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Exchange Rate Pass-Through: A Generalization

The extent of exchange rate pass-through has been playing an increasingly pivotal role in the transmission of exchange rate shocks and adequate policy responses. We develop a model of exchange rate pass-through that allows the stochastic process of exchange rate to include the lagged values of the velocity of money. We show that the likelihood and extent of pass-through is sensitive to the lagged response.

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1. Introduction

As the economic boundaries between nations continue to erode rapidly, through various regional integration agreements that have reduced the barriers to international trade and investment as well as important technical innovations in telecommunications and information technology that have facilitated the coordination of international production networks, the degree of exchange rate pass-through is becoming increasingly critical to the transmission of shocks and adequate policy responses.¹ This has been attracting the attention of several influential researchers during the last couple of decades since the pioneering works by Baldwin (1988) and Baldwin and Krugman (1989).

The motivation, for constructing a model of exchange rate pass-through that allows the exchange rate to follow a more general stochastic process (of which the Brownian motion is a special case²), stems primarily from the assessment by Choudhri, Faruquee and Hakura (2005) that none of the existing models of exchange rate pass-through perform well in quantifying the dynamic response of prices to exchange rate shocks and the insight from Devereux (2006) that the form of the exchange rate rule is critical for the transmission mechanism. While the boundaries of extensions of such stochastic processes have been constrained by the possibility of finding closed-form general solutions³ it is

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important to keep in mind that the stochastic fundamentals encompass a wide range of variables including changes in real domestic output, money demand, foreign interest rate, foreign price level, foreign money supply etc. --- as a matter of fact anything other than money supply and expected movements in the exchange rate that could possibly affect the exchange rate. As such there is no compelling reason to rule out the possibility that the fundamentals respond with a *lag* to at least a subset of these variables and it is not surprising that the discomfort among many economists with the specification that current exchange rates depend only on current fundamentals has found its expression in notable theoretical as well as empirical works. For illustration, consider long-horizon predictability of exchange rates through analysis of equations of the form:

$$\Delta_k s_t = \alpha + \beta s_{t-k} + \gamma v_{t-k} + \varepsilon_t$$

where v_{t-k} is an exchange rate fundamentals term (e.g. that suggested by the monetary class of models, $v_{t-k} \equiv [(m_{t-k} - m_{t-k}^*) - (y_{t-k} - y_{t-k}^*)]$ with m and y denoting money and income respectively) and ε_t is a disturbance term. In a series of forecasting tests over long horizons several researchers have reported that the goodness of in-sample fit and the estimated value of γ is found to rise as the lag length k rises.⁴

In the 1990s changes in exchange rates have had surprisingly small effects on prices even in small open economies where imported products are a large fraction of final consumption and intermediate inputs to production. Event studies by Cunningham and Haldane (1999) of the 1992 depreciation and 1996 appreciation in the United Kingdom, the 1992 depreciation in Sweden, and the 1999 depreciation in Brazil showed a remarkable small pass-through of exchange rate changes to prices. In the case of the United Kingdom, neither the 20% depreciation in 1992 nor the 20% appreciation in 1996

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caused retail price inflation to deviate noticeably from the 2.5 percent trend. The same was true for the 1992 depreciation in Sweden. In addition, after the depreciation in Brazil in early 1999 there was a much smaller pass through than in earlier periods when inflation was much higher. Research at the Reserve Bank of Australia on the pass-through of exchange rate changes in Australia following the Asian financial crisis in 1997 and 1998 shows that the “price movements at the docks appear to have had little or no impact at the retail level, where prices of imported items have generally continued to decline.” McCarthy (1999) reported a decline in exchange rate pass-through for all nine of the OECD countries examined in the period 1983 through 1998 compared with the period 1976 through 1982. According to those estimates the pass-through declined by 50 % or more in the United States, the United Kingdom, France, and Japan.⁵

The transmission mechanism through which a change in the current exchange rate affects price can be influenced by changes in market concentration due to entry and exit decisions of firms: firm entry and exit decision are associated, due to the role of sunk costs, with hysteresis in trade flows and large exchange rate changes can be associated with little or no movement in trade flows and prices so that exchange rate pass-through to prices is incomplete. We demonstrate that lagged velocity of money can influence the transmission mechanism, through which a movement in exchange rate can affect price, by showing that the size of exchange rate shocks for which trade flows and prices remain unchanged depends crucially on the exchange rate’s response to lagged velocity. Intuitively, the effect that any movement in the current exchange rate has on price depends critically on the response of output which, in turn, depends on the entry and exit decisions of firms. The decision of a typical firm to enter or exit the market depends on

the range within which the exchange rate is contained which we show is sensitive to the lagged velocity of money. In effect, lagged velocity can influence the transmission mechanism through which a movement in exchange rate can affect prices.

In an attempt to explain the 1980s dollar cycle⁶, Baldwin (1988) had constructed a model of exchange rate pass-through where he assumed perfect foresight that led to an elegant formulation of the transmission mechanism. In a subsequent piece, Baldwin and Krugman (1989) replaced the finite-horizon perfect foresight framework by an indefinite-horizon, stochastic exchange rate set-up where, the levels of the real exchange rate, at successive instants of time, are independently and identically distributed. Dixit (1989) incorporated the possibility that the fundamentals follow a Brownian motion: the continuous-time analog of a pure random walk. We generalize the exchange rate process so that the exchange rate responds to current as well as lagged values of the velocity of money.⁷ We show that the size of the exchange rate shocks for which trade flows and prices remain unchanged depends crucially on the exchange rate's response to lagged velocity.⁸

The rest of the paper is organized as follows. The next section offers a brief review of the key contributions relevant to our work. Section 3 lays out our model and its solution and discusses the implications of our model. Section 4 concludes.

2. Landmarks

The rich and growing literature on exchange rate pass-through focuses on the extent to which an exchange rate movement impacts traded goods prices as opposed to being absorbed in producer profit mark-ups. Analysis of such linkages have followed numerous paths, ranging from early macro-economic debate on exchange rates and monetarism, to

market integration or segmentation associated with the law of one price as well as the role of market micro-structure in the ability and desire of producers to price discriminate. Though an exhaustive account of this large body of literature is beyond the scope of our current paper, in this section, we choose to review some landmark contributions that portray a sufficiently illustrative image of the major accomplishments in this direction of research.

Conventional wisdom on exchange rate pass-through dates back to Dornbusch (1976) where he showed that a one-time increase in the stock of a currency will only lead to a temporary real depreciation but, in the long run, the shock is neutral (i.e. has no real effects). As such, in models built in this tradition, real exchange rate shocks have been perceived to create only temporary real effects and no effect at all on the underlying structure of the economy.

Baldwin (1988) was the first to highlight the idea that large exchange rate swings can leave a persistent effect (hysteresis) that continues even after the cause that brought it about has been removed. He developed a finite-horizon perfect-foresight model in which a foreign firm can enter the domestic market only by incurring a sunk cost. To remain in the market, a fixed maintenance cost is required each period. The paper showed that in this case a temporary rise in the exchange rate, if sufficiently large, would induce foreign firms to enter the domestic market. Since the entry costs are sunk, not all of the new entrants exit when the exchange rate returns to its original level. Krugman and Baldwin (1989) extended this model by replacing the finite-horizon, perfect-foresight framework by an indefinite-horizon, stochastic exchange rate set-up where, the levels of the real exchange rate, at successive instants of time, are independently and identically

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distributed. They examined the feedback from entry and exit decisions to exchange rate itself and showed that even in a multi-industry case large exchange rate shocks have persistent effects in a way small shocks do not.

Froot and Kelmerer (1989) drew attention to the dynamic demand-side effects in an oligopolistic market and demonstrated how the degree of exchange rate pass-through may change substantially over time. In their model, future demand for the product of each firm depends on its current market share. Since this induces stickiness in demand, firms' current strategic choices affect future as well as current profits. This approach emphasized not only the extent to which foreign firms will pass through current exchange rate changes but the mechanism through which exchange rate pass-through is affected by expected changes in future exchange rates as well.

Dixit (1989) took an innovative approach to modeling exchange rate pass-through by allowing the exchange rate to follow a Brownian motion. The most important benefit of the Brownian motion specification was that firms' entry and exit choices could be thought of as options: greater exchange rate volatility makes the entry and exit options more valuable and, therefore, less readily exercised. In consequence, a sufficiently large exchange rate shock, even if it is temporary, results in a permanent change in the level of imports and a permanent reduction in the degree of exchange rate pass-through.

Obstfeld and Rogoff (1995) modeled exchange rate dynamics in an infinite-horizon setting of monopolistic competition and sticky nominal prices that allowed the scope of integrating exchange rate pass-through with the current account. They showed that money supply shocks can have real effects that last well beyond the time frame of nominal rigidities because of induced short-run wealth accumulation through the current

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account. As such, in the literature that has followed this benchmark, where monopolistic producers preset their prices, a special role is given to the degree of exchange rate pass-through.

Bacchetta and Wincoop (2002), in view of the emerging evidence on the limited pass-through to consumer prices relative to import prices, constructed a model where foreign exporting firms sell intermediate goods to domestic firms who assemble the imported intermediate goods and, in turn, sell final goods to consumers. In effect, they demonstrate the extreme possibility that the exchange rate pass-through to import prices is complete while the pass-through to consumer prices is nil.

Devereux and Yetman (2003) showed, in a model of a small open economy in which exchange rate pass-through is determined by the frequency of price changes of importing firms, that there should be a positive, but non-linear, relationship between pass-through and mean inflation, and a positive relationship between pass-through and exchange rate volatility.

Devereux, Engel and Storgaard (2004) have developed a model of endogenous exchange rate pass-through within an open economy macroeconomic framework, where both pass-through and the exchange rate are simultaneously determined, and interact with one another. They show that pass-through is related to the relative stability of monetary policy: countries with relatively low volatility of money growth will have relatively low rates of exchange rate pass-through, while countries with relatively high volatility of money growth will have relatively high pass-through.

Corsetti and Dedola (2005) examined the transmission mechanism in the presence of international price discrimination by firms where profit-maximizing monopolistic firms

drive a wedge between prices across countries, optimally dampening the response of import and consumer prices to exchange-rate movements. They derive general equilibrium expressions for the pass-through into import and consumer prices, tracing the differential impact of real and monetary shocks on marginal cost and markup fluctuations through the exchange rate.

Choudhri, Faruqee and Hakura (2005) have provided the most comprehensive and thorough assessment of the performance of a variety of models (that have their roots in the works cited above) in explaining the exchange rate pass-through. They have used quantitative versions of the models from different strands of the literature to derive the dynamic response of various prices to an exchange rate shock and have compared the predicted responses with the evidence.

Devereux (2006), in one of his more recent contributions, has established a relationship between exchange rate policy and price flexibility in a model where price flexibility is an endogenous choice of profit-maximizing firms. In effect, he has developed a model that link nominal price flexibility to exchange rate regimes. Integrating the micro-economic decision of a firm to invest in price flexibility into a two-country macroeconomic model, he shows that the form of the exchange rate rule is critical for the link between exchange rate regimes and price flexibility.

3. Model, Solution and Implications

In this section we first present our model⁹ of exchange rate pass-through that allows the stochastic process of exchange rate to include lagged values of the velocity of money. Let us consider a simple flexible-price log-linear monetary model of exchange rate. Expressing all variables in natural logarithms, the spot exchange rate (s), defined as the

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domestic price of foreign currency, at any instant (t) is a linear function of the fundamentals (labeled as velocity (v)) exogenous to the monetary authority, domestic money supply (m), and expected future movements in the exchange rate ($E[ds/dt]$):

$$(1) \quad s(t) = m(t) + v(t) + \gamma E[ds(t) | \psi(t)] / dt$$

where γ is the semi-elasticity of money demand with respect to interest rate and $\psi(t)$ is the information set at time t which includes the current value of the aggregate fundamentals ($m(t) + v(t)$) as well as any explicit or implicit restrictions placed by authorities on the future evolution of the fundamentals.

The velocity term is assumed to evolve according to the following stochastic process:

$$(2) \quad dv(t) = [\alpha v(t) + \beta v(t - \tau)]dt + \sigma dz(t) \quad (t, \tau \geq 0)$$

with the initial condition $v(r) = \mathcal{G}(r) \quad \forall r \in [-\tau, 0]$

where α , β and σ are known real constants, dz is the increment of a standard uni-dimensional Wiener process $W = (W(t), F(t), t \geq 0)$ defined on a probability space (Ω, F, P) with the filtration $(F(t), t \geq 0)$ and $\mathcal{G}(r)$ is the initial (given) process $\forall r \in [-\tau, 0]$.

The innovation of our model lies in its inclusion of the lagged velocity term ($v(t - \tau)$) in the stochastic process. When $\beta \neq 0$ the possibility of lagged response is allowed with τ indicating the order of the lag. We will demonstrate how the hysteresis in trade is affected by this possibility.

Following Dixit (1989), the import-competing industry has stable demand and supply functions in terms of the home-currency. The import demand function is

$$p = P(q)$$

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where $P(q) = U'(q)$, $P' < 0$, $U(0) = 0$, $U' > 0$, and $U'' \leq 0$.

There are N risk-neutral rational foreign firms that are potential suppliers of the product of the import-competing industry. Each foreign firm has a fixed scale and a fixed coefficient technology, is a price-taker in the host-market, and must incur a sunk capital cost of θ_1 , in the host currency, at the point of entry into this market that can not be recouped if the firm should decide to quit at a later date. In addition, each foreign firm must incur a cost of θ_2 , in the host currency, to shut down. Each firm in the host-market sells one unit of the homogeneous product per unit of time. The variable cost of supplying this unit to the host market is w_n , in the foreign currency, for the n -th foreign firm. The firms are sorted in the order such that w_n is increasing in n . The market price, when n foreign firms have entered the host market, is

$$(3) \quad p_n = U(n) - U(n-1) \quad \forall \quad 1 \leq n \leq N$$

Then,

$$(3a) \quad U(n) = \begin{cases} \sum_{j=1}^n p_j & \forall 1 \leq n \leq N \\ 0 & \text{for } n = 0 \end{cases}$$

Define

$$(3b) \quad w_n = \omega(n) - \omega(n-1)$$

Then,

$$(3c) \quad \omega(n) = \sum_{j=1}^n w_j \quad \text{where } \omega' > 0 \text{ and } \omega'' > 0.$$

Suppose, at $t = 0$, we start with a number $n_0 = n$ of foreign firms in the host market.

Let ρ be the discount rate used by the foreign firms. If, at any instant $t = i$,

$[\Delta n_i]_+$ denotes the increase in the number of foreign firms and $[\Delta n_i]_-$ the decrease in it, then the equilibrium will emerge¹⁰ from solving the maximand:

$$(4) \quad E \left\{ \int_0^{\infty} \{s(t)U(n(t)) - \omega(n(t))\} e^{-\rho t} dt - \sum_i s_i \{ \theta_1 [\Delta n_i]_+ + \theta_2 [\Delta n_i]_- \} e^{-\rho i} \right\}$$

Let the outcome of this dynamic programming problem, the Bellman value function, be written as $V_n(s)$. The flow dividend is $[sU(n) - \omega(n)]$. Equating the sum of the flow dividend and the capital gain to the normal return $\rho V_n(s)$ we obtain:

$$(5) \quad [sU(n) - \omega(n)] + E[dV_n(s)/dt] = \rho V_n(s)$$

To find the general solution first consider the solution to the deterministic equation:

$$(6) \quad \begin{array}{ll} \dot{v}(t) = \alpha v(t) + \beta v(t - \tau) & (t, \tau \geq 0) \\ \text{where } v(t) = g(t) & \forall t \in [-\tau, 0] \end{array}$$

where $\dot{v}(t) = \frac{dv(t)}{dt}$ and $g(\cdot)$ is a given function on $[-\tau, 0]$.

Equation (6) can be solved step by step on the interval $[k\tau, (k+1)\tau]$ for $k \geq 0$ to yield the fundamental solution (v_0):

$$(6a) \quad \begin{array}{ll} v_0(t) = \sum_{k=0}^{\lfloor t/\tau \rfloor} \frac{\beta^k}{k!} (t - k\tau)^k e^{\alpha(t-k\tau)} & (t, \tau \geq 0) \\ \text{for } g(t) := 1_{(0)}(t), & t \in [-\tau, 0] \end{array}$$

where $\lfloor t/\tau \rfloor$ is the maximum non-negative integer less than t/τ and $1_A(\cdot)$ is the indicator function of the set A .

Therefore the general solution (v_g) of equation (6) is:

$$(6b) \quad v_g(t) = v_0 g(0) + \beta \int_{-\tau}^0 v_0(t-r-\tau) g(r) dr \quad (t, \tau \geq 0)$$

$$\forall t \in [-\tau, 0] \text{ and } g \in \mathcal{L}^1(-\tau, 0)$$

where $L^n(a,b)$ is the linear space of real values with respect to the Lebesgue-measure n -summable functions on $[a,b]$.

The unique general solution¹¹ (v) of (2) is then given by

$$(6c) \begin{cases} v(t) = v_0 \mathcal{G}(0) + \beta \int_{-\tau}^0 v_0(t-r-\tau) \mathcal{G}(r) dr + \sigma \int_0^t v_0(t-r) dz(r) dr & (t, \tau \geq 0) \\ v(t) = \mathcal{G}(t) & \forall t \in [-\tau, 0] \text{ and } \mathcal{G} \in \mathcal{L}^1(-\tau, 0) \end{cases}$$

By Ito's lemma,

$$(6d) \quad E[ds(t) | \psi(t)] / dt = \frac{\sigma^2}{2} s_{vv}$$

Substituting (6d) in (1), we have

$$(6e) \quad s(t) = m(t) + v(t) + \gamma \frac{\sigma^2}{2} s_{vv}$$

Therefore, the general solution of (5) takes the form

$$(7) \quad V_n(s(t)) = B_1(n) s(t)^{-\lambda_1} + B_2(n) s(t)^{\lambda_2} + \frac{s(t)U(n)}{\rho - \alpha - \beta} - \frac{\omega(n)}{\rho}$$

where

$$s(t) = m(t) + v(t) + A \left[e^{v(t) \sqrt{\frac{2}{\gamma \sigma^2}}} - e^{-v(t) \sqrt{\frac{2}{\gamma \sigma^2}}} \right]$$

$$A = \frac{-1}{\sqrt{\frac{2}{\gamma \sigma^2}} \left[e^{v(t) \sqrt{\frac{2}{\gamma \sigma^2}}} - e^{-v(t) \sqrt{\frac{2}{\gamma \sigma^2}}} \right]}$$

$$v(t) = \begin{cases} v_0 \mathcal{G}(0) + \beta \int_{-\tau}^0 v_0(t-r-\tau) \mathcal{G}(r) dr + \sigma \int_0^t v_0(t-r) dz(r) dr & (t, \tau) \geq 0 \\ v(t) = \mathcal{G}(t) & \forall t \in [-\tau, 0] \end{cases}$$

and $\mathcal{G} \in \mathcal{L}^1(-\tau, 0)$

B_1 and B_2 are constants to be determined¹², and λ_1 and λ_2 are roots of the quadratic equation in y :

$$f(y) = \left(\frac{1}{2}\right)\sigma^2 y(y-1) + (\alpha + \beta)y - \rho = 0$$

The solution (7) will be stationary $\forall \tau > 0$, if ¹³

$$\alpha + \beta < \rho$$

and

$$\alpha - \beta < 0$$

i.e. if the *sum* of the speed of mean-reversion (α) and the speed of lagged response (β) is less than the discount rate and the speed of mean-reversion (α) is less than the speed of lagged response (β) then the solution will be stationary. This follows directly from

$$-\lambda_1 < 0 \text{ and } \lambda_2 > 1$$

since

$$f(0) = -\rho < 0$$

$$f(1) = -(\rho - \alpha - \beta) < 0$$

and

$$f''(y) > 0.$$

Finally, consider the entry and exit decisions. Let s_{n+} be the exchange rate at which it is optimal for the n -th firm to enter the market. Then s_{n+} must satisfy:

$$(8a) \quad V_{n-1}(s_{n+}) = V_{n-1}(s_{n+}) - \theta_1 s_{n+} \quad (\text{value-matching condition})$$

$$(8b) \quad V'_{n-1}(s_{n+}) = V'_n(s_{n+}) - \theta_1 \quad (\text{smooth-pasting condition})$$

Substituting (7) into (8a) and (8b), we obtain

$$(8c) \quad b_{1n} s_{n+}^{-\lambda_1} - b_{2n} s_{n+}^{\lambda_2} + \frac{P_n s_{n+}}{(\rho - \alpha - \beta)} - \frac{W_n}{\rho} - \theta_1 s_{n+} = 0$$

$$(8d) \quad -\lambda_1 b_{1n} s_{n+}^{-\lambda_1-1} - \lambda_2 b_{2n} s_{n+}^{\lambda_2-1} + \frac{P_n}{(\rho - \alpha - \beta)} - \theta_1 = 0$$

where, $B_1(n) = \sum_{j=1}^n b_{1j}$ and $B_2(n) = \sum_{j=1}^n b_{2j}$.

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Analogously, let s_{n-} be the exchange rate at which it is optimal for the n -th firm to exit the market. Then s_{n-} must satisfy:

$$(8e) \quad b_{1n}s_{n-}^{-\lambda_1} - b_{2n}s_{n-}^{\lambda_2} + \frac{P_n s_{n-}}{(\rho - \alpha - \beta)} - \frac{w_n}{\rho} + \theta_2 s_{n-} = 0$$

$$(8f) \quad -\lambda_1 b_{1n}s_{n-}^{-\lambda_1-1} - \lambda_2 b_{2n}s_{n-}^{\lambda_2-1} + \frac{P_n}{(\rho - \alpha - \beta)} + \theta_2 = 0$$

In effect, we have a system of 4 equations (8c – 8f) to solve for 4 unknowns ($b_{1n}, b_{2n}, s_{n+}, s_{n-}$) for each $n = 1, 2, \dots, N$.

To visualize a general property characterizing the solution to this system that reveals the mechanism through which exchange rate swings can leave a persistent effect, we can rewrite (7) as:

$$(9) \quad V_n(s(t)) = \sum_{j=1}^n \left(b_{1j}s(t)^{\lambda_1} + \frac{s(t)p_j}{\rho - \alpha - \beta} - \frac{w_j}{\rho} \right) + \sum_{j=1}^n b_{2j}s(t)^{\lambda_2}$$

where

$$s(t) = m(t) + v(t) + A \begin{bmatrix} e^{v(t)\sqrt{\frac{2}{\gamma\sigma^2}}} & -e^{-v(t)\sqrt{\frac{2}{\gamma\sigma^2}}} \end{bmatrix}$$

$$A = \frac{-1}{\sqrt{\frac{2}{\gamma\sigma^2}} \begin{bmatrix} e^{v(t)\sqrt{\frac{2}{\gamma\sigma^2}}} & -e^{-v(t)\sqrt{\frac{2}{\gamma\sigma^2}}} \end{bmatrix}}$$

$$v(t) = \begin{cases} v_0 \mathcal{G}(0) + \beta \int_{-\tau}^0 v_0(t-r-\tau) \mathcal{G}(r) dr + \sigma \int_0^t v_0(t-r) dz(r) dr & (t, \tau) \geq 0 \\ v(t) = \mathcal{G}(t) & \forall t \in [-\tau, 0] \end{cases}$$

and $\mathcal{G} \in \mathcal{L}^1(-\tau, 0)$

It may be noted that, in what follows, the main result of our paper hinges critically on (9).

For $\beta = 0$, (9) boils down to a special case

$$(9a) \quad V_n(s(t)) = \sum_{j=1}^n \left(b_{1j} s(t)^{\lambda_1} + \frac{s(t) p_j}{\rho - \alpha} - \frac{w_j}{\rho} \right) + \sum_{j=1}^n b_{2j} s(t)^{\lambda_2}$$

which replicates the solution proposed by Dixit (1989).

Define

$$(10) \quad V_n^{IN}(s(t)) = \left(b_{1n} s(t)^{\lambda_1} + \frac{s(t) p_n}{\rho - \alpha - \beta} - \frac{w_n}{\rho} \right) \quad \text{and}$$

$$(11) \quad V_n^{OUT}(s(t)) = b_{2n} s(t)^{\lambda_2}$$

where V_n^{IN} represents the capitalized marginal contribution of the n -th firm if it were in the market forever plus the value of the option to exit and V_n^{OUT} represents the opportunity cost of activating the n -th firm. Hence, (8c – 8f) boil down to:

$$(12a) \quad \left. \begin{aligned} V_n^{OUT}(s_{n+}) &= V_n^{IN}(s_{n+}) - \theta_1 s_{n+} \\ V_n^{OUT'}(s_{n+}) &= V_n^{IN'}(s_{n+}) - \theta_1 \end{aligned} \right\} \quad \text{(Entry)}$$

$$(12b) \quad \left. \begin{aligned} V_n^{IN}(s_{n-}) &= V_n^{OUT}(s_{n-}) - \theta_2 s_{n-} \\ V_n^{IN'}(s_{n-}) &= V_n^{OUT'}(s_{n-}) - \theta_2 \end{aligned} \right\} \quad \text{(Exit)}$$

Define $\phi_n(s) = V_n^{IN} - V_n^{OUT}$.

Figures 1 and 2 below plot $\phi_n(s)$ against s . Note that the shape of $\phi_n(s)$ follows directly from (10) and (11). From (12a), s_{n+} is determined where $\theta_1 s$ is tangent to $\phi_n(s)$. From (12b), s_{n-} is determined where $-\theta_2 s$ is tangent to $\phi_n(s)$.

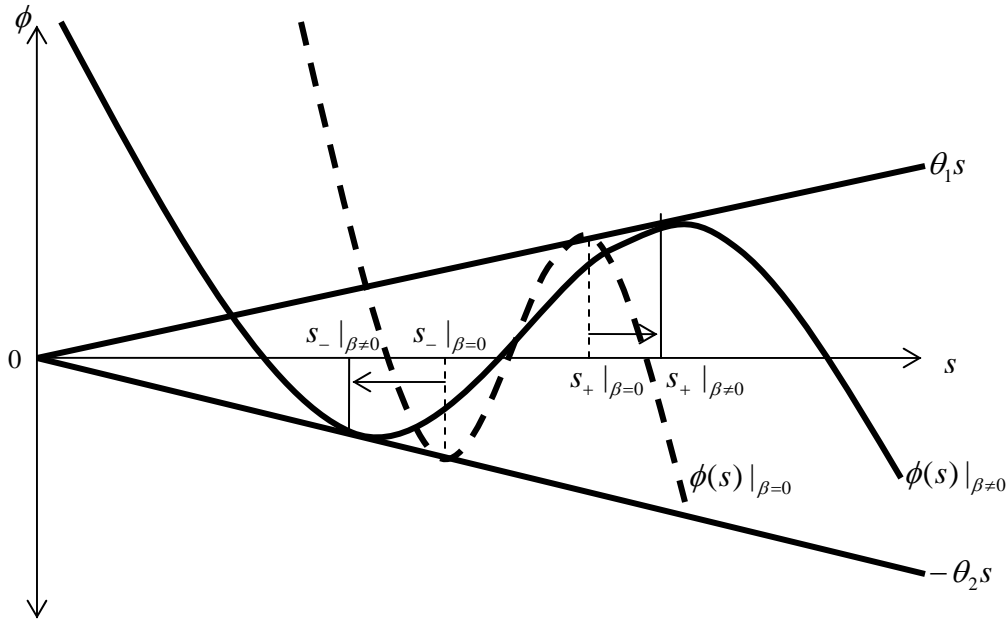


Figure 1. Hysteresis-augmenting effect of lagged velocity of money

Figure 1 demonstrates the possibility that lagged response can augment hysteresis. Suppose the current exchange rate has reached a new high and current fundamentals have a positive effect on the current exchange rate but lagged values of the velocity of money have a negative effect. Consider the n -th foreign firm's decision to enter the market. The potential entrant weighs its future profits against the value of its option to postpone entry. In the presence of lagged values of the velocity of money, the future path of the exchange rate will be less favorable. The reverse argument holds for a firm contemplating to leave the market when current exchange rate has reached a lower level. As such, the band formed by s_{n+} and s_{n-} is widened by any lag in the response which, in this case, reinforces any effect that mean-reversion has and magnifies hysteresis. Our first corollary follows.

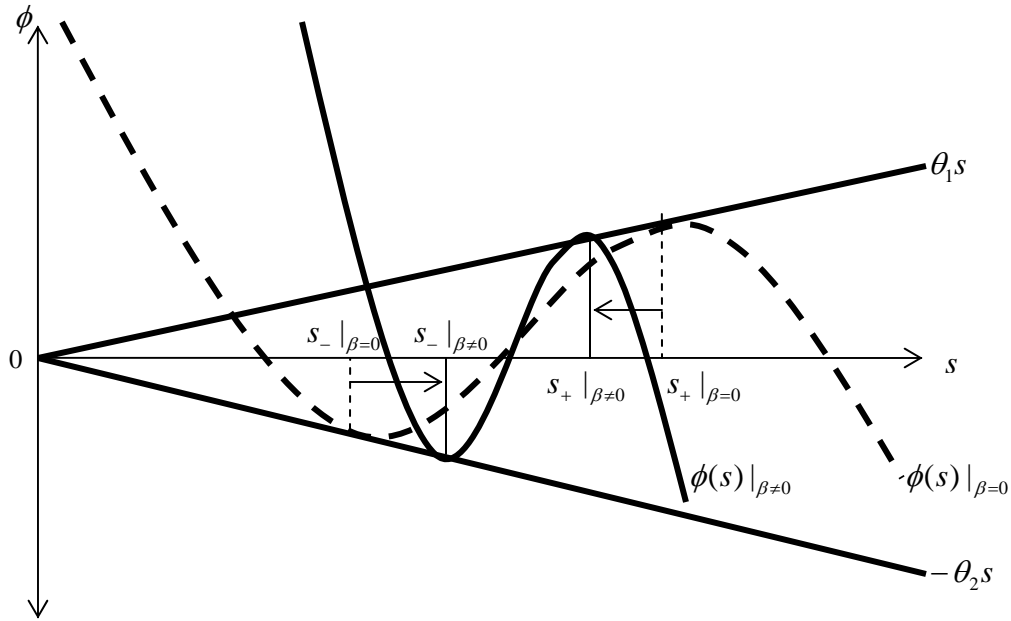


Figure 2. Hysteresis-dampening effect of lagged velocity of money

Figure 2 demonstrates the possibility that lagged response can dampen hysteresis. Suppose the current exchange rate has reached a new low and current fundamentals have a negative effect on the current exchange rate but lagged fundamentals have a positive effect. Consider the n -th foreign firm's decision to exit the market. In the presence of lagged response of fundamentals, the future path of the exchange rate will be more attractive. The reverse argument holds for a firm contemplating to enter the market when current exchange rate has reached a higher level. As such, the band formed by s_{n+} and s_{n-} is shrunk by any lag in the response of fundamentals which, in this case, offsets (more than or partially or depending on relative magnitudes of the speed of mean-reversion (α) and the speed of lagged response (β)) any effect that mean-reversion has and reduces hysteresis.

Our main result follows.

Theorem. $s_+|_{\beta \neq 0} \begin{matrix} > \\ < \end{matrix} s_+|_{\beta=0} >$ and $s_-|_{\beta \neq 0} \begin{matrix} < \\ > \end{matrix} s_-|_{\beta=0} >$ if $\alpha \begin{matrix} > \\ < \end{matrix} 0$ and $\beta \begin{matrix} < \\ > \end{matrix} 0$.

Thus, lagged fundamentals can have persistent effects on domestic prices through their effect on exchange rates. The mechanism through which a change in the exchange rate affects the price depends on the number of firms. This critically depends on whether the movement in the exchange rate is contained within the band formed by s_{n+} and s_{n-} which, we have shown, is sensitive to the direction and magnitude of the lagged response of fundamentals. Pass-through occurs when the current exchange (s) rate rises beyond the entry threshold (s_{n+}) or drops below the exit cut-off (s_{n-}): when s rises above s_{n+} , new firms will enter and prices will fall; when s falls below s_{n-} , existing firms will exit and prices will rise. When the band shrinks (expands) due to lagged response of fundamentals the likelihood as well as the extent of pass-through rises (declines).

In sum, the effect that any movement in the current exchange rate has on price depends critically on the response of output. The change in output depends on decisions of existing firms to exit the market and of new firms to enter. The micro-economic decision of a typical firm to enter or exit the market depends on the width of the band within which the exchange rate is contained. We have shown that the width of this band is sensitive to any lag in the velocity of money. As such, lagged velocity can influence the transmission mechanism through which a movement in exchange rate can affect price.

4. Conclusion

This paper is a natural follow up of continued efforts to capture the essence of the transmission mechanism that is reflected in exchange rate pass-through. By allowing the

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stochastic process to include lagged values of the velocity of money it not only adds analytical content to and complements the existing literature but also enhances the practical relevance of the logical structure that has dominated this area of research for more than three decades. It is well-known that the explanatory power of existing economic models of nominal and real exchange rates is even worse than that of a simple random walk which requires no information on fundamentals. Since the superiority of the random walk holds just as well for conditional out-of-sample forecasts, the apparent lack of success of the existing models of exchange rate has widely been attributed to the absence of a link between current fundamentals and contemporaneous movements in the exchange rate. On the other hand, none of the existing models of exchange rate pass-through is known to perform well, in isolation, when attempting to quantify the dynamic response of prices to exchange rate shocks.

In this paper, we demonstrate that lagged velocity of money can influence the transmission mechanism, through which a movement in exchange rate can affect price, by showing that the size of exchange rate shocks for which trade flows and prices remain unchanged depends critically on the response of exchange rate to lagged velocity. The effect that any movement in the current exchange rate has on price depends critically on the response of output which, in turn, depends on the entry and exit decisions of firms. The entry or exit decision of a firm depends on the range within which the exchange rate is contained which we show depends on the lagged velocity of money. We have thus shown, by explicitly modeling the role of lagged values of the velocity of money in the transmission of exchange rate shocks, how lagged response can affect (dampen or augment) the likelihood and extent of exchange rate pass-through. A particularly

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challenging extension our model, we are working on, would be to embed it in a general equilibrium setting in which the exchange rate is endogenous and depends on other endogenous fundamental variables and their lags. A couple of other interesting extensions can include merging our model with the constructs of Beladi and Chao (2003), Marjit et al. (2007) and Batra and Beladi (2008).

Appendix

A solution will be stationary if its finite-dimensional distributions are invariant under time translations i.e. $v(t)$ is stationary if

$$P(v(t + t_k) \in A_k, k = 1, 2, \dots, n) = P(v(t_k) \in A_k, k = 1, 2, \dots, n)$$

$\forall t > 0, n > 1, t_k \geq -\tau$ and all Borel sets $A_k, k = 1, 2, \dots, n$.

Suppose $W = (W(t), F(t), t \geq 0)$ is extended to a Weiner process $W = (W(t), F(t), t \in R)$ with R as time axis.

The characteristic function $h(\cdot)$ of (2) is defined by:

$$(2a) \quad h(\lambda) = \lambda - \alpha - \beta e^{-\lambda\tau}, \quad \lambda \in C.$$

A characteristic root of (2) is a solution of $h(\lambda) = 0$. Let Λ be the set of all characteristic roots of (2):

$$(2b) \quad \Lambda := \{\lambda \in C \mid h(\lambda) = 0\}$$

Let $x_0 = x(\alpha, \beta, \tau) := \max\{\text{Re } \lambda \mid \lambda \in \Lambda\}$. It can be shown¹⁴ that for every $x > x_0 \exists$ constants $K_0 = K_0(x) > 0$ such that

$$(2c.1) \quad |v_0(t)| \leq K_0 e^{x_0 t}, \quad t \geq 0$$

It follows immediately that for every $x > x_0 \exists$ constants $K_1 = K_1(x) > 0$ such that

$$(2c.2) \quad |\dot{v}_0(t)| \leq K_1 e^{x_0 t}, \quad t \geq 0$$

Hence, for every $V \in \mathcal{L}(-\tau, 0)$ and every $x > x_0$ there exists a constant $C = C(g, v)$ such that

$$(2c.3) \quad |v_g(t)| \leq C e^{x_0 t}, \quad t \geq 0$$

Now define

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$$S := \{(y, x) \in \mathbb{R}_2 : y < 1, y + x < 0, -x < \theta \sin \theta + y \cos \theta\}$$

where $\theta = \theta(y)$ is the root of

$$\theta = y \cdot \tan \theta \quad , \quad 0 < \theta < \pi \text{ if } y \neq 0$$

and

$$\theta = \frac{\pi}{2} \text{ if } y = 0$$

Lemma 1. $x_0(\alpha, \beta, \tau) < 0$ iff $(\alpha\tau, \beta\tau) \in S$.

Proof. Let $y = \alpha\tau$, $x = \beta\tau$ and $\mu = \lambda\tau$. Then λ is a root of $h(\lambda) = 0$ iff μ is a root of $\tilde{h} = 0$ where

$$\tilde{h}(\mu) = \mu - y - xe^{-\mu}, \quad \mu \in \mathbb{C}$$

Hence

$$x_0(\alpha, \beta, \tau) = \tilde{x}_0(\alpha\tau, \beta\tau) / \tau$$

where $\tilde{x}_0(y, x) := \max\{\text{Re } \mu : \tilde{h}(\mu) = 0\}$.

Since \tilde{h} is the characteristic function of $\dot{v}(t) = yv(t) + xv(t-1)$, $t \geq 0$ it follows¹⁵ that

$\tilde{x}_0(y, x) < 0$ iff $(y, x) \in S$.

[QED]

Lemma 2. The following properties are equivalent for equation (2):

- a) There exists a stationary solution;
- b) All characteristic roots of equation (2) have a negative real part: $x_0(\alpha, \beta, \tau) < 0$;
- c) $(\alpha\tau, \beta\tau) \in S$;
- d) The fundamental solution $v_0(\cdot)$ of (2) is square integrable:

$$(2d) \quad \sigma_0^2 = \int_0^{\infty} v_0^2(r) dr < \infty$$

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Proof. The equivalence of b) and c) follow immediately from Lemma 1. Then proving b) \Leftrightarrow d) and d) \Leftrightarrow a) should establish Lemma 2.

Let us split the proof for b) \Leftrightarrow d) into the following cases:

Case I: $x_0 < 0$

In this case (2d) follows from (2b) by choosing $x \in (x_0, 0)$. Suppose (2d) holds. Then from (6b), using Schwartz' inequality, it follows that every solution g of (6) with initial function $g \in L^2(-\tau, 0)$ is square integrable over R_+ .

Case I: $x_0 \geq 0$

In this case \exists a characteristic root λ_0 with $\text{Re } \lambda_0 \geq 0$. Then $\varphi(t) = e^{\lambda_0 t}$, $t \geq -\tau$ is a solution of (6) with $\varphi|_{[-\tau, 0]} \in L^2(-\tau, 0)$. But φ is *not* square integrable on R_+ . Hence, $x_0 < 0$ must hold.

It now remains to be shown that d) \Leftrightarrow a).

Suppose d) holds. Then, by assumption, the following integral exists:

$$Y_t = \int_{-\infty}^t v_0(t-r) dz(r) dr \quad r t \in R$$

for which $EY_t = 0$. Calculating the characteristic function it can be shown that $\forall t_1 < t_2 < \dots < t_n$ the random vector $(Y(t_1), Y(t_2), \dots, Y(t_n))$ is normally distributed with a variance-covariance matrix (γ_{ij}) where

$$(4e) \quad \gamma_{ij} = \int_0^{\infty} v_0(|t_i - t_j| + r) v_0(r) dr, \quad i, j = 1, 2, \dots, n$$

In particular, Y is continuous and stationary. It is straightforward to verify that Y satisfies equation (2) by plugging it in that equation and using the result that $v_0(\cdot)$ is the fundamental solution of (6). Therefore d) \Rightarrow a).

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Next suppose a) holds. Consider a stationary solution V which is continuous and is represented by (6d). Let V^0 be defined as:

$$V^0(t) = v_0(t)V(0) + \beta \int_{-\tau}^0 v_0(t-r-\tau)V(r)dr, \quad t \geq 0$$

Then

$$E[e^{i\lambda V(t)}] = E[e^{i\lambda V^0(t)}] e^{\left(\frac{-\lambda^2}{2} \int_0^t v_0^2(r)dr\right)}, \quad t \geq 0, \lambda \in R$$

Since $\int_0^t v_0(t-r)dz(r)dr \in N\left(0, \int_0^t v_0^2(r)dr\right)$ and $(W(t), t \geq 0)$ and $F(t)$ are independent, it follows that $E[e^{i\lambda V(t)}]$ is independent of t by stationarity. Hence, (4d) holds. Therefore a) \Rightarrow d).

[QED]

Lemma 3. [Existence] A stationary solution for (2) exists $\forall \tau > 0$ if $\alpha + \beta < \rho$ and $\alpha - \beta < 0$.

Proof. Follows directly from Lemma 2.

Lemma 4. [Uniqueness] If a stationary solution $X = (X(t), t \geq -\tau)$ exists for (2) then it is *unique*.

Proof. Let V be the solution of (2). If a stationary solution exists for (2) then, from (6d), it follows that

$$V(t) = V^0(t) + \Gamma(t)$$

where

$$V^0(t) = v_0(t)V(0) + \beta \int_{-\tau}^0 v_0(t-r-\tau)V(r)dr, \quad t \geq 0$$

and
$$\Gamma(t) = \sigma \int_0^t v_0(t-r) dz(r) dr$$

Because of the existence of a stationary solution we have $x_0 < 0$ by Lemma 3. From (2c) it follows that:

$$\lim_{t \rightarrow \infty} V^0(t) = 0 \quad P - a.s.$$

In particular, $V(t) \in N(0, \sigma_0^2)$.

Similarly, by calculating the characteristic function $E \left[e^{i \sum_{k=1}^n \lambda_k \Gamma(t+t_k)} \right]$, it can be shown that

the finite dimensional distributions of Γ tend to a normal distribution with zero mean and variance-covariance matrix (γ_{ij}) given by (2e).

It follows that the distribution of $(V(t-t_1), V(t-t_2), \dots, V(t-t_n))$ tends to a normal distribution with zero mean and variance-covariance matrix (γ_{ij}) given by (2e) as $t \rightarrow \infty$ where $n \in N$, t_1, t_2, \dots, t_n are fixed with $0 \leq t_1 < t_2 < \dots < t_n$.

Therefore, X is the unique stationary solution of (2) and it follows a Gaussian process with zero mean and a variance-covariance function $K(\cdot)$ given by

$$K(t) = \begin{cases} \int_0^\infty v_0(t+r)v_0(r)dr, & t \geq 0 \\ K(-t), & t < 0 \end{cases}$$

[QED]

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Endnotes

¹ See Campa and Goldberg (2005) and Campa and Gonzalez (2006) for some compelling evidence. Campa and Goldberg (2005) measured the extent of exchange rate pass-through for 23 OECD countries. Campa and Gonzalez (2006) estimated industry-specific rates of the pass-through of exchange rate changes into the prices of imports made by euro area countries originating outside the area. Also, several Latin American countries pursued exchange-rate-based stabilizations in the 1990s with the hope that the exchange rate commitment would feed directly into private sector expectations and price setting behavior.

² See Frankel and Meese (1987).

³ See the discussion in section 3.3 of Froot and Obstfeld (1991).

⁴ See Sarno (2005) for and an insightful view on this strand of literature.

⁵ These observations draw heavily on Taylor (2000) who highlights the accumulation of a large body of compelling evidence reflecting a systematic reduction in pass-through over time. While a full account of the literature on exchange rates is beyond the scope of this paper, some notable recent works include Gopinath *et al.* (2010), Garcia-Solanes and Torrejon-Flores (2010), Mallick and Marques (2010), Maria-

Dolores (2010), Marques and Garcia-Solanes (2010), Muller-Plantenberg (2010), Terra and Valladares (2010), Yoshida (2010) and Devereux (2009).

⁶ The decade of the 1980s witnessed both the rise (1980 – 1985) and fall (1985 – 1995) of the U.S. dollar against most major currencies.

⁷ See Chakrabarti (2009) which captures the implications of such a stochastic process for target zones.

⁸ Taylor (2000) shows that pass-through is systematically related to the inflation environment.

⁹ Our mathematical proofs draw heavily on Hayes (1950), Hale (1977), Dixit (1989), Froot and Obstfeld (1991) and Kuchler and Mensch (1992).

¹⁰ See Lucas and Prescott (1971, 1974) and Dixit (1989).

¹¹ A path-wise continuous stochastic process $v(t)$ on a probability space (Ω, F, P) is a solution of (2) if

i) $v(t)$ is $F(t)$ -measurable $\forall t \geq 0$;

ii) $v(t) = \mathcal{G}(0) + \int_0^t (\alpha v(r) + \beta v(r - \tau)) dr + \sigma_z(t)$ *P*-a.s., $t \geq 0$; and

iii) v satisfies: $v(r) = \mathcal{G}(r) \forall r \in [-\tau, 0]$

A solution $v(t)$ is unique when for every solution $g(t)$ of (2):

$$P\left(\sup_{t \geq \tau} |g(t) - v(t)| > 0\right) = 0$$

¹² It may be noted that $B_1(0) = 0 = B_2(N)$.

¹³ See appendix.

¹⁴ For a proof see chapter 1 in Hale (1977).

¹⁵ For a proof see p. 339 of Hale (1977). See also Hayes (1950).