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Intergenerational Bargaining and Indeterminacy of Equilibria

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Intergenerational Bargaining and Capital Formation

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Abstract

Most studies that use an overlapping generations setting assume complete depreciation of capital or complete reversibility with partial depreciation. These assumptions are generally made to make the analysis more tractable. Departing from a simple version of the Diamond (1965) model, I demonstrate that relaxing these assumptions can generate multiplicity of equilibria - an issue that should be treated seriously in models that examine policy implications in overlapping generations production economies. More specifically, in this setting, used capital is traded by two different generations, who bargain over its price. While a higher degree of bargaining power to buyers promotes short-term growth through a lower cost of capital, it can also lead to development traps.

JEL Codes: E13, E21, D42

Keywords: Economic Development, Intergenerational Trade, Bargaining

1 Introduction

Previous studies that use overlapping generations models with production such as Diamond (1965) assume that capital is either completely reversible and/or completely depreciates in the production process. By imposing the assumption of complete depreciation, one does not need to worry about the transfer of capital over time. By comparison, if capital does not fully depreciate but is completely reversible, old capital goods can be traded across generations at the same price at which they were purchased in the previous period - assuming that it takes one period to build capital.

I extend the Diamond (1965) model by assuming that capital partially depreciates in the production process and is not completely reversible. This generates an opportunity for capital to be traded between two heterogenous groups of people: potential old sellers and potential young buyers. One may also treat intergenerational trade as a stock market that permits the transfer of capital

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across generations.[1,2] Agents bargain over a price at which trade may or may not take place in equilibrium. Following Townsend (1978), bilateral exchange is costly.

The outside option to each type of agent is as follows. Buyers have the ability to invest in capital from scratch at a price of one unit of goods. On the other hand, sellers can liquidate their matured capital into some amount of goods below unity.

In this setting, I demonstrate that multiple steady-states can arise when the extent of bargaining power to buyers is sufficiently high and trading costs are over an intermediate range. Specifically, there can be up to three steady-states that differ by their level of capital and price of capital goods. In the low capital economy, the price of capital is significantly low to induce the participation of sellers. Therefore, intergenerational trade never takes place. In the steady-state with an intermediate level of capital, trade takes place. However, the level of market capitalization is relatively low. Finally, trade unambiguously occurs in the economy with a significant level of capital formation. In this manner, trade takes place when market capitalization is high enough. Consequently, the level of economic activity is essential for financial market developments. In addition, financial development is also important for economic development.[3]

While intergenerational trade unambiguously leads to a higher level of capital formation and development, the net gains from trade are not determined. That is, it could lead to significant gains or little gains in capital accumulation. Finally, development traps can arise. If trade leads to little gains, a deviation from the steady-state could either lead to a higher level of capital formation or to the break down of capital markets.[4]

As multiple steady-states can occur when intergenerational bargaining is present bears important implications for work that uses an overlapping generations setting to address policy issues. For example, I demonstrate that a higher cost of trade (which can be thought as a higher tax or a less efficient financial system) has a non-linear effect on capital formation. Specifically, in economies with low levels of capital formation (less developed economies), higher transactions costs have significant adverse effects on asset prices and capital formation. The effects are much less significant in advanced economies.

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[1] Levine (1991) and Greenwood and Smith (1997) examine the liquidity role of the stock market. In their setting, the stock market provides a mechanism for members of the same generation to share liquidity risk. By comparison, the stock market in this model extends the life of capital goods beyond that of its owners. Thus, it provides intergenerational liquidity.

[2] Bencivenga, Smith, and Starr (1995) study a two-period overlapping generations economy where the maturity of capital can exceed the life of investors. In their setting, intergenerational trade of capital in progress occurs in a centralized market. However, capital completely depreciates in production.

[3] A large amount of work has been devoted to examine the relationship between financial development and economic activity. See for example, King and Levine (1993), Levine and Zervos (1998), and Levine (1997).

[4] This result is consistent with recent work by Minier (2003) who finds a non-linear relationship between financial development and economic growth. Specifically, the author finds that a positive correlation between financial development and economic growth only appears in economies with high degrees of stock market capitalization.

The paper is organized as follows. In Section 2, I describe the model and study equilibrium behavior in two different economies. In the first economy capital is exogenously not traded across generations, while in the second economy, a stock market is operating. The choice to participate in a stock market is endogenized in section 3. I offer concluding remarks in Section 4. Most of the technical details are presented in the Appendix.

2 Environment

Consider a Diamond (1965) discrete-time production economy populated by an infinite sequence of two-period lived overlapping generations. At the beginning of each period, t, with $t = 0, 1, ..,$ a continuum of young agents is born with unit mass.

Except for the initial old population, agents do not receive any physical endowments. However, each agent is born with one unit of labor effort which she supplies inelastically when young and is retired when old. Moreover, individuals only derive utility from old-age consumption, c, with preferences $u(c) = c$ and they do not discount the future.

The economy's single perishable consumption good is produced by a representative firm that has access to a constant returns to scale technology. The firm uses labor, L_t, and capital, K_t, to produce Y_t units of output. Total output per worker produced in period t is given by a standard Cobb-Douglas production function of the form, $y_t = k_t^\alpha$, where $k_t \equiv K_t/L_t$ is the capital-labor ratio and α is the capital share of total output.[5] Since the population size is constant and equal to one, all lower case variables also reflect their aggregate levels. Additionally, a fraction, δ of the capital stock is destroyed in the production process.

Due to perfect competition in factors market, labor and capital earn their marginal products, with:

$$w_t = w(k_t) = (1-\alpha) k_t^\alpha \tag{1}$$

and

$$r_t = \alpha k_t^{\alpha-1} \tag{2}$$

where w_t and r_t are respectively the rental rates of labor and capital in period t.

Capital is generated in the following manner. All savings of young workers are invested in capital goods. One unit of goods invested in capital by a young agent in period t becomes one unit of capital next period. Thus, the price of one unit of *new* capital is one unit of goods. Denote the per capita amount of investment in new capital goods by i_t. In contrast to standard neoclassical

[5] The results in this manuscript hold under a general production technology satisfying standard Inada conditions.

models, capital is assumed to be partially reversible after it matures. In particular, one unit of matured capital can be converted back into $\lambda \in [0,1]$ units of goods.[6] If $\lambda = 1$, capital is completely reversible as in Diamond (1965) and $\lambda = 0$, reflects the case where capital is completely irreversible.

As some capital remains in the hands of old agents, there is an opportunity for trade between young and old agents. Such trading opportunities clearly depend on the availability and the competitive structure of markets. Moreover, agents' portfolio composition, which consists of how much new and old capital to acquire, also depends on the availability and functioning of the market. As a benchmark, I assume that a re-sale market for capital goods does not exist. I refer to such situation as financial autarky.

I proceed to describe the timing of the events. Suppose each period is divided into two sub-periods. At the beginning of the first sub-period of period t, young agents are born and old agents receive their capital stock, k_t, which they acquired in the previous period. Subsequently, production takes place and factors are paid. Furthermore, young agents make their consumption and savings choice - which is trivial in this setting.

In sub-period 2, young buyers of capital are matched with old sellers in pairs. Following Townsend (1978), bilateral exchange is costly. In particular, I assume that young buyers incur a lump sum resource cost of τ units of goods from transacting with a seller.[7] Additionally, a buyer always meets a seller with probability one.[8] Once two trading parties meet, the price of used capital, p_t^k, is determined in a Nash bargaining game. Let $\theta \in (0,1]$, reflect the bargaining power of a buyer. Once the offer is accepted, old agents consume and die, while young agents leave the market.

2.1 Financial Autarky

In absence of a market for used capital, all young age income is invested (saved) in new capital goods, with

$$s_t = w_t = i_t \tag{3}$$

where s_t, is the level of savings of a typical young agent. Moreover, the amount of capital goods at the beginning of each period is strictly determined by the level of investment in the previous period as in models with complete depreciation:

[6] Equivalently, agents incur adjustment costs equal to $1 - \lambda$ units of goods per unit of capital.

[7] In Townsend (1978), each trading party incurs a fixed cost when exchange takes place. However, such an assumption is not essential for the results in this paper.

[8] As I discuss below, although there is one type of capital and agents are identical, a competitive equilibrium generates a continuum of solutions due to the age heterogeneity between traders and the irreversibility of investment. In addition, one may integrate a role for financial intermediation in this setting. Specifically, old agents have an incentive to coalite in order to gain bargaining power. Young agents' best response in this case is to also form a coalition, which can also help young traders economize on transactions costs. Under such a setting, bargaining takes place between two coalitions that behave in the interest of their members. It can be easily verified that similar insights can be generated by applying these changes. I elaborate on this issue in section 4 below.

$$k_{t+1} = i_t \tag{4}$$

because $(1-\delta) k_t$ units of capital are converted back into $(1-\delta) \lambda k_t$ units of goods.

In this manner, the return on a unit of capital (savings) in period t, is:

$$R_t = r_t + \lambda (1-\delta) \tag{5}$$

and therefore, the consumption of an old agent in period t is:

$$c_t = [r_t + \lambda (1-\delta)] k_t \tag{6}$$

2.1.1 Equilibrium Under Financial Autarky

I proceed to characterize the equilibrium behavior of the economy. In equilibrium, labor receives its marginal product, (1), and the labor market clears, with $L_t = 1$. Moreover, competition in factors market implies that capital earns its marginal product, (2). Therefore, using (2) and (5), the equilibrium return to capital and the consumption of a typical agent are respectively:

$$R_t = f'(k_t) + \lambda (1-\delta) \tag{7}$$

and

$$c_t = [f'(k_t) + \lambda (1-\delta)] k_t \tag{8}$$

Finally, using (3) and (4), the equilibrium behavior of the economy is summarized by:

$$k_{t+1} = w(k_t) \equiv \Psi(k_t) \tag{9}$$

which is the same law of motion in a standard Diamond (1965) economy, when the savings choice is trivial, population growth is zero, and with complete depreciation of capital.

The law of motion of capital, (9) is illustrated in Figure 1 below. It is easy to verify that a unique non-trivial steady-state exists, denoted as A,, with a corresponding capital stock, k^{NT}. Specifically, imposing steady-state on (9) and using a Cobb-Douglas production function of the form described above, $k^{NT} = (1-\alpha)^{\frac{1}{1-\alpha}}$. Furthermore, this steady-state is globally stable.

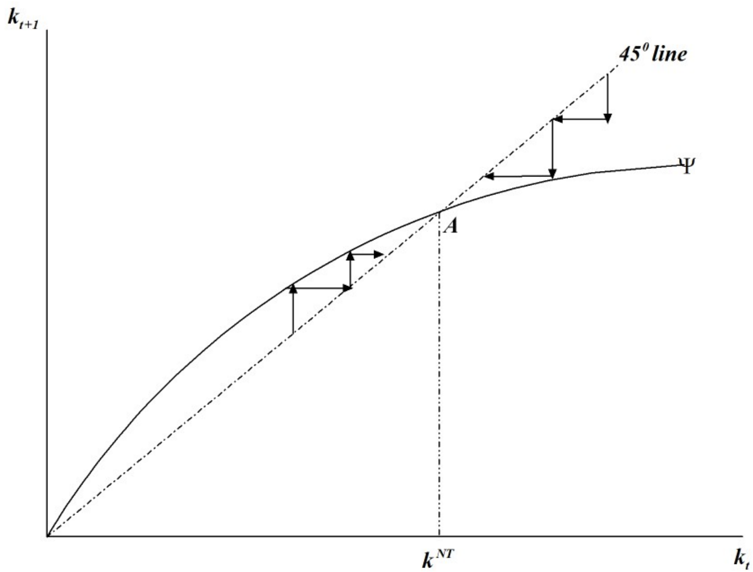

Figure 1: Law of Motion of Capital without Intergenerational Trade

2.2 An Economy with Intergenerational Trade

Unlike financial autarky, suppose young agents have the ability to purchase undepreciated capital from old agents by incurring a lump sum cost, τ. In particular, after production takes place in period t, $(1-\delta)\,k_t$ units of capital may be traded in a secondary market or stock market at a price, p_t^k units of goods per unit. In this manner, young age income is allocated between investing in new capital goods and the purchase of old capital, with:

$$s_t = w_t = i_t + p_t^k\,(1-\delta)\,k_t + \tau \tag{10}$$

where the gross return to a unit of capital, in period t is:

$$R_t = r_t + p_t^k\,(1-\delta) \tag{11}$$

Moreover, the quantity of capital available in the subsequent period is:

$$k_{t+1} = i_t + (1-\delta)\,k_t \tag{12}$$

and the consumption of an old agent in period t is:

$$c_t = \left[r_t + p_t^k\,(1-\delta)\right]k_t \tag{13}$$

The determination of the price of capital, p_t^k:

I proceed by examining the feasible range of prices at which trade in the market for capital goods prevails. A buyer has to choose between buying old capital for $(1-\delta)\, p_t^k k_t + \tau$ units of goods and investing capital from scratch by paying $(1-\delta)\, k_t$ units of goods since the price of one unit of new capital is one unit of goods. Thus, a buyer will trade if:

$$(1-\delta)\, p_t^k k_t + \tau \leq (1-\delta)\, k_t \tag{14}$$

Equivalently, the price that induces a buyer to trade has to satisfy:

$$p_t^k \in \left[0, 1 - \frac{\tau}{(1-\delta)\, k_t}\right]$$

By comparison, a seller receives $(1-\delta)\, p_t^k k_t$ in the market compared to $(1-\delta)\, \lambda k_t$ from scrapping capital. Therefore, he is willing to trade only if:

$$p_t^k \geq \lambda \tag{15}$$

In this manner, the irreversibility of investment, generates a continuum of prices at which trade can occur in a competitive equilibrium. Therefore, in this setting, I allow p_t^k, to be pinned down through bargaining between young buyers and old sellers in sub-period two of period t.

Consider trade in period t between an old seller with a capital stock k_t and a young buyer with income $w\,(K_t)$. Define u_t^s and u_{t+1}^b to be the expected payoffs from trading to a seller and buyer, respectively. The payoff to a seller is her consumption level if she sells capital goods in the market in period t. The expected payoff to a buyer is the present value of her consumption in $t+1$ if she purchases capital goods in secondary markets in period t, with an expected return, R_{t+1}^e.

Moreover, let $\underline{u}_t^s$ and $\underline{u}_t^b$ reflect the threat points to a seller and a buyer, respectively. The threat point to a seller is her the reservation utility in period t if she liquidates capital, while that to a buyer is the present value of her consumption in $t+1$ if all capital is generated from scratch. The expressions for $\underline{u}_t^s$, u_t^s, $\underline{u}_t^b$, and u_t^b are:

$$\underline{u}_t^s = (r_t + \lambda\,(1-\delta))\, k_t \tag{16}$$

$$u_t^s = \left(r_t + p_t^k\,(1-\delta)\right) k_t \tag{17}$$

$$\underline{u}_{t+1}^b = R_{t+1}^e k_{t+1}^{NT} \tag{18}$$

$$u_{t+1}^b = R_{t+1}^e k_{t+1}^{T} \tag{19}$$

where $k_{t+1}^{NT} = w\,(K_t)$ and using (10) and (12), $k_{t+1}^{T} = \left[w\,(K_t) + \left(1 - p_t^k\right)(1-\delta)\, k_t - \tau\right]$, which reflect the level of capital accumulated between t and $t+1$ under no-trade and trade, respectively.

Using (16) – (19), the net gains from trade to each market participant are:

$$\left(u_t^s - \underline{u}_t^s\right) = \left(p_t^k - \lambda\right)\left(1-\delta\right)k_t \tag{20}$$

$$\left(u_{t+1}^b - \underline{u}_{t+1}^b\right) = \left(r_{t+1} + p_{t+1}^k\left(1-\delta\right)\right)\left[\left(1-p_t^k\right)\left(1-\delta\right)k_t - \tau\right] \tag{21}$$

The price at which used capital is traded at a particular point in time is the generalized Nash solution to the following problem:

$$\underset{p_t^k}{Max}\left(u_{t+1}^b - \underline{u}_{t+1}^b\right)^{\theta}\left(u_t^s - \underline{u}_t^s\right)^{1-\theta} \tag{22}$$

subject to $u_t^s \geq \underline{u}_t^s$ and $u_{t+1}^b \geq \underline{u}_{t+1}^b$. It can be easily verified that the solution to the problem is:[9]

$$p_t^k \equiv p_t^k\left(k_t\right) = \left(\theta\lambda + \left(1-\theta\right)\left(1 - \frac{\tau}{\left(1-\delta\right)k_t}\right)\right) \tag{23}$$

It is clear from (23) that the price of capital goods is increasing in the amount traded. This takes place because the average cost of trading falls with k. Therefore, buyers are willing to pay a higher price, which raises the average price at which trade occurs. Furthermore, the price of capital is lower when capital investment is more sunk (more irreversible), as sellers are willing to accept a lower price for their used capital.

2.2.1 Equilibrium Under Intergenerational Trade

In equilibrium, factors of production earn their marginal product, (1) and (2). Moreover, the resale price of a unit of capital is expressed by (23). Upon using this information, the return to savings in an economy with intergenerational trade is:

$$R_t = f'\left(k_t\right) + \left(\theta\lambda + \left(1-\theta\right)\left(1 - \frac{\tau}{\left(1-\delta\right)k_t}\right)\right)\left(1-\delta\right) \tag{24}$$

In contrast to standard neoclassical models such as Diamond (1965), the amount of capital goods traded in period t has an ambiguous effect on the return to capital. This is due to the positive effect of the volume of trade on the re-sale value of capital, and thus its return.

Using (13) and (24), the consumption of an old investor is:

$$c_t = \left[f'\left(k_t\right) + \left(\theta\lambda + \left(1-\theta\right)\left(1 - \frac{\tau}{\left(1-\delta\right)k_t}\right)\right)\left(1-\delta\right)\right]k_t \tag{25}$$

Finally, using (10), (12), and (23), the law of motion of capital is such that:

[9] Given that capital is homogeneous, agents have no market power in the rental market. A similar outcome is obtained if for example secondary markets are intermediated. While intermediation can generate bargaining power in the market for used capital, each agent receives the capital and rents it to firms by herself. Hence, rental markets are still perfectly competitive in this case as well.

$$k_{t+1} = w(k_t) + \left(1 - p_t^k(k_t)\right)(1-\delta) k_t - \tau \equiv \Phi\left(k_t, p_t^k\right) \tag{26}$$

In contrast to Diamond (1965), a change in the capital stock in period t affects k_{t+1} through three different channels. The standard channel occurs through wages. In particular, a higher level of capital formation in period t, raises young agents' level of savings directly, which promotes capital formation. However, a change in k_t affects the total amount of used capital goods traded. As I explained above, trade will only occur if $p_t^k < 1 - \frac{\tau}{(1-\delta)k_t} < 1$. In this manner, agents incur a lower average cost of capital due to buying more units in secondary markets at a price below unity. This enables them to expand their level of investment and increase capital formation in $t+1$. Finally, by (23), the price of capital goods is higher under a higher amount traded, which has adverse effects on k_{t+1}.[10]

Using (23) into (12), the locus characterizing the behavior of the economy at a certain point in time if intergenerational trade occurs is:

$$k_{t+1} = w(k_t) + \theta\left[(1-\lambda)(1-\delta) k_t - \tau\right] \equiv \Phi(k_t) \tag{27}$$

The locus defined by (27) satisfies the following. First, $\frac{dk_{t+1}}{dk_t} > 0$ and $\frac{d^2 k_{t+1}}{dk_t^2} = w''(k_t) < 0$. Moreover, $k_{t+1} = -\theta T$ for $k_t = 0$ and $k_t = \tilde{k}_t$ when $k_{t+1} = 0$, where $\tilde{k}_t$ is the unique solution to $\frac{1}{\theta} w(k_t) + (1-\lambda)(1-\delta) k_t = \tau$. In this manner when $\Phi(k_t)$ intersects the 45^0 line, it does so at least once as illustrated in Figure 2 below. I examine existence and uniqueness conditions in the following Proposition.

Proposition 1. *Suppose* $\tau < \tau_1 = \frac{1}{\theta}\left(\frac{(1-\alpha)^{\frac{2-\alpha}{\alpha}}\alpha}{[1-(1-\lambda)(1-\delta)\theta]}\right)^{\frac{\alpha}{1-\alpha}}$. *Under this condition, two steady-states in an economy with intregenerational trade exist. By comparison, a steady-state does not exist if* $\tau > \tau_1$.

Because capital is traded across generations, strategic complementarities from investment in physical capital occur. In particular, the investment decision by one agent raises the marginal return to investment for other agents of the same generation. For instance, a higher investment by a young agent in period t, raises the total availability of capital in $t+1$, which increases wages and the ability of buyers in $t+1$ to pay a higher price for used capital goods. Due to the presence of strategic complementarities, multiple steady-states can exist.[11]

[10] Imposing steady-state on (26), we get:

$$w(k) = \delta k + p^k(k)(1-\delta) k + \tau$$

which implies that a young agent will purchase the first $(1-\delta)k$ units of capital at p^k per unit and pay a total amount of $p^k(1-\delta)k + \tau$. The remaining $\delta k = I$ units of capital (new investment) is acquired at a price of one.

[11] Please refer to Cooper and John (1988) for the link between strategic complementarity and multiplicity of equilibria.

Under the condition in Proposition 1, the locus defined by (27) intersects the 45^0 line twice and two non-trivial steady-state equilibria exist.[12] In steady-state C, the amount of capital traded, k_H^T, is significant and the market value of capital is substantial. Moreover, as observed in Figure 2, k_H^T is asymptotically stable.

By comparison, a coordination failure occurs in economy B, where individual agents fail to realize the potential gains from a higher level of investment, which leads to a low level of capital formation in the long run, k_L^T. Further, the price of capital goods is low and agents receive a low level of consumption compared to economy C. Finally, all trajectories starting with an initial capital stock, $k_0 \in \left[\tilde{k}_t, k_L^T\right)$, converge to $\tilde{k}_t$, and the trajectories starting with $k_0 > k_L^T$, converge to k_H^T, hence economy B is unstable.

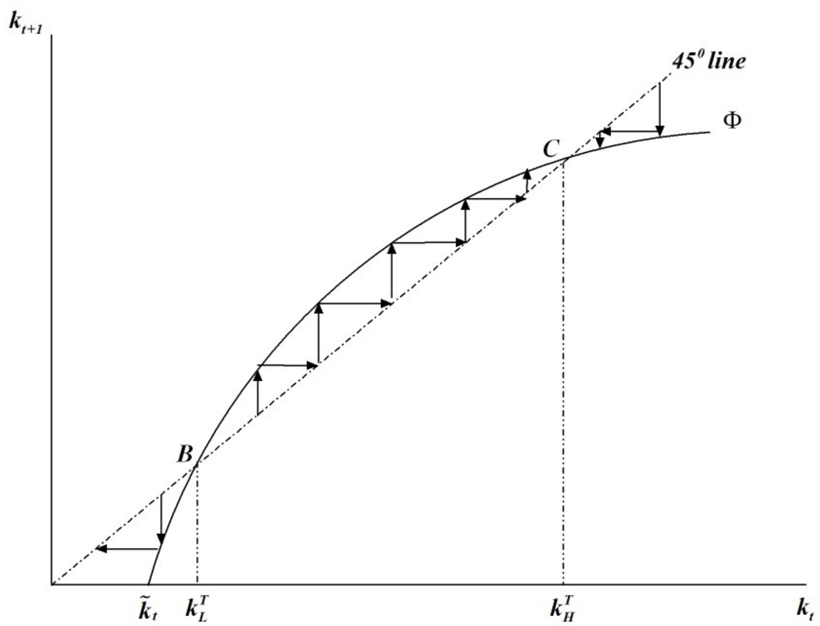

Figure 2: Law of Motion of Capital with Intergenerational Trade

It can be easily verified from (9) and (27), that an economy where capital is traded across generations grows faster in the short run relative to an economy without a stock market. Specifically, $\left|\frac{k_{t+1}}{k_t}\right|^T > \left|\frac{k_{t+1}}{k_t}\right|^{NT}$ for all $k_t > \tilde{k}_t$, where

[12] As I demonstrate in the following section, low transactions costs are necessary but not sufficient to generate multiplicity of equilibria when the choice of trade is endogenized.

the superscript, $i = T, NT$, denotes the outcome under trade and no-trade, respectively. More importantly, growth is accelerated in the stock market economy when the extent of bargaining power to buyers is higher. Intuitively, a high degree of bargaining power to buyers exerts a downwards pressure on the price of capital goods, which raises the marginal gains from a higher capital stock in period t, as more resources can be devoted towards new capital investment, which spurs growth.

3 Endogenous Formation of Markets

While the previous two sections treated the decision to trade as exogenous, I proceed to endogenize the choice to trade used capital between two generations. As discussed above, a necessary condition for trade to occur is that $1 - \frac{\tau}{(1-\delta)k_t} \geq \lambda$. That is, the upper bound on the price of capital must be the highest price at which buyers are willing to trade old capital goods. Define $\underline{k_t}$ such that the condition above holds with equality, with $\underline{k_t} = \frac{\tau}{(1-\lambda)(1-\delta)}$. The following result follows:

Lemma 1.
i. If $k_t > \underline{k_t}$*, old capital is traded at* $p_t^k \in \left(\lambda, 1 - \frac{\tau}{(1-\delta)k_t}\right)$ *and the behavior of the economy is characterized by* (26)*.*
ii. if $k_t < \underline{k_t}$*, trade does not take place and the locus* (9) *describes the behavior of the economy.*
iii. If $k_t = \underline{k_t}$*, old capital is traded only if* $p_t^k = \lambda = 1 - \frac{\tau}{(1-\delta)k_t}$*.*

For a given price of capital goods, p_t^k, the average cost to a buyer from transacting with a seller is strictly decreasing in the volume of trade. Therefore, there exists a level of capital, $\underline{k_t}$, at which buyers and sellers are both indifferent between trading and not trading. As indicated in point *iii* in the Lemma, the price of old capital goods at that level of capital is $p_t^k = \lambda = 1 - \frac{\tau}{(1-\delta)k_t}$. Moreover, for all $k_t < \underline{k_t}$, $p_t^k < \lambda$, and sellers will not trade. Finally, it can be easily verified that $p_t^k \in \left(\lambda, 1 - \frac{\tau}{(1-\delta)k_t}\right)$ and $\lambda < 1 - \frac{\tau}{(1-\delta)k_t} < 1$, if $k_t > \underline{k_t}$. Therefore, capital is traded across generations.

Furthermore, $\Phi(k_t)$ and $\Psi(k_t)$ intersect at $\underline{k_t} = \frac{\tau}{\left(1-p_t^k\right)(1-\delta)} = \frac{\tau}{(1-\lambda)(1-\delta)}$, which is the point of indifference to market participants between trading and not trading at a particular price. Define A^I to be the point at which $\Phi(k_t)$ and $\Psi(k_t)$ intersect. It is clear that $\Phi'(k_t) > \Psi'(k_t)$ for all $k_t \geq 0$. In this manner, A^I can be located either above or below the 45^0 line.

In equilibrium, all markets clear and prices are determined. In particular, factors are paid their marginal product, (1) and (2). Equilibrium in the labor market requires that $L_t = 1$. For all $k_t > \underline{k_t}$, intergenerational trade takes place at the equilibrium price, (23). At this price, all undepreciated capital is traded. Moreover, asset and goods markets clear, with (27) summarizing the behavior of the economy at a particular point in time. In contrast, for all $k_t < \underline{k_t}$, capital is not traded across generations and the output market clears when (9) is satisfied. The following proposition examines conditions under which multiple steady-states could arise in this setting.

Proposition 2.

a. Suppose $\tau < \tau_0$, where $\tau_0 = (1-\lambda)(1-\delta)(1-\alpha)^{\frac{1}{1-\alpha}}$. Under this condition, a steady-state where intergenerational trade takes place exists and is unique.

b. Suppose $\tau \in (\tau_0, \tau_1)$, where $\tau_1 > \tau_0$ is defined above.

i. If $\theta > \tilde{\theta}$, where $\tilde{\theta} : \Phi'(k_t)|_{k_t = \underline{k_t}} = 1$. Under these conditions, three steady-state equilibria exist. In one equilibrium, capital is not traded across generations and in the other two equilibria, intergenerational trade of capital occurs.

ii. If $\theta < \tilde{\theta}$, a steady-state where capital is not traded across generations exists and is unique.

c. Suppose $\tau > \tau_1$. Under this condition, a steady-state where secondary capital markets are closed exists and is unique.

The result in Proposition 1 is illustrated in Figures 3-5. Interestingly, it demonstrates that agents' degree of bargaining power bears significant consequences for the number of steady-states. Although a higher degree of bargaining power to buyers leads to faster growth, it can also cause multiple steady-state equilibria to arise when transactions costs are over an intermediate range. To begin, define τ_0 such that $k^{NT} = \underline{k_t}$. For all $\tau < \tau_0$, $k_H^T > k^{NT} > \underline{k_t} > k_L^T$ and A^I lies above the 45^0 line. Under this condition, economy C in which the market for capital goods is operative exists and is unique as illustrated in Figure 3 below. This steady-state has $k_t = k_{t+1} = k_H^T$ and is asymptotically stable.

Next, as previously defined, τ_1 is such that Φ intersects the 45^0 line twice, with $\tau_1 > \tau_0$. For all $\tau > \tau_0$, the point of indifference is located below the 45^0. Therefore, the number of steady-states depends on whether $k^{NT} \lesseqgtr k_H^T$ when $\tau \in (\tau_0, \tau_1)$. It is easily verified that Φ rotates counter clockwise around A^I under a higher θ. Thus, when buyers have significant bargaining power, as under case *bi* in the Proposition, Φ intersects the 45^0 twice to the right of $\underline{k_t}$ and we have $k^{NT} < \underline{k_t}$. In this manner, three steady-states exist, two steady-states where trade occurs, k_L^T and k_H^T, and one steady-state where capital is not traded across generations, k^{NT}, with $k^{NT} < \underline{k_t} < k_L^T < k_H^T$, as illustrated in Figure 4 below. By comparison, if sellers enjoy a high degree of bargaining power, as under case *bii*, we have $k_L^T < k_H^T < k^{NT} < \underline{k_t}$. As it can be seen in Figure 5, there is a unique steady-state under which trade of used capital goods does not occur that exists. Finally, if $\tau > \tau_1$, Φ does not intersect the 45^0 line and a steady-state where intergenerational trade is absent exists and is unique.

Discussion

When the cost of trading used capital is small, the volume of trade required to induce buyers to participate in secondary capital markets is relatively low.

Despite that, coordination failure in an economy like B, renders the amount of investment to be inefficiently low. This in turn lowers the price of capital goods to a level where sellers refuse to trade. In this case, trade always dominates no trade in economy C and the steady-state is unique. Conversely, agents will never trade in equilibrium when the level of transactions costs is significant.

However, when the level of transactions costs is over an intermediate range, the number of steady-state equilibria depends on agents' degree of bargaining power.[13] Specifically, If sellers have significant bargaining power, they will charge a high price for capital goods in the market. For a given amount traded in period t, buyers will have less resources to devote towards capital investment. This obviously lowers the gains from trade and the capital stock in period $t+1$. Specifically, when θ is sufficiently low, it is cheaper for agents to invest in capital from scratch. As a result, buyers refuse to trade and used capital is not traded in equilibrium.

Alternatively, suppose buyers have significant bargaining power as in case *bi*, with $\theta > \tilde{\theta}$. A lot of bargaining power to buyers drives the price of capital goods down, which makes k_{t+1} highly sensitive to changes in k_t. Additionally, the gains from trade to buyers are substantial, which permits an economy like B to have a larger capital stock than economy A, where intergenerational trade is absent. In this manner, three non trivial steady-state equilibria exist. As observed in Figure 4, k^{NT} and k_H^T are asymptotically stable. However, the steady-state with a low level of capital formation under which trade takes place, k_L^T, is a source. This implies that economy B is subject to development traps. If the level of capital is sightly above k_L^T, the economy converges to a higher stage of development where secondary capital markets are active and market capitalization is high. In contrast, if the level of capital is sightly below k_L^T, capital markets will breakdown as the volume of trade and the price of capital decline significantly up to the point where trade no longer takes place. That is, the economy moves in the long run from B to A.

Notably, the dynamics of different equilibria discussed above suggests that the relationship between transactions costs (or financial development) and the level of economic activity becomes non-linear when buyers have significant bargaining power. In particular, when transactions costs are low ($\tau < \tau_0$), the steady-state is unique and trade always occurs in equilibrium. It is easily verified that Φ shifts downwards for a given k_t under a higher τ. Intuitively, for a given level of capital in period t, a higher cost of trade implies a higher amount of resources is being devoted towards secondary trading. Therefore, less resources are available to finance new investment in capital goods, which reduces the total amount of capital available in $t+1$. Thus, higher transactions costs hamper capital formation in the long-run when transactions costs are low.

However, when multiple steady-states arise under relatively high degrees of bargaining power to buyers, the impact of a change in τ becomes indeterminate as in case *bi* above. Specifically, higher transactions costs cause a significant

[13] In a model with microfoundations for money balances and no capital accumulation, Rupert, Schindler, and Wright (2001) also find that multiple steady-states can exist when buyers (money holders) have a large degree of bargaining power.

deterioration in the level of economic activity in the low-capital economy, k_L^T, as it converges to k_t^{NT}. By comparison, lower transactions costs lead to substantial gains in this economy as it will converge to the steady-state with a high level of capital formation. The impact of a change in τ on the high-capital economy is more predictable and less significant.

Finally, the parameter space under which multiple steady-states exist increases significantly when capital becomes less reversible (lower λ). The proposition also indicates that in standard neoclassical models where capital is completely reversible ($\lambda = 1$), the steady-state is always unique. As capital becomes more irreversible in a setting where decentralized trade occurs between two different generations, young buyers gain more market power, which raises the scope of multiplicity of equilibria.

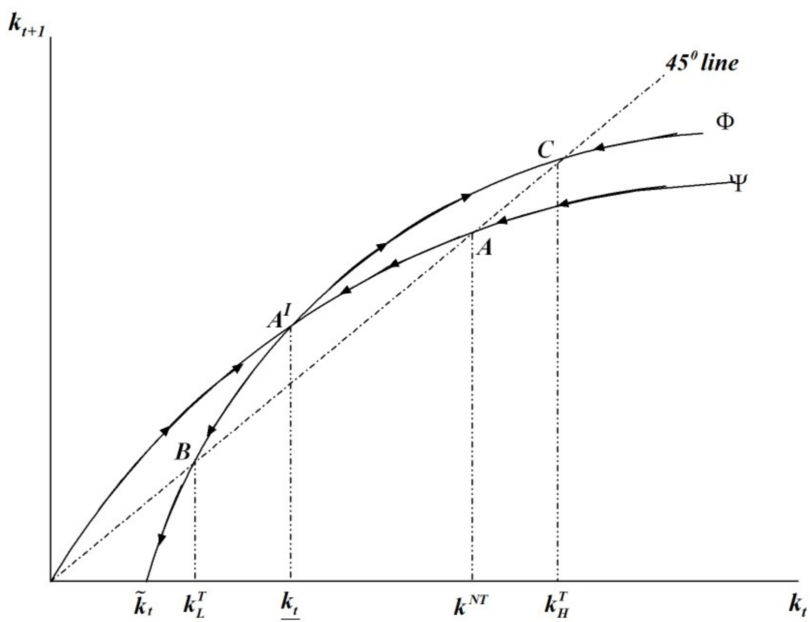

Figure 3: Case a: Unique Steady-State with Trade

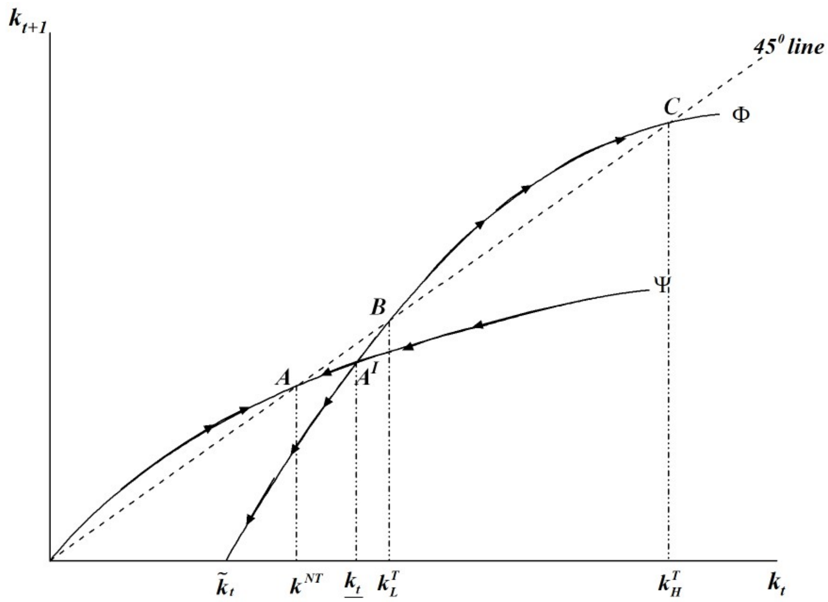

Figure 4: Case b, Three Steady-States

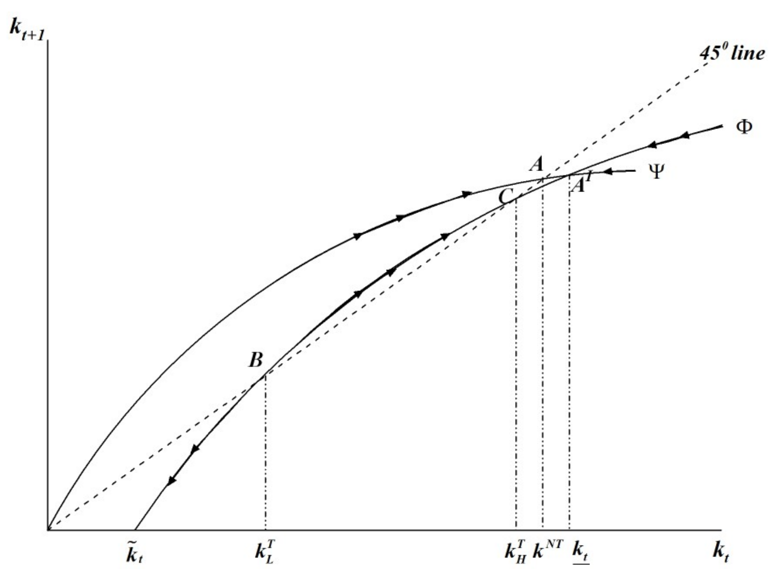

Figure 5: Case b, Unique Steady-State

4 Conclusions

This manuscript examines a two-period overlapping generations production economy, where capital investment is irreversible and partially depreciates in the production process. As capital does not completely depreciate, there is an opportunity for used capital goods to be traded between two different generations. Moreover, given that capital is irreversible, its price cannot be pinned down in a competitive equilibrium, despite that capital is homogenous. In this setting, I allow to price of used capital goods to be determined through bargaining between young buyers and old sellers. Given that buyers have to incur a resource cost from trading in secondary capital markets (or stock market), trade may not occur in equilibrium.

Although trading capital goods across generations fosters economic growth in the short-term, multiple steady-state equilibria can exist. Therefore, trading capital across generations could lead to significant gains or little gains in capital accumulation. Furthermore, development traps can arise. If trade leads to little gains, a deviation from the steady-state could either lead to a higher level of capital formation or to a steady-state where the stock market is closed.

The analysis provided in this paper can be extended in a number of ways. For example, one may allow differentiated capital goods to be traded, which gives capital traders market power in the rental market. Additionally, a role for financial intermediation can be endogenously examined, enabling us to examine the interaction between financial intermediation and capital market developments.

References

Bencivenga, V., B.D. Smith, and R. Starr, 1995. Transactions Costs, Technological Choice, and Endogenous Growth. Journal of Economic Theory 67, 153-177.

Cooper, R. and A. John, 1988. Coordinating Coordination Failures in Keynesian Models. Quarterly Journal of Economics 103, 441-463.

Diamond, P.A., 1965. National Debt in a Neoclassical Growth Model. American Economic Review 55, 1126-1150.

Greenwood, J. and B. Smith, 1997. Financial Markets in Development and the Development of Financial Markets. Journal of Economic Dynamics and Control 21, 145-181.

King, R.G. and R. Levine, 1993. Finance and Growth: Schumpeter Might be Right. Quarterly Journal of Economics 108, 717-737.

Levine, R., 1991. Stock Markets, Growth, and Tax Policy. Journal of Finance 46, 1445-1465.

Levine, R., 1997. Financial Development and Economic Growth: Views and Agenda. Journal of Economic Literature XXXV, 688–726.

Levine, R. and S. Zervos, 1998. Stock Markets, Banks, and Economic Growth. American Economic Review 88, 537-558.

Minier, J.A., 2003. Are small stock markets different? Journal of Monetary Economics 50, 1593–1602.

Rupert, P., M. Schindler, and R. Wright, 2001. Generalized Search-Theoretic Models of Monetary Exchange. Journal of Monetary Economics 48, 605-622.

Townsend, R.M, 1978. Intermediation with Costly Bilateral Exchange. The Review of Economic Studies 45, 417-425.

5 Technical Appendix

1. Proof of Proposition 1. Imposing steady-state on (27) and using functional form, yields the following polynomial in k:

$$\mu(k) \equiv (1-\alpha) k^{\alpha} - [1-(1-\lambda)(1-\delta)\theta] k = \theta\tau \tag{28}$$

It is easily verified that μ satisfies the following: First, $\mu(k) \leq (>) 0$ for all $k \geq (<) \left(\frac{(1-\alpha)}{1-(1-\lambda)(1-\delta)\theta}\right)^{\frac{1}{1-\alpha}}$ and $\mu(0) = 0$. Moreover, $\frac{d\mu}{dk} \geq (<) 0$ if $k \leq (>) \left(\frac{(1-\alpha)\alpha}{1-(1-\lambda)(1-\delta)\theta}\right)^{\frac{1}{1-\alpha}} = \hat{k}$. In this manner, $\mu(k)$ interests the $\theta\tau$ line if it does so at the inflection point. That is, $\mu\left(\hat{k}\right) \geq \theta\tau$. Upon substituting the expression for $\hat{k}$ into μ, (28) has two solutions when $\tau < \tau_1 = \frac{\alpha^{\frac{\alpha}{1-\alpha}}(1-\alpha)^{\frac{2-\alpha}{1-\alpha}}}{\theta[1-(1-\lambda)(1-\delta)\theta]^{\frac{\alpha}{1-\alpha}}} = \tau_1$, and no solution when $\tau > \tau_1$. Let k_L^T and k_H^T be the real positive roots of the polynomial, (28), with $k_L^T < k_H^T$. This completes the proof of Proposition 1.

1. Proof of Proposition 2. The number of equilibria depends on the location of k^{NT} and the point of indifference, A^I, with a corresponding capital stock, $\underline{k}$, relative to k_L^T and k_H^T. From the work in the text, $\underline{k}$ is such that $k_{t+1} = w(k_t)$. The point of indifference, A^I, lies above the 45^0 line if $\frac{k_{t+1}}{k_t}|_{k_t=\underline{k}} > 1$. Upon using the functional form for the production function and substitute for $\underline{k}$, $\frac{k_{t+1}}{k_t}|_{k_t=\underline{k}} > 1$ when $\tau < \tau_0 = (1-\lambda)(1-\delta)(1-\alpha)^{\frac{1}{1-\alpha}}$. Under this condition, $\Phi(k_t)$ intersects the 45^0 line twice, which also implies that $\tau_0 < \tau_1$. In this manner, $k^{NT} \in \left(k_L^T, k_H^T\right)$, which also implies that $k_L^T < \underline{k} < k^{NT}$.

Next, suppose, $\tau \in (\tau_0, \tau_1)$, which implies that $\frac{k_{t+1}}{k_t}|_{k_t=\underline{k}} < 1$ ($\underline{k} > k^{NT}$) and that (28) has two solutions. I proceed to show that $k^{NT} < \underline{k_t} < k_L^T < k_H^T$ when $\theta > \tilde{\theta}$. From the the characterization of $\mu(k)$ above, $\underline{k} < k_L^T$, if $\mu'(k)|_{k=\underline{k}} > 0$.[14] It is easily verified that $\mu'(k)|_{k=\underline{k}} = \frac{\alpha(1-\alpha)[(1-\lambda)(1-\delta)]^{1-\alpha}}{\tau^{1-\alpha}} - [1-(1-\lambda)(1-\delta)\theta]$. Therefore, $\mu'(k)|_{k=\underline{k}} > 0$ if $\theta > \frac{\left(1-\frac{\alpha\tau_0^{1-\alpha}}{\tau^{1-\alpha}}\right)}{(1-\lambda)(1-\delta)} = \tilde{\theta}$, where $\tilde{\theta} : \mu'(k)|_{k=\underline{k}} = 0$. Under this condition, $k^{NT} < \underline{k_t} < k_L^T < k_H^T$, which is case *bi* in the proposition.

Next, suppose $\theta < \tilde{\theta}$. Under this condition, $\underline{k_t} > k_H^T$. However, $k^{NT} > k_H^T$ if $\mu\left(k^{NT}\right) < \theta T$. Upon substituting for k^{NT} into (28), the condition becomes:

$$(1-\alpha)(1-\alpha)^{\frac{\alpha}{1-\alpha}} - [1-(1-\lambda)(1-\delta)\theta](1-\alpha)^{\frac{1}{1-\alpha}} < \theta\tau$$

simplifying, this condition becomes, $\tau > \tau_0$. Therefore, for all $\tau \in (\tau_0, \tau_1)$ and $\theta < \tilde{\theta}$, $k_L^T < k_H^T < k^{NT} < \underline{k_t}$ as in case *bii*. This completes the Proposition 2.

[14] This condition is equivalent to $\Phi'(k_t)|_{k_t=\underline{k_t}} > 1$.