Endogenous Financial Structure and Monetary Policy

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Abstract

The objective of this manuscript is study the linkages between the structure of the financial system and monetary policy. In contrast to previous studies with money, the stucture of the financial system is endogenously determined and depends on economic conditions. I show that as economies become more market oriented, it is optimal to set lower inflation targets in order to improve risk sharing that gets distorted by higher stock market participation. Furthermore, I demonstrate that the optimal financial structure depends on the value of money. In particular, it is optimal to promote participation in equity markets when inflation is low - hence to have a market-oriented system. However, when inflation is high, it is optimal to allow banks to play a bigger role in the economy relative to financial markets.

JEL Codes: E21, E31, E44, G11

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1 Introduction

The structure of the financial system varies significantly across countries. For example, the financial system in Europe and Japan is often described as a bank-based system given the large role banks play in allocating savings and investments relative to equity markets. By comparison, the financial system in the United States relies more on financial markets (a market-based system) in allocating resources.[1] Such differences in financial structure across countries triggered a debate among researchers over the past decade as to which type of financial system allocates resources more efficiently and spurs economic growth. For instance, Allen and Gale (1997) study an overlapping generations economy

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[1] Allen et al. (2004) provide a detailed comparison of the financial systems in Europe, the United States, and Asia. In a unique dataset, more recent work by Beck et al. (2009) document differences in financial structure across all countries. The authors highlight that many countries are moving towards allowing markets to play a bigger role in the financial system. This trend is even more significant for high income countries.

where financial intermediaries provide risk sharing services while financial markets enable intertemporal risk smoothing. In their setting, competition between financial markets and banks lead to disintermediation. As a result, they find that a mixed system where both banks and markets operate does not improve welfare compared to a financial system where only markets are operative. Boot and Thakor (1997) obtain similar results regarding disintermediation. More recent work by Deidda and Fattouh (2008) also shows that the process of disintermediation as the financial system becomes more market oriented might hinder economic growth. However, other studies such as Fecht et al. (2008) and Mattana and Panetti (2014) find that a higher stock market participation (due to lower entry costs) reduces the liquidity of the financial system as agents invest more in capital goods, which stimulates economic growth.[2]

Given the important role the financial system plays in the transmission of monetary policy, a number of recent studies attempt to study the linkages between monetary policy and financial structure. For example, Antinolfi and Kawamura (2008) examine an overlapping generations economy with money where markets and banks provide different economic functions. In their setting, banks insure depositors against aggregate liquidity shocks, while markets (market for Arrow securities) permit financial intermediaries to insure against productivity shocks. Moreover, the central bank provides zero nominal interest rate loans to intermediaries. As in Allen and Gale (1997, 2004), Antinolfi and Kawamura (2008) focus on how the interaction between markets and different institutions affects resource allocation. In particular they show that in an economy where banks, markets, and a central bank are present, resources are allocated efficiently. In addition, Ghossoub and Reed (2013) study the effects of monetary policy at different stages of financial and economic development. The authors show that adding a market for equity may result in multiple steady-states and the effects of monetary policy vary at each steady-sate.[3]

Notably, one common element between the literature cited above is that the structure of the financial system is exogenously imposed on the economy. Therefore, monetary policy cannot influence the degree of reliance of agents on financial markets relative to financial institutions to allocate resources. The objective of this manuscript is to study the linkages between financial structure and monetary policy in a dynamic general equilibrium setting where financial structure is endogenously determined. In this manner, the paper is able to address some important issues such as the effects of monetary policy on the socially optimal financial structure. This work also sheds some light on how optimal monetary policy can vary with the structure of the financial system. I proceed to provide more details about the model.

I examine a two-period overlapping generations production model. The economy is populated by a unit mass of ex-ante identical agents and a large number of banks. Following Townsend (1987) and Schreft and Smith (1997), agents are

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[2] Chakraborty and Ray (2006) find a non-trivial relationship between financial structure and economic growth. This issue has also been addressed in empirical studies by Levine (2002) and Demirgüç-Kunt and Maksimovic (2002).

[3] Other work also includes Huybens and Smith (1999) and Ghossoub and Reed (2015).

born on one of two geographically separated, yet symmetric locations and only value their old age consumption. At the beginning of each period, agents work when young and invest all their savings in the economy' s assets: fiat money and capital investment.

With some probability, agents must relocate to the other location after they make their portfolio choice. Whether agents are movers (relocate) or non-movers (do not relocate) is private information. Due to private information and limited communication relocated agents must liquidate their assets (physical capital) into cash to be able to consume. Ex-ante, agents can choose between intermediating their savings and investing directly in asset markets. Banks take deposits, insure their depositors against relocation shocks, and invest in the economy's assets to maximize profits. In equilibrium, all savings will be intermediated.

In contrast to Schreft and Smith (1997), claims on capital can be traded in exchange for currency after the relocation shock is realized. Due to private information, not all workers can distinguish between a real and a fake investment. Workers that know they will not relocate can pay a fee and become sophisticated, enabling them to distinguish between fake and real investments.[4] On the other hand, banks cannot distinguish between sophisticated and non-sophisticated agents. Therefore, a sophisticated non-relocated agent can pull her funds (cash) early from the banking system and trade claims on capital goods (stocks). In this manner, the extent of participation in equity markets as a hedge against liquidity risk compared to the banking system is an endogenous outcome and depends on market conditions. Finally, there is a government that targets the rate of money creation (steady-state inflation) and rebates seigniorage revenue to young workers in lump-sum transfers.

In this environment, a higher cost of becoming sophisticated lowers the participation in equity markets and therefore has adverse effects on capital formation. The lower level of capital investment puts downwards pressures on wages and welfare. However, given that agents are less reliant on equity markets and therefore keep their deposits longer in banks, risk sharing provided by the banking system improves. The socially optimal (welfare maximizing) structure of the financial system weighs these gains and losses. Interestingly, I demonstrate that the stance of monetary policy affects the shape of the financial system.

To begin, I show that inflation has a non-monotonic effect on the structure of the financial system. In particular, when inflation is initially low, raising the rate of money creation encourages stock market participation as investors seek a higher return. However, when inflation rises above a certain threshold, further increases in the inflation rate lowers the number of sophisticated agents as banks provide better insurance against relocation shocks compared to markets. As I demonstrate in the text, a market-oriented financial system leads to a higher total welfare than a bank-oriented system when inflation is relatively low. This happens because in a low inflationary environment, financial intermediaries in a bank-based system provide an inefficient amount of insurance to their de-

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[4]This follows Fecht et al. (2008). As in Diamond (1997), limited participation permits both financial intermediaries and stock markets to coexist.

positors and under-invest in capital formation compared to a market-oriented financial system. However, the ability of banks in a market-based system to insure their depositors deteriorates much faster with inflation as more agents become sophisticated and pull their funds early when inflation increases from initially low levels. Consequently, in a high inflationary environment, a bank based system dominates a market oriented financial system on welfare grounds. More importantly, it is optimal to minimize the cost of participating in equity markets when inflation is low. On the other hand, when inflation is high, moving towards a market-based system hinders total welfare and further reduction in the cost of becoming sophisticated fails to raise welfare above the level achieved under a more bank-based financial system. Interestingly, it is optimal to have a pure bank-based financial system when inflation is high enough. Furthermore, I show that a bank-based financial system can lead to the same level of welfare when inflation is high as in a market-oriented system with low inflation.

Next, I study how optimal monetary policy varies according to the structure of the financial system. In this economy, higher inflation rates stimulate capital formation when inflation is initially low. However, once inflation is high enough, further increases in the rate of money creation become detrimental for capital formation. Furthermore, as in standard random relocation models such as Schreft and Smith (1997), risk sharing deteriorates when inflation increases. In this manner, higher inflation rates involve a trade off, especially when inflation is initially low. The situation is exacerbated at initially low levels of inflation as higher inflation stimulates stock market activity, which hampers the ability of banks to provide risk sharing services. The optimal rate of money creation balances these trade-offs. As economies become more market oriented, it is optimal to set lower inflation targets in order to improve risk sharing that gets distorted by higher stock market participation.

The results in this manuscript provide insights into explaining cross country differences in the structure of the financial sector. In particular, in countries where inflation is inherently elevated as in many developing countries, it is optimal to rely on banks as the main conduit to allocate resources. However, in advanced economies like the United States where inflation is low, it is optimal to encourage equity market participation. In addition, many countries appear to be promoting a bigger role for stock markets. My work suggests that the impact of financial structure on economic welfare is not trivial and depends on monetary policy. For instance, when the economy is initially heavily dependent on financial intermediation relative to markets, promoting a bigger role for financial markets initially lowers total welfare. This happens because the gain in capital formation under a more market based system may not be enough to offset the drop in risk sharing. However, further reduction in the cost of market participation raises total welfare. Therefore, policymakers should take drastic measures by lowering transaction costs significantly and setting low inflation targets in order to reap the benefits from a more market oriented financial system.

The remainder of the paper is as follows. An outline of the physical environment of the model is discussed in Section 2. In section 3, I study the behavior of different agents in the economy. Section 4 examines the general equilibrium

effects of monetary policy and financial structure in the steady-state. Section 5 addresses the welfare implications of the model, including the optimal financial structure and the optimal monetary policy rule under different financial structures. I conclude in Section 6. All technical details are provided in the appendix.

2 Environment

Consider a discrete-time economy with two geographically separated locations or islands. Let $t = 1, 2, .., \infty$, index the time period. At the beginning of each time period, a unit mass of ex-ante identical workers are born on each island.

Workers are born with one unit of labor effort which they supply inelastically when young and are retired when old. In addition, agents derive utility from consuming the economy's single consumption good when old (c_{t+1}). The preferences of a typical worker are expressed by $u(c_{t+1}) = \frac{c_{t+1}^{1-\theta}}{1-\theta}$, where $\theta > 1$ is the coefficient of risk aversion.

The consumption good is produced by a representative firm using capital and labor as inputs. The production function is of the Cobb-Douglas form, $Y_t = AK_t^\alpha L_t^{1-\alpha}$, where K_t, Y_t, and L_t are period t aggregate capital stock, output, and labor, respectively. In addition, A is a technology parameter and $\alpha \epsilon (0,1)$ reflects capital intensity. Denoting capital per worker as $k_t \equiv \frac{K_t}{L_t}$, output per worker is expressed as, $y_t = Ak_t^\alpha$. One unit of the consumption good invested by a young worker in period t yields one unit of capital in $t+1$ and zero units if the process is interrupted early. Define i_t as the amount goods invested in capital formation per worker. Further, I assume that the capital stock depreciates completely in the production process. Therefore, $i_t = k_{t+1}$.

The market for factors of production is assumed to be perfectly competitive. Therefore, the real rental rates of labor and capital in period t are respectively:

$$w_t = (1 - \alpha) Ak_t^\alpha \equiv w(k_t) \tag{1}$$

and

$$R_t = \alpha Ak_t^{\alpha-1} \tag{2}$$

In addition to investing in physical capital, agents can hold fiat money. Denote the aggregate nominal monetary base available in period t by M_t. Assuming that the price level is common across locations, I refer to P_t as is the dollar value of a unit of goods in period t. In this manner, one unit of real cash held in period t, yields $\frac{P_t}{P_{t+1}}$ in real return the following period. At the initial date 0, the generation of old workers at each location is endowed with the initial aggregate stocks of capital and money (K_0 and M_0). Since the population of workers is equal to one, these variables also represent their values per worker.

Following previous work such as Schreft and Smith (1997, 1998), private information and limited communication between locations require workers to use cash if they move to a different location. Moreover, workers in the economy

are subject to relocation shocks. After exchange occurs in the first period, a fraction $\pi \in (0, 1)$ of agents is randomly chosen to relocate. While π is known at the beginning of the period, agents are privately informed about their types at the end of period.

The final agent in this economy is a government (or central bank) that adopts a constant money growth rule. Denote the real money stock per person in period t by $m_t = \frac{M_t}{P_t}$. The evolution of real money balances between periods $t - 1$ and t is expressed as:

$$m_t = \sigma \frac{P_{t-1}}{P_t} m_{t-1} \tag{3}$$

where $\sigma > 0$ is the constant gross rate of money creation (or destruction when $\sigma < 1$) The government rebates seigniorage income equally to young workers through lump-sum transfers. Denote the total amount of transfers at the beginning of period t by τ_t, where

$$\tau_t = \frac{\sigma - 1}{\sigma} m_t \tag{4}$$

2.1 Investments

In this economy, before the relocation shock is realized, agents can invest in two assets: Fiat money and capital. Unlike standard relocation models, agents can trade claims to physical capital in exchange for cash in secondary markets after they learn their type. In particular, one claim for a unit of capital is exchanged for z_t units of real cash or (q_t dollars).

As in Fecht et al. (2008), investing in claims is risky. With some probability $\rho \in (0, 1)$, the claim could turn out to be fake and yield nothing. Alternatively, if the claim is not fake, agents receive $\frac{R_{t+1}}{z_t}$ with probability $(1 - \rho)$. As in Diamond (1997), there is limited participation in financial markets. In particular, a subset $(1 - \lambda_t)$ of agents can validate the authenticity of a claim by incurring an exogenous cost to their utility. These agents are called sophisticated investors. The cost $\chi > 1$ is proportional to the absolute value of an agent's expected utility, amounting to $(\chi - 1) |Eu|$. By incurring the cost, a sophisticated investor guarantees $\frac{R_{t+1}}{z_t}$ units of goods from the claim in $t + 1$. On the other hand, agents that remain unsophisticated, of whom there are λ_t, expect a return of $(1 - \rho) \frac{R_{t+1}}{z_t}$. Throughout the analysis, I assume that $(1 - \rho) \frac{R_{t+1}}{z_t} < \frac{P_t}{P_{t+1}}$. Therefore, unsophisticated non-movers will never have an incentive to participate in equity markets. Given that a subset of agents cannot insure themselves against idiosyncratic risk, financial intermediaries can coexist with equity markets and play important economic functions.

2.2 Timing of the Events

Let each period be divided into two sub-periods. At the beginning of period t, the stock of capital, k_t is in the hands of old sophisticated non-movers and

banks. The economy's factors of production are rented and paid their marginal product according to (1) and (2) and production occurs. Banks announce deposit contracts, consisting of a gross return, r_t^m if an agent is a mover (relocates) and r_t^n if an agent turns out to be a non-mover. At the same time, agents decide whether to become sophisticated or not. Define λ_t to be the fraction of agents who choose not to become sophisticated. As I discuss below, all young agents deposit their income (wages and government transfers) into the banking system. Each bank decides how much cash and physical capital to invest in.

In sub-period 2 of period t, the relocation shock is realized. Relocated agents pull their funds from the banking system and are paid in cash. In addition, sophisticated non-movers pretend to be movers and also receive cash from banks, which they trade with banks for claims to capital.

3   Trade

In this section, I begin by analyzing the behavior of workers and that of financial intermediaries.

3.1   The Price of Capital in Secondary Markets

After the shock is realized in period t, sophisticated agents can trade claims on capital for cash in secondary markets. As pointed out above, one claim on capital is exchanged for q_t units of currency or $z_t = \frac{q_t}{P_t}$ units of real cash. In equilibrium, sophisticated agents invest in both cash and physical capital if arbitrage opportunities in primary and secondary markets are absent. The following result is established:

Proposition 1. *In equilibrium, the nominal price of a claim on capital traded in period t is: $q_t = P_t$.*

The intuition behind Proposition 1 is straightforward. If capital is cheaper to acquire in secondary markets, $q_t < P_t$ ($z_t < 1$), agents will choose not hold capital at the beginning of the period. Similarly, if $q_t > P_t$ agents prefer to invest only in capital as money is cheaper to obtain in secondary markets. In both cases, the price cannot support an equilibrium where secondary markets are active. Therefore, an interior solution, where $q_t = P_t$ (or $z_t = 1$) must prevail in equilibrium.

3.2   A Typical Bank's Problem

At the beginning of each period t, banks announce a gross real return per unit of deposit, r_t^m if depositors withdraw early and r_t^n if they do not. In this environment, banks are Nash competitors and therefore take the real return offered by other banks as given. At the same time, that banks announce their

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deposit contracts, agents decide whether to become sophisticated or not. In equilibrium, a fraction, $(1 - \lambda_t)$ of agents chooses to become sophisticated.

All agents work when young and earn the market wage rate, w_t, which they combine with government transfers, τ_t, to constitute their savings. Agents can decide to hold assets directly (self insure) or intermediate their savings. Given that unsophisticated agents have no incentive to participate in equity markets, all their savings are intermediated as banks provide insurance against idiosyncratic risk to risk averse agents. Sophisticated agents on the other hand, can deposit at banks and earn r_t^m if they turn out to be movers. If a sophisticated agent finds out that she is a non-mover, she will pretend to be a mover and collect $r_t^m P_{t+1}$ dollars from the banks which she uses to buy $r_t^m \frac{P_{t+1}}{P_t}$ units of capital, which enables her to consume $r_t^m R_{t+1} \frac{P_{t+1}}{P_t}$ for every unit of deposits. Therefore, sophisticated agents deposit at the bank only if

$$r_t^m \geq \frac{P_t}{P_{t+1}} \tag{5}$$

Finally, sophisticated non-movers pull their funds early if:

$$r_t^m R_{t+1} \frac{P_{t+1}}{P_t} \geq r_t^n \iff \frac{r_t^n}{r_t^m} \leq I_t \tag{6}$$

where $I_t = R_{t+1} \frac{P_{t+1}}{P_t}$ is the nominal return to capital goods. Throughout the analysis I focus on equilibria where money is dominated in rate of return. That is, $I_t \geq 1$.

Note that unsophisticated non-movers will not pull their deposits early given that:

$$(1 - \rho) r_t^m R_{t+1} \frac{P_{t+1}}{P_t} < r_t^n \tag{7}$$

as $(1 - \rho) R_{t+1} \frac{P_{t+1}}{P_t} < 1$ is assumed to hold.

Perfect competition in the banking system implies that banks will make their portfolio choice, which consists of real cash reserves, m_t and capital investment, i_t to maximize the expected utility of unsophisticated depositors:

$$\underset{r_t^m, r_t^n, m_t, i_i}{Max} \frac{\pi \left(r_t^m \left(w_t + \tau_t \right) \right)^{1-\theta} + (1 - \pi) \left(r_t^n \left(w_t + \tau_t \right) \right)^{1-\theta}}{1 - \theta} \tag{8}$$

subject to the following resource constraints. First, deposits received by a bank are allocated towards real money balances, m_t and investment in capital goods, i_t.

$$w_t + \tau_t = m_t + i_i \tag{9}$$

Moreover, all payments made to early withdrawers are made in cash. In particular, the bank pays its π movers and $(1 - \pi)(1 - \lambda_t)$ sophisticated non-movers using the cash it held from the beginning of the period and the cash it obtains

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from selling claims to a fraction, $\delta_t \epsilon (0,1)$ of its capital investment in secondary markets to sophisticated non-movers. As discussed above, each claim to capital is sold for P_t dollars. Thus, the following resource constraint has to be met:

$$\pi r_t^m (w_t + \tau_t) + (1 - \pi)(1 - \lambda_t) r_t^m (w_t + \tau_t) = m_t \frac{P_t}{P_{t+1}} + \delta_t i_i \frac{P_t}{P_{t+1}}$$

which can also be written as:

$$[\pi \lambda_t + (1 - \lambda_t)] r_t^m (w_t + \tau_t) = m_t \frac{P_t}{P_{t+1}} + \delta_t i_i \frac{P_t}{P_{t+1}} \tag{10}$$

In period $t+1$, unsophisticated non-movers get paid from the return on the remaining capital stock rented to firms:

$$(1 - \pi) \lambda_t r_t^n (w_t + \tau_t) = (1 - \delta_t) R_{t+1} i_t \tag{11}$$

Finally, unsophisticated non-movers should not have an incentive to pull their funds early by pretending to be movers. Therefore, the following self-selection constraint needs to hold:

$$r_t^m \le r_t^n \tag{12}$$

In sum, the bank maximizes (8) subject to: (6) and (9)$-$(12). Upon substituting the binding constraints, the problem can be written as:

$$\underset{r_t^m, r_t^n}{Max} \frac{\pi \left(r_t^m (w_t + \tau_t)\right)^{1-\theta} + (1-\pi)\left(r_t^n (w_t + \tau_t)\right)^{1-\theta}}{1 - \theta}$$

subject to the following resource constraint expressed in per unit of deposits:

$$1 = [\pi \lambda_t + (1 - \lambda_t)] r_t^m \frac{P_{t+1}}{P_t} + (1 - \pi) \lambda_t \frac{r_t^n}{R_{t+1}} \tag{13}$$

The solution to the problem yields:

$$r_t^m = \left[\frac{(1-\pi)}{\pi^{\frac{1}{\theta}}} \left(\frac{R_{t+1}}{\lambda_t}\right)^{\frac{1-\theta}{\theta}} \left(\frac{[1 - \lambda_t(1-\pi)]}{\frac{P_t}{P_{t+1}}}\right)^{\frac{1}{\theta}} + \frac{[1 - \lambda_t(1-\pi)]}{\frac{P_t}{P_{t+1}}} \right]^{-1} \tag{14}$$

and

$$r_t^n = \left[(1-\pi) \frac{\lambda_t}{R_{t+1}} + \frac{(\pi \lambda_t)^{\frac{1}{\theta}}}{[1 - \lambda_t(1-\pi)]^{\frac{1-\theta}{\theta}} R_{t+1}^{\frac{1}{\theta}} \left(\frac{P_{t+1}}{P_t}\right)^{\frac{1-\theta}{\theta}}} \right]^{-1} \tag{15}$$

where the relative return to depositors is such that:

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$$\frac{r_t^n}{r_t^m} = \left[\frac{\frac{1}{\lambda_t} - (1-\pi)}{\pi}\right]^{\frac{1}{\theta}} I_t^{\frac{1}{\theta}} \tag{16}$$

which indicates that for a given number of unsophisticated agents, depositors receive less insurance against liquidity risk when the nominal return to capital increases. Moreover, banks are able to provide better risk sharing if fewer agents become sophisticated (λ_t is higher) and withdraw their funds early. Additionally, it can be easily verified that complete risk sharing cannot be achieved when some agents withdraw early. That is, $\frac{r_t^n}{r_t^m} > 1$ whenever $I_t \geq 1$ and $\lambda_t \in (0,1)$.[5]

I proceed to solve for the equilibrium number of unsophisticated depositors. In equilibrium, agents will choose to participate in equity markets up to the point where they are indifferent between becoming sophisticated and remaining unsophisticated. That is:

$$\frac{\pi \left(r_t^m (w_t + \tau_t)\right)^{1-\theta} + (1-\pi) \left(r_t^n (w_t + \tau_t)\right)^{1-\theta}}{1-\theta} = \chi \frac{\pi \left(r_t^m (w_t + \tau_t)\right)^{1-\theta} + (1-\pi) \left(r_t^m (w_t + \tau_t) I_t\right)^{1-\theta}}{1-\theta} \tag{17}$$

Upon using (14) and (15) into (17) and some simplification, we get:

$$\frac{r_t^n}{r_t^m} = \left[\frac{\chi}{I_t^{\theta-1}} + \frac{\pi}{1-\pi} (\chi - 1)\right]^{-\frac{1}{(\theta-1)}} \tag{18}$$

which states that banks provide more insurance against liquidity shocks if the cost of becoming sophisticated is higher. Intuitively, fewer agents participate in equity when it becomes more costly. As more agents are keeping their deposits longer, banks can offer better insurance to its depositors. Further, from (16) and (18), the fraction of agents that choose not to participate in equity markets is:

$$\lambda_t = \pi^{-1} \left[\frac{1}{I_t \left[\frac{\chi}{I_t^{\theta-1}} + \frac{\pi}{1-\pi} (\chi - 1)\right]^{\frac{\theta}{(\theta-1)}}} + \frac{1-\pi}{\pi}\right]^{-1} \tag{19}$$

which is increasing in χ as discussed above. Interestingly, a change in the nominal return to capital has an ambiguous impact on the extent of participation in equity markets. The following Proposition provides a characterization of λ followed by the intuition.

Proposition 1. *Suppose $\chi \geq \bar{\chi}$, where $\bar{\chi} : (\chi - 1)\chi^{\frac{1}{\theta-1}} \theta^{\frac{\theta}{\theta-1}} = (\theta - 1)\frac{1-\pi}{\pi}$. Under this condition, $\lambda_t = 1$. In comparison, suppose, $\chi \in (1, \bar{\chi})$, $\frac{d\lambda_t}{dI_t} (<) \geq 0$*

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[5] This necessarily happens if agents incur a non-negative cost for becoming sophisticated. If becoming sophisticated is costless ($\chi = 1$), sophisticated agents become indifferent between pulling their deposits early and keeping them at the bank. Only then, banks can provide complete risk sharing.

for $I \leq (>) \tilde{I}$, where $\tilde{I} = \left((\theta - 1) \frac{1-\pi}{\pi} \frac{\chi}{(\chi - 1)} \right)^{\frac{1}{\theta - 1}}$. Moreover, $\lambda_t = 1$ if $I_t \leq \underline{I}$ or $I_t \geq \bar{I}$, with $\underline{I} < \tilde{I} < \bar{I}$.

Intuitively, if the cost of becoming sophisticated is too high, no one chooses to participate in secondary markets. However, if the cost is low enough, the extent of participation depends on the return from capital. In particular, a change in the nominal return to capital has a non-monotonic impact of stock market participation. In this setting, an increase in the nominal return to capital affects the choice of becoming sophisticated in two ways. First, sophisticated agents directly receive a higher return from participating in secondary markets as I increases, which encourages participation in secondary markets. However, banks offer a lower relative return to depositors who are relocating, thus lowering income earned in period t from early withdrawal. When the nominal return in equity markets is initially low, stock market participation is also low and depositors receive too much insurance. As the nominal return to capital increases, more agents have an incentive to become sophisticated. However, once the return to capital exceeds a certain threshold, $\tilde{I}$, the number of agents who are sophisticated is significant and the return to movers is relatively low, hence fewer depositors have an incentive to participate in equity markets as the return to capital increases.

Furthermore, using (9) and the fact that payments to movers are made out of cash, the demand for money by a bank is such that:

$$m_t = \frac{w_t + \tau_t}{1 + \frac{1-\pi}{\pi} \left[\left[\frac{1 - \lambda_t(1-\pi)}{\pi} \right]^{\frac{1}{\theta}} \left(\frac{\lambda_t}{I_t} \right)^{\frac{\theta}{\theta - 1}} + (1 - \lambda_t) \right]} \tag{20}$$

Equivalently, denote the fraction of deposits allocated towards money balances by γ_t with $\gamma_t = m_t / (w_t + \tau_t)$, then:

$$\gamma_t = \frac{1}{1 + \frac{1-\pi}{\pi} \left[\left[\frac{1 - \lambda_t(1-\pi)}{\pi} \right]^{\frac{1}{\theta}} \left(\frac{\lambda_t}{I_t} \right)^{\frac{\theta}{\theta - 1}} + (1 - \lambda_t) \right]} \tag{21}$$

Because agents are highly risk averse, the bank allocates a larger fraction of its deposits towards money balances under a higher nominal interest rate and for a given number of sophisticated agents.[6] Intuitively, the higher nominal rate of return on capital yields both a substitution effect and an income effect. The substitution effect occurs because the higher return to capital raises the cost of holding money and lowers its demand. On the other hand, the higher interest rate implies that banks can obtain the same amount of interest income by acquiring a lower amount of capital. In this manner, the income effect leads to an increase in the demand for money.

In addition, a change in the number of sophisticated agents has an ambiguous effect on the demand for money balances. First as the number of sophisticated

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[6] Schreft and Smith (1998) obtain a similar relationship in a pure banking economy.

agents increases (λ is lower), the bank faces higher levels of early capital liqui-
dation and therefore a higher marginal cost of holding money. This stimulates
the bank to hold more capital and less cash. However, there is less need to make
payments to unsophisticated non-movers as more agents become sophisticated.
Therefore, the marginal cost of holding cash falls, which raises the demand
for cash reserves. When stock market participation is initially high, the first
effect dominates and further stock market participation encourages banks to
hold more liquid portfolios. The opposite happens when participation in equity
markets is initially small. The following Lemma summarizes this result:

Lemma 1. $\frac{\partial \gamma_t}{\partial \lambda_t} \leq (>) 0 \; if \; \lambda_t \leq (>) \hat{\lambda}.$

4 General Equilibrium

I proceed to characterize the equilibrium behavior of the economy under which
both banks and markets are operative. In equilibrium, all markets will clear. In
particular, labor receives its marginal product, (1) and $L_t = 1$. Furthermore,
upon using the expression for transfers, (4) into the money demand equation,
the demand for money can be written as:

$$m_t = \frac{w_t}{\left[\left[\frac{1-\lambda_t(1-\pi)}{\pi}\right]^{\frac{1}{\theta}} \left(\frac{\lambda_t}{I_t}\right)^{\frac{\theta}{\theta-1}} + (1-\lambda_t)\right] \frac{1-\pi}{\pi} + \frac{1}{\sigma}} \tag{22}$$

where the number of sophisticated agents is given by (19).

Using $(1) - (2)$ along with the money demand equation, (22), and the de-
finition of I into the evolution equation of money, (3), money market clearing
requires that the following condition holds:

$$\mu (I_{t+1}) = \sigma \frac{\alpha A k_{t+1}^{\alpha-1}}{I_t} \mu (I_t) \frac{k_t^{\alpha}}{k_{t+1}^{\alpha}} \tag{23}$$

where $\mu (I_t) = m_t/w_t$

Finally, using (4), (9), and (22), the supply for capital by banks is:

$$k_{t+1} = \left(1 - \frac{1}{\sigma} \frac{1}{\left[\left[\frac{1-\lambda_t(1-\pi)}{\pi}\right]^{\frac{1}{\theta}} \left(\frac{\lambda_t}{I_t}\right)^{\frac{\theta}{\theta-1}} + (1-\lambda_t)\right] \frac{1-\pi}{\pi} + \frac{1}{\sigma}}\right) w_t \tag{24}$$

Conditions $(22) - (24)$ characterize the behavior of the economy at a particular
point in time.

4.1 Steady-State Analysis

In this manuscript I focus on the behavior of the economy in the steady-state. Upon imposing steady-state on (23) and (24), the following two loci characterize the behavior of the economy in the long-run.

$$I = \sigma \alpha A k^{\alpha - 1} \qquad (25)$$

and

$$\Omega\left(k\right) \equiv \frac{k}{w} = 1 - \frac{1}{1 + \left[\left[\frac{1-\lambda(1-\pi)}{\pi}\right]^{\frac{1}{\theta}} \left(\frac{\lambda}{I}\right)^{\frac{\theta}{\theta-1}} + (1-\lambda)\right] \frac{1-\pi}{\pi} \sigma} \qquad (26)$$

where the gross inflation rate in the steady-state is such that $\frac{P_{t+1}}{P_t} = \sigma$ and the number of unsophisticated agents is given by (19).

The locus defined by (25) reflects the demand for capital, which is strictly decreasing in the return to capital. The following Lemma characterizes the behavior of the supply of capital, (26).

Lemma 2. *The locus defined by (26) is such that $\frac{dk}{dI} < 0$ for $I \in \left[\underline{I}, \bar{I}\right]$ and $\left(\underline{I}, \bar{k}\right)$ and $\left(\bar{I}, \underline{k}\right)$ are two points on the locus.*

Intuitively, a change in the return to capital affects the portfolio of banks in a number of ways. First, as agents are highly risk averse, banks seek to hold a more liquid portfolio when the nominal return to capital is higher. However, a change in the return to capital affects the incentive to participate in equity markets. In particular, when the nominal return to capital is initially low, the level of participation in the stock market is pretty small. Therefore, a higher nominal return to equity stimulates more agents to become sophisticated, which in turn raises the need by banks to invest in capital. Moreover, from our discussion of Proposition 1, stock market participation drops when the nominal return to capital exceeds a threshold level. Overall, as we demonstrate in the appendix, banks supply less capital when the nominal return to capital is higher. An illustration of (25) and (26) is provided in Figure 1 below. I proceed to establish existence of steady-state equilibria in the following Proposition:

Proposition 2. Existence of Steady-States where Banks and Markets are Active.
    a. Suppose $\sigma \in (\underline{\sigma}, \overline{\sigma})$. Under this condition, an equilibrium with active equity markets and banks exists and is unique.
    b. Suppose $\sigma > \overline{\sigma}$ and $\chi \in (\chi_0, \bar{\chi})$. Under this condition, two equilibria where both markets and banks are active exist.

In this manuscript, I focus on equilibria where both equity markets and banks are operative. This also implies that both money and capital need to be held in

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equilibrium. From the characterization of λ, in Proposition 1, the equity market is active when $I \in \left(\underline{I}, \bar{I}\right)$. As I demonstrate in the appendix, sophisticated non-movers withdraw early and the self-selection constraint is satisfied when markets are active. That is, conditions (6) and (12) hold when $I \in \left(\underline{I}, \bar{I}\right)$. Therefore, it suffices to find conditions under which the capital market clears over that range of I. Upon examination of Figure 1 below, both loci intersect once at a point like E, if an excess demand (supply) for capital occurs at $\underline{I}$ $(\bar{I})$. This in turn requires the rate of money growth to be within the range highlighted in Proposition 2.

Furthermore, if $\sigma > \bar{\sigma}$, it is possible that both loci, (25) and (26) intersect twice as illustrated in Figure 2 below. This necessarily happens when the cost of becoming sophisticated is high enough as highlighted in case b in Proposition 2. In economy E_1, capital formation is high and the return to capital is low. Moreover, stock market participation is also high. In contrast, very few agents choose to become sophisticated in economy E_2. In such an economy, capital investment is low and the return to capital is high. As an example, consider the following parameters: $A = 1$, $\chi = 1.14$, $\theta = 5$, $\pi = .7$, and $\alpha = .33$. Under these parameter values, a unique steady-state exists when $\sigma = 1.01$, with $k = 0.06$ and $\lambda = 0.77$. However, two equilibria emerge when $\sigma = 1.1$. In the economy with a high capital stock, $k = 0.069$ and $\lambda = 0.72$. The low capital economy is such that: $k = 0.028$ and $\lambda = 0.994$. Numerical work suggests that the parameter space under which multiple equilibria exist is small or might require unrealistic parameter values. Therefore, throughout the remainder of the paper, I focus on case a above, where a unique equilibrium exists.
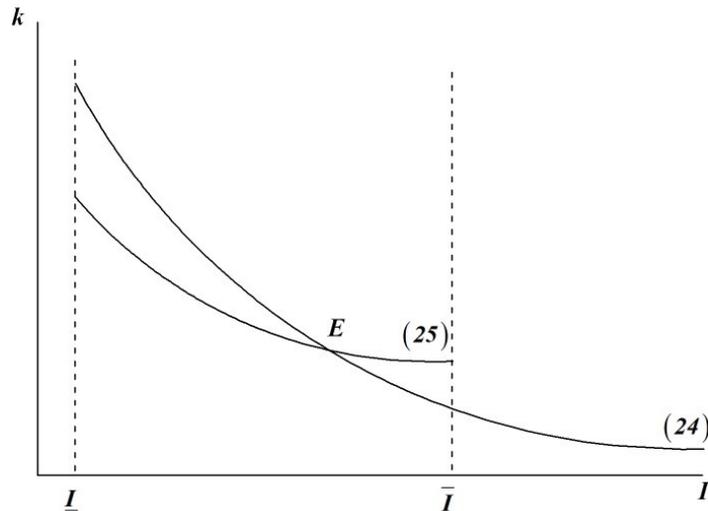
Figure 1. Unique Steady-State Equilibrium

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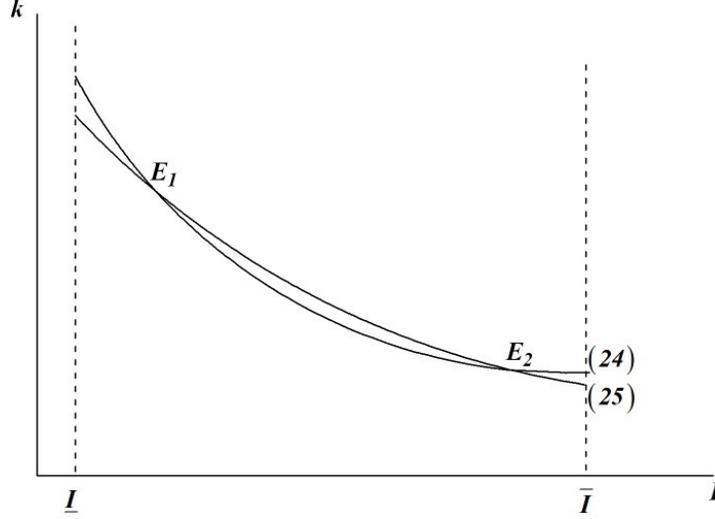
Figure 2. Multiple Steady-State Equilibria

I proceed to examine the effects of a change in the cost of becoming sophisticated on various economic outcomes.

Proposition 3. $\frac{dk}{d\chi} < 0$, $\frac{dI}{d\chi} > 0$, $\frac{\partial \lambda}{\partial \chi} > 0$, and $\frac{\partial \frac{r_t^n}{r_t^m}}{\partial \chi} < 0$.

It is easy to verify that the supply of capital, represented by locus (26) shifts downwards for a given nominal return to capital under a higher value of χ. Intuitively, a higher cost of becoming sophisticated lowers stock market participation, which encourages banks to reduce their capital investment. The lower stock of capital raises its return. Furthermore, as fewer agents withdraw their funds early, banks can provide better insurance against relocation shocks for a given return to capital. In this manner, while a more market oriented financial system stimulates capital formation, this comes at the cost of providing less insurance to depositors against liquidity risk.

The effects of monetary policy are analyzed in the following Proposition.

Proposition 4. $\frac{dk}{d\sigma} \geq (<) 0$ *if* $\sigma \leq (>) \hat{\sigma}_1$. *In addition,* $\frac{dI}{d\sigma} > 0$, *and* $\frac{d\lambda}{d\sigma} \leq (>) 0$ *if* $\sigma \leq (>) \tilde{\sigma}$.

Interestingly, the result in Proposition 4 suggests that monetary policy has non-monotonic effects on capital formation. In particular, a higher rate of money creation stimulates capital formation when inflation is initially low. However, once inflation crosses a certain threshold level, $\hat{\sigma}_1$, further increases in the inflation rate becomes detrimental for the capital stock and output. Intuitively, a

change in the rate of money creation affects the economy through three primary channels. First, young depositors receive higher transfers from the monetary authority when the rate of money growth increases. This translates into a higher deposit base, which raises the ability of banks to invest in the economy's assets. Second, inflation raises the nominal return to capital, which lowers the return to relocated agents. Given the agents are highly risk averse, the bank will hold more of the liquid asset (cash) and less capital to accommodate its depositors.

Finally, Proposition 4 points to an interesting relationship between inflation and the structure of the financial system. In particular, whether a higher inflation rate leads to a more bank-based or a more market-based system is not clear and depends on the initial rate of inflation. From our discussion of Proposition 1, when the nominal return to capital (inflation) is initially low, raising the nominal return to capital stimulates stock market participation and capital holding by the bank. However, when the nominal return to capital is high enough, further increases in inflation discourages agents from becoming sophisticated. As less capital is traded in secondary markets, the bank holds less of it ex-ante. Overall, a higher rate of money growth promotes capital accumulation and stock market participation when inflation is initially low. However, when inflation is high enough, a lower value of money adversely affects capital formation and discourages participation in equity markets.

The results presented in Proposition 4 are consistent with the literature that finds a nonlinear relationship between inflation and the real economy such as Bullard and Keating (1995).[7] However, we also identify inflation thresholds for stock market participation.[8] Using a dataset that covers 175 countries over 1960-1997 period, Demirgüç-Kunt and Levine (2001) construct an index of financial structure, which reflects the importance and development of the banking sector relative to the stock market. A country with a higher value of the index is categorized as more bank-based. Moreover, they divide countries into three groups based on their financial structure: underdeveloped, market-based, and bank-based. A country is considered underdeveloped if it has below the median values of both bank and stock market development indicators. Furthermore, for economies with a developed financial system, countries with a financial structure index value above the mean are classified as bank-based. Notably, the average inflation rate for bank-based economies is 3.91% compared to 4.31% for market based economies. In comparison, inflation averaged 25.23% in economies with underdeveloped financial systems. There results suggest that inflation is not significantly different between market-based and bank-based systems. Interestingly, this manuscript captures all these observations. However, our theoretical work points out to nonlinearity in the relationship between inflation and the structure of the financial system, which could explain the statistical results obtained by Demirgüç-Kunt and Levine (2001). Therefore, further empirical

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[7] Other work includes Fischer (1993), Ghosh and Phillips (1998), Khan and Senhadji (2001), and Boyd et al. (2001).

[8] While I am unable to compare $\hat{\sigma}_1$ and $\tilde{\sigma}$ analytically, numerical work suggests that $\hat{\sigma}_1 > \tilde{\sigma}$. Therefore, it takes a much lower inflation rate to hamper stock market activity compared to that which negatively influences the economy.

investigation is needed on this front.

5 Welfare Analysis

In this section, I attempt to answer the following two questions. First, what is the socially optimal financial structure? More importantly, should monetary policy be designed according to the structure of the financial system? Following previous work such as Williamson (1986) and Fecht et al. (2008), I use the steady-state expected utility of a typical generation of depositors as a proxy for welfare. Given the analytical complexity of this exercise, I proceed numerically.

I begin by examining the optimal monetary policy. In particular, the monetary authority chooses the rate of money creation, σ to maximize (8) subject to (19), (25), and (26). In this setting and from the result in Proposition 4, a higher rate of money creation involves a trade off, especially when inflation is initially low. On one hand, more inflation stimulates capital formation and welfare through the Tobin effect. On the other hand, higher rates of money creation lead to lower risk sharing. The situation is exacerbated at initially low levels of inflation as higher inflation stimulates stock market activity, which hampers the ability of banks to provide risk sharing services. The optimal rate of money creation balances these trade-offs.

To shed more light on the optimal monetary policy, I begin with the following set of baseline parameter values: $A = 1$, $\chi = 1.01$, $\theta = 2$, $\pi = .4$, and $\alpha = .4$.[9] In Figure 3 below, I illustrate the relationship between inflation and total welfare for two economies: a mixed economy (where both markets and banks are operative) and an economy where markets are closed (pure banking). As shown in the Figure, the effects of inflation on welfare are non-monotonic and the optimal rate of money growth in the mixed economy is 4.3%. In comparison, the optimal rate of money creation in the pure banking economy is 17%.

Interestingly, a market-oriented financial system leads to a higher total welfare when inflation is relatively low. The intuition is fairly simple. When inflation is low, financial intermediaries in a bank-based system provide an inefficient amount of insurance to their depositors and under-invest in capital formation compared to a market-oriented financial system. However, the ability of banks in a hybrid financial system to insure their depositors deteriorates much faster with inflation as more agents become sophisticated and pull their funds early when inflation increases (within a certain range). Consequently, in a high inflationary environment, a bank based system dominates a market oriented financial system on welfare grounds.

Figure 4 below sheds more light on this issue by showing how optimal monetary policy varies with the extent of financial market participation. As economies become more market oriented (due to a lower value of χ), it is optimal to set lower inflation targets. This needs to happen in order to improve risk sharing that gets distorted by a higher stock market participation. The result

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[9] The results are robust over a large set of parameters.

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is in line with the numerical work from Figure 3.[10]
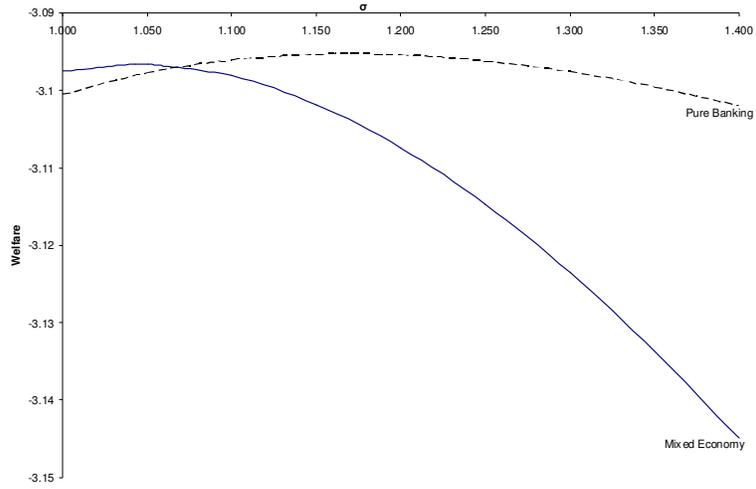
Figure 3. Optimal Monetary Policy
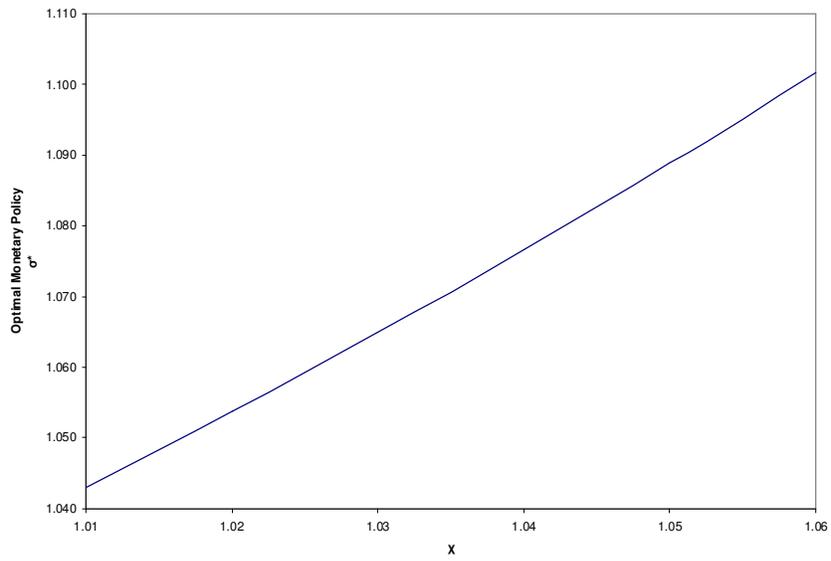
Figure 4. Optimal Monetary Policy Under Different Financial Structures

[10]In the case where agents are highly risk averse and participation costs are high, it may even be optimal to inflate the economy to the extent where markets are completely closed.

Finally, what is the optimal structure of the financial system? That is, for a given rate of money creation, what is the welfare maximizing value of χ? The work above suggests that the optimal structure of the financial system should depend on monetary policy. Using the same set of parameters from the previous example, Figure 5 illustrates the relationship between total welfare and the cost of becoming sophisticated when inflation is 5% and 10%. Notably, the impact of χ on welfare is not straightforward. In particular, when inflation is relatively low (5%) and starting from a bank oriented system (high χ), moving into a more market-based financial system initially lowers total welfare. As the gain in capital formation under a more market based system may not be enough to offset the drop in risk sharing. However, further reduction in the cost of market participation raises total welfare. Under the case where $\sigma = 1.05$, it is optimal to minimize the cost of participating in equity markets ($\chi^* = 1$), which coincides with $\lambda = .945$.

However, when inflation is 10%, moving towards a market-based system hinders total welfare and further reduction in the cost of becoming sophisticated fails to raise welfare above the level achieved under a more bank-based financial system. Interestingly, it is optimal to have a pure bank-based financial system when $\sigma = 1.1$. Moreover, in the economy with high inflation, a bank-based system leads to similar total welfare compared to a more market-oriented system with low inflation.
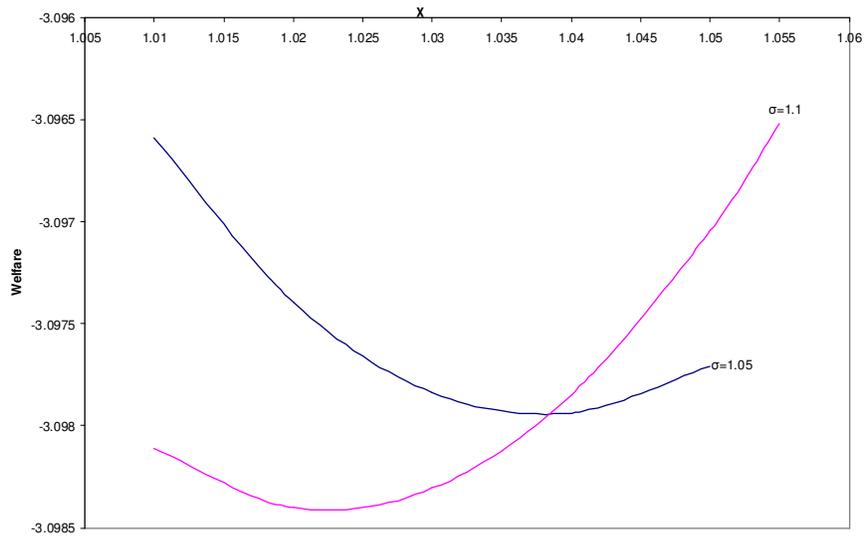
Figure 5. Optimal Financial Structure

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6   Conclusion

The structure of the financial system varies significantly across countries. For example, the financial system in Europe and Japan is often described as a bank-based system while the U.S. financial system is more market oriented. Recent trends also point out that equity markets are playing a bigger role in various financial systems, especially in high income countries. It is hard to believe that the stance of monetary policy does not influence the shape of the financial sector. In addition, given the important role the financial sector plays in the transmission of monetary policy, financial structure should matter for the formulation of policy. In order to address these issues, I develop a dynamic general equilibrium model with important roles for money, banks, and equity markets. More importantly, the participation in equity markets is an endogenous outcome and depends on economic conditions. In this setting, I demonstrate that policy makers should set low inflation targets in more market oriented economies in order to improve risk sharing that gets distorted by higher stock market participation. Furthermore, I show that the optimal financial structure depends on the inflation rate. In particular, it is optimal to minimize the cost of participating in equity markets when inflation is low. However, when inflation is high, a more bank-oriented financial system leads to higher total welfare.

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Technical Appendix

1. Proof of Proposition 1. From the expression for λ_t, (19) define $z\left(I_t\right) = \frac{1}{I_t\left[\frac{\chi}{I_t^{\theta-1}}+\frac{\pi}{1-\pi}(\chi-1)\right]^{\frac{\theta}{(\theta-1)}}} = \frac{\left[\chi I_t^{1-\theta}+\frac{\pi}{1-\pi}(\chi-1)\right]^{\frac{\theta}{1-\theta}}}{I_t}$, where (19) can be written as:

$\lambda_t = \frac{1}{\pi}\frac{1}{z(I_t)+\frac{1-\pi}{\pi}}$. It is trivial to show that: $z'\left(I_t\right) < 0$ if:

$$I_t > \left((\theta-1)\frac{1-\pi}{\pi}\frac{\chi}{(\chi-1)}\right)^{\frac{1}{\theta-1}} = \tilde{I}$$

which clearly implies that $\frac{d\lambda_t}{dI_t}$ $(<)\geq 0$ for $I \leq (>)\tilde{I}$.

Moreover, $\lim_{I_t\to\infty}\lambda_t \to \frac{1}{(1-\pi)}$ and at $I_t = 0$, $\lambda_t = \frac{1}{(1-\pi)} > 0$. This clearly implies that λ_t is U shaped. Furthermore, simple algebra implies that $\lambda\left(\tilde{I}\right) \leq 1$ if

$$(\chi-1)\chi^{\frac{1}{\theta-1}}\theta^{\frac{\theta}{\theta-1}} < (\theta-1)\frac{1-\pi}{\pi} \tag{27}$$

where the term on the left-hand-side of (27) is increasing in χ. Therefore, their exists a $\bar{\chi} > 1$ where $\bar{\chi}$: (27) holds with equality and for all $\chi \geq \bar{\chi}$, $\lambda = 1$. Given the characterization of λ, if $\chi < \bar{\chi}$, the polynomial $\lambda\left(I_t\right) = 1$ has two roots, $\underline{I}$, and $\bar{I}$, with $\underline{I} < \tilde{I} < \bar{I}$. Moreover, $\lambda\left(I\right) \in (0,1)$ if $I \in \left(\underline{I},\bar{I}\right)$. Finally, the self-selection constraint, (18) needs to hold. Therefore, we need $I_t \geq \left(\frac{\chi}{1-\frac{\pi}{1-\pi}(\chi-1)}\right)^{\frac{1}{\theta-1}} = \underline{I}_0$.

It is trivial to show that $\lambda\left(\underline{I}_0\right) > 1$ and that $\underline{I}_0 < \tilde{I}$ if $\chi < \frac{1-\pi}{\pi}\frac{\theta-1}{\theta}+1 = \bar{\chi}_0$. If this condition does not hold, a banking equilibrium with active markets does not exist. It remains to show that $\bar{\chi}_0 > \bar{\chi}$. This condition holds if the condition in (27) holds at $\bar{\chi}_0$. Upon substituting the expression for $\bar{\chi}_0$ into (27), we get: $\bar{\chi}_0^{\frac{1}{\theta-1}}\theta^{\frac{1}{\theta-1}} > 1$ which always holds given that $\bar{\chi}_0 > 1$ and $\theta > 1$. As a result, when $\chi < \bar{\chi}$, $1 < \underline{I}_0 < \underline{I} < \tilde{I} < \bar{I}$ and λ behaves as described above. This completes the proof of Proposition 1.

2. Proof of Lemma 2. From the capital supply locus, (26), it is clear that for $I \in \left[\tilde{I},\bar{I}\right)$, $\frac{dk}{dI} < 0$ since $\frac{d\lambda}{dI} > 0$. I proceed to demonstrate that $\frac{dk}{dI} < 0$ when $I \in \left(\underline{I},\tilde{I}\right]$.

$$\Omega\left(k\right) \equiv \frac{k_{t+1}}{w_t} = 1 - \frac{1}{\left[1-\left(1-\frac{1}{\left[\chi+\frac{\pi}{1-\pi}(\chi-1)I^{\theta-1}\right]^{\frac{1}{\theta-1}}}\right)\lambda\right]\frac{1-\pi}{\pi}\sigma+1}$$

Define $\mu\left(I\right) = \left(1 - \frac{1}{\left[\chi+\frac{\pi}{1-\pi}(\chi-1)I^{\theta-1}\right]^{\frac{1}{\theta-1}}}\right)\lambda\left(I\right)$, with $\frac{dk}{dI} < 0$ if $\mu'\left(I\right) > 0$. Using the expression for λ and some algebra:

$$\mu'(I) = \frac{\pi I^{-\frac{1}{\theta}} I^{\theta-2}}{I^{\frac{1}{\theta-1}}} \left(\left(-\theta\chi + I^{\frac{1}{\theta}} \right) \left(1 - \frac{1}{I^{\frac{1}{\theta}\frac{1}{\theta-1}}} \right) + \frac{(\chi-1)}{1-\pi} I^{\frac{1}{\theta}} \right)$$

Therefore, $\mu'(I) \geq 0$ if:

$$\left(-\theta\chi + I^{\frac{1}{\theta}} \right) \left(1 - \frac{1}{I^{\frac{1}{\theta}\frac{1}{\theta-1}}} \right) + \frac{(\chi-1)}{1-\pi} I^{\frac{1}{\theta}} \geq 0 \tag{28}$$

where the polynomial, (28) is increasing in I. It suffices to show that this condition holds at $I = 1$. Evaluating (28) at $I = 1$, the condition is satisfied if $\frac{(\chi-1)}{1-\pi} \geq 0$ which always holds. From the proof of Proposition 1, we know that $\underline{I}$ and $\bar{I}$ are both above unity. Therefore, $\mu'(I) \geq 0$ over the feasible range and $\frac{dk}{dI} < 0$. This completes the proof of Lemma 2.

3. Proof of Proposition 2. Evaluating (25) and (26) at the lower and upper bounds on I, an excess demand occurs at $I = \underline{I}$ if:

$$\psi \equiv \frac{1-\pi}{\pi} \frac{\alpha}{(1-\alpha)\underline{I}} \left[\sigma \frac{1-\pi}{\pi} + \underline{I}^{\frac{\theta-1}{\theta}} \right] \geq 1$$

which holds when $\sigma \geq \underline{\sigma}$, where $\underline{\sigma}$ is such that the above holds with equality. Analogously, an excess supply of capital is present at $\bar{I}$ If $\psi < 1$. This requires that $\sigma < \bar{\sigma}$, where $\bar{\sigma} : \psi = 1$ at $I = \bar{I}$. Consequently, when $\sigma \in (\underline{\sigma}, \bar{\sigma})$, an equilibrium with active equity markets and banks exists and is unique. In addition, when $\sigma > \bar{\sigma}$, an excess demand for capital prevails at $\bar{I}$. Given that the locus, (26) shifts downwards under a higher value of χ, it can intersect twice with (25) if $\chi \epsilon [\chi_0, \bar{\chi})$, χ_0 is such that both loci intersect at $\bar{I}$. This completes the proof of Proposition 2.

4. Proof of Proposition 4. From the equilibrium conditions, (25) and (26), denote the demand and supply for capital by k^D and k^S, respectively. It is trivial to show that $\frac{\partial k^D}{\partial \sigma} > 0$ and $\frac{\partial k^S}{\partial \sigma} > 0$, which results in an ambiguous impact on the equilibrium level of capital and its nominal return. If $\frac{\partial k^S}{\partial \sigma} > \frac{\partial k^D}{\partial \sigma}$, the capital stock will unambiguously rise and the nominal return to capital falls. Partially differentiating k^D and k^S with respect to σ to get:

$$\frac{\partial k^D}{\partial \sigma} = \frac{1}{1-\alpha} \left(\frac{\alpha A}{I} \right)^{\frac{1}{1-\alpha}} \sigma^{\frac{\alpha}{1-\alpha}}$$

and

$$\frac{\partial k^S}{\partial \sigma} = A\left[(1-\alpha)A\sigma\right]^{\frac{\alpha}{1-\alpha}} \frac{\left(\left[\left(\left[\frac{\frac{1}{\lambda}-(1-\pi)}{\pi} \right]^{\frac{1}{\theta}} \frac{1}{I^{\frac{\theta-1}{\theta}}} - 1 \right) \lambda + 1 \right] \frac{1-\pi}{\pi} \right)^{\frac{1}{1-\alpha}}}{\left(\left[\left(\left[\frac{\frac{1}{\lambda}-(1-\pi)}{\pi} \right]^{\frac{1}{\theta}} \frac{1}{I^{\frac{\theta-1}{\theta}}} - 1 \right) \lambda + 1 \right] \frac{1-\pi}{\pi}\sigma + 1 \right)^{\frac{2-\alpha}{1-\alpha}}}$$

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With some simplifying algebra, $\frac{\partial k^S}{\partial \sigma} \geq \frac{\partial k^D}{\partial \sigma}$ if:

$$I\frac{(1-\alpha)}{\alpha}\frac{\left(\left[\left(\left[\frac{\frac{1}{\lambda}-(1-\pi)}{\pi}\right]^{\frac{1}{\theta}}\frac{1}{I^{\frac{\theta-1}{\theta}}}-1\right)\lambda+1\right]\frac{1-\pi}{\pi}\right)}{\left(\left[\left(\left[\frac{\frac{1}{\lambda}-(1-\pi)}{\pi}\right]^{\frac{1}{\theta}}\frac{1}{I^{\frac{\theta-1}{\theta}}}-1\right)\lambda+1\right]\frac{1-\pi}{\pi}\sigma+1\right)} \geq 1 \qquad (29)$$

which clearly indicates that for a given I, $\frac{\partial k^S}{\partial \sigma} \geq \frac{\partial k^D}{\partial \sigma}$ when $\sigma \leq \hat{\sigma}_0$, where $\hat{\sigma}_0$ satisfies the above with equality. Furthermore, there exists a $\hat{\sigma}_1$, where $\hat{\sigma}_1 > \hat{\sigma}_0$, beyond which $\frac{dk}{d\sigma} < 0$. For $\sigma \leq \hat{\sigma}_0$, $\frac{dk}{d\sigma} > 0$ and $\frac{dI}{d\sigma} < 0$. In comparison, $\frac{dk}{d\sigma} > 0$ and $\frac{dI}{d\sigma} > 0$ if $\sigma\epsilon\,(\hat{\sigma}_0,\hat{\sigma}_1)$. Moreover, $\frac{dk}{d\sigma} < 0$ and $\frac{dI}{d\sigma} > 0$ if $\sigma > \hat{\sigma}_1$. I proceed to show that $\hat{\sigma}_0 < \underline{\sigma}$. Therefore, the nominal return to capital is rising with the inflation rate in the feasible domain. Specifically, evaluate (29) at $\sigma = \underline{\sigma}$, where $I = \underline{I}$ and $\lambda = 1$. Condition (29) does not hold if:

$$\sigma \geq \frac{\pi}{1-\pi}\left(\frac{(1-\alpha)}{\alpha}\frac{1-\pi}{\pi}\underline{I}-\underline{I}^{\frac{\theta-1}{\theta}}\right) = \underline{\sigma}$$

This automatically implies that $\hat{\sigma}_0 < \underline{\sigma}$ and $\frac{dI}{d\sigma} > 0$ for all $\sigma > \underline{\sigma}$. In addition, $\frac{dk}{d\sigma} \geq (<)\,0$ *if* $\sigma \leq (>)\,\hat{\sigma}_1$. The impact of σ on λ follows directly from the work in Proposition 1 given that I is strictly increasing in σ. This completes the proof of Proposition 4.