		3.426	3.949	4.487
k	0.626	0.645	0.659		0.474	0.480	0.484
m	0.335	0.337	0.338		0.158	0.160	0.161
$w+\tau$	0.962	0.982	0.997		0.633	0.640	0.645
w	0.552	0.557	0.562		0.501	0.503	0.504
r	0.474	0.465	0.459		0.568	0.564	0.561
c^m	0.222	0.193	0.169		0.105	0.091	0.081
c^n	0.223	0.220	0.217		0.359	0.361	0.362
u	-1.502	-1.549	-1.590		-1.331	-1.362	-1.391
Π	0.130	0.136	0.140		0.000	0.000	0.000
r^n/r^m	1.001	1.140	1.285		3.426	3.949	4.487

Table 2. Example 2, Low π