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## Central Bank Liquidity Provision and Financial Sector Competition

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## Abstract

This paper presents a general equilibrium production economy where money is essential and financial intermediaries provide important economic functions. In this setting, we study the effects of liquidity provision by the monetary authority under two different banking structures: a perfectly competitive and a fully concentrated banking system. When the banking sector is perfectly competitive, liquidity injections through an open market purchase stimulate capital investment and production. Interestingly, an expansionary monetary policy can become contractionary when the banking sector is fully concentrated. This necessarily happens in economies where government liabilities constitute a large fraction of total deposits and inflation is high. Moreover, we demonstrate that imperfect banking competition is a source of indeterminacy of dynamical equilibria. More specifically, the economy can display Hopf bifurcation. However, monetary policy plays an important role in controlling deterministic cycles and endogenous volatility that could arise under a fully concentrated banking sector.

*JEL Classification:* E43, E52, L11

*Keywords:* Monetary Policy, Open Market Operations, Banking Competition

## 1 Introduction

It is widely accepted that the banking sector plays an important role in the transmission of monetary policy.<sup>1</sup> On the verge of creating a monetary union in Europe, in the late 1990s, questions were raised whether monetary policy will

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<sup>1</sup>See for example previous studies by Bernanke and Blinder (1992), Bernanke and Gertler (1995), Kashyap and Stein (2000), Kishan and Opiela (2000), Nilsen (2002), and Gan (2007).

have symmetric effects across members of the European Union. For instance, Kashyap and Stein (1997, 2000) highlight the importance of banking structure as a primary factor of asymmetry in the effects of monetary policy across members of the single European currency. Specifically, Kashyap and Stein (1997) point out that the effects of monetary policy through the bank lending channel should be weaker in economies with a more concentrated banking sector such as Greece, compared to Germany that has a much less concentrated banking system.<sup>2</sup>

This issue received considerable attention lately as fiscal problems in some member countries of the European monetary union like Greece spilled over and threatened the whole banking sector across the entire Euro zone. The European central bank's response in 2010 and 2011 was to inject banks with more liquidity to alleviate the stress in the market and encourage lending.<sup>3</sup> These events raise two very important questions: given that the banking structure differs across Euro member countries, does liquidity provision by the European central bank have symmetric effects? That is, do the effects of open market operations depend on the competitive structure of the banking sector? Furthermore, does the industrial organization of the banking sector have implications for the determinacy of equilibria and economic stability?

This manuscript attempts to shed some light on these issues by comparing the effects of central bank liquidity provision under two different financial structures. Specifically, we consider a two-period overlapping generations production economy inhabited by two types of agents, depositors and bankers. Following Townsend (1987), depositors are born on one of two geographically separated, yet symmetric locations. With some probability, depositors must relocate to the other location after they make their portfolio choice. Due to private information and limited communication relocated agents must liquidate their assets (bonds and capital) into cash to be able to consume. As banks can completely diversify idiosyncratic risk, all savings are intermediated. Therefore, bankers take deposits, insure their depositors against relocation shocks, and invest in the economy's assets to maximize profits. Finally, there is a government that adjusts the amount of liabilities (bonds and cash) to satisfy its budget. As in Wallace (1984), the monetary authority conducts policy by changing the bonds to money ratio (or the degree of liquidity in the banking sector).

We begin our analysis by assuming that the banking sector is perfectly competitive. As deposits and capital markets are competitive, banks do not earn profits in equilibrium. Therefore, each bank makes its portfolio choice to maximize the expected utility of its depositors. Under a technical condition, a steady-state exists and is unique. Moreover, there is a unique trajectory leading the economy to its stationary level. When the banking sector is competitive, liquidity injections raise the ability of banks to invest in capital formation.

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<sup>2</sup>Data from the European Central bank point out to significant differences in the degree of banking concentration across members of the single European currency. For instance, in 2009, the concentration ratio, as measured by the assets of the largest 5 banks to total assets, was 0.934 in Estonia compared to 0.25 in Germany.

<sup>3</sup>For more information, one may refer to the European central bank's annual report as well as Mario Draghi, president of the ECB's introductory statement on December 2011.

Therefore, the level of output increases under an expansionary monetary policy.

We proceed to examine an economy in which the banking sector is fully concentrated. Following Ghossoub (2012), the banker has market power in both deposit and capital markets. In particular, the bank offers deposit contracts to its depositors by extracting all the surplus from the market. Additionally, the banker exerts its market power in the rental market for capital goods. Interestingly, when the banking sector is fully concentrated, two steady-state equilibria can exist. In one steady-state, investment activity is low as the bank is allocating a large fraction of its deposits towards government liabilities. Moreover, nominal interest rates and inflation are high. The other steady-state has a high level of capital formation and low inflation and nominal interest rates.

Notably, when the banking sector is not competitive, the effects of monetary policy are non-monotonic. More importantly, the impact of liquidity injections or contractions by the monetary authority on the level of output depends on the level of capital formation and inflation in the economy. For example, a lower debt to reserves ratio raises the level of output in an economy with a relatively high level of capital investment and a low inflation rate. By comparison, an expansionary monetary policy is contractionary if the level of capital formation is low and the inflation rate is high enough.

Unlike a competitive banking system, monetary policy affects the economy through three primary channels. First, an expansionary monetary policy reduces the amount of government debt in the bank's portfolio, which increases its ability to invest in capital goods. In addition, a higher degree of liquidity positively affects the amount of insurance received by depositors. As the bank has market power in the deposit market, it responds by holding less cash reserves as there is a lower need for insurance against relocation shocks. Finally, an expansionary monetary policy reduces the marginal return the bank earns from capital investment, which provides an incentive for the bank to reduce its investment activity in order to maximize profits.

When the level of capital formation is high, the bank is providing a lot of insurance to its depositors against relocation shocks. More importantly, the marginal gains from restricting capital investment under a higher degree of liquidity is small. Consequently, the first two effects dominate, and an expansionary monetary policy raises capital investment and reduces nominal interest rates as under a perfectly competitive banking system.

By comparison, if markets are highly distorted by market power, the level of investment in the economy is low. Moreover, the bank is making a small amount of profits and charging high interest rates. Therefore, changes in the degree of liquidity have significant adverse effects on the marginal revenue from capital. As we demonstrate in the text, these effects dominate the direct impact on the bank's balance sheet and any gains from a reduction in the marginal cost of investing in capital that come about under an expansionary policy. As a result, capital investment decreases.

We conclude our work by conducting stability analysis. Interestingly, when two steady-states exist, they can both be saddle path stable. In this manner, although the effects of monetary policy can become ambiguous under an im-

perfectly competitive banking sector, lack of competition can preserve the local determinacy present under a competitive banking system. However, if the level of investment under an imperfectly competitive banking sector is sufficiently low, orbits in the neighborhood of the steady-state can display stable spirals. Further, the economy can display Hopf bifurcations. More importantly, injecting a large amount of liquidity in the economy by the monetary authority shields it against endogenous fluctuations and deterministic cycles.

Interestingly, our results are consistent with recent empirical evidence that finds significant differences in the effects of monetary policy across Europe. For example, over the period 1982–1998, Ciccarelli and Rebucci (2006) find that output is much more responsive to a common monetary shock in Germany compared to Spain, which has a more concentrated banking system. Similar results were also obtained by Cecchetti (1999), that suggest weaker effects of monetary policy in economies with a more concentrated banking sector.<sup>4</sup> These asymmetries are usually attributed to the ability of large banks to raise external funds when facing a tight monetary policy compared to financially constrained small banks. Our work demonstrates that the degree of banking competition is important for monetary policy. However, the ability of a fully concentrated banking system to overcome a tight monetary policy also depends on other distortions in the economy such as the amount of government liabilities in the bank’s portfolio and the inflation rate in the economy.

#### *Related Literature*

Although the banking sector in most countries has become more concentrated in the past few decades, there has been very little work done to explore the consequences of banking competition on the conduct of monetary policy in a general equilibrium setting.<sup>5,6</sup> Among the few papers we are aware of, Williamson (1986) studies an endowment economy where banks have market power in credit markets, while the market for deposits is perfectly competitive. He demonstrates that monetary policy is not superneutral when the banking sector is not competitive. In his setting, cash and deposits are perfect substitutes. Therefore, a higher rate of money creation lowers the return to cash, which stimulates deposits and promotes lending activity.

Recent work by Ghossoub (2012) and Ghossoub, Laosuthi, and Reed (2012) demonstrate how the effects of a change in the rate of money creation can

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<sup>4</sup>More recent work by Olivero et al. (2011) finds support for a negative correlation between banking concentration and the effects of monetary policy on bank lending in Asia and Latin America. Furthermore, analyzing local markets in the United States, Adams and Amel (2011) find that bank lending is more responsive to changes in the federal funds rate in areas where the banking sector is less concentrated.

<sup>5</sup>Previous studies that discuss the acceleration of financial consolidation across countries include Berger et al. (1999), Amel et al. (2004), Berger et al. (2004), Beck et al. (2004), and more recent work by Goddard et al. (2007).

<sup>6</sup>There is a large literature that emphasizes the the role of banking competition in financial stability. See for example, the work by Matutes and Vives (2000), Allen and Gale (2004), Boyd et al. (2004), and Boyd and De Nicoló (2005) among others.

vary significantly depending on the competitive structure of the banking sector. Specifically, in a two sector monetary growth economy, Ghossoub (2012) shows that the standard Tobin logic can fail to hold in an economy with a fully concentrated banking sector when the degree of liquidity risk in the economy is significant. That is, investment activity could fall under a higher rate of money creation.

While these studies are important, central banks in developed countries conduct monetary policy through open-market operations. Therefore, it is important to understand the implications of banking competition for monetary policy from this angle.<sup>7</sup> This manuscript provides a first attempt at examining how the effects of monetary policy through open-market operations depend on the industrial organization of the banking sector. Specifically, we compare the effects of open market operations under two different financial structures, a perfectly competitive banking sector and a monopolistic banking sector.

Our analysis indicates that the competitive structure of the banking sector bears significant consequences for the number of equilibria in the economy and for monetary policy. Although liquidity injections by the central bank can generate more stability in economies where the banking sector is not competitive, their effect on output becomes ambiguous. That is, increasing the amount of liquidity by the central bank can have adverse consequences on output when the banking sector is fully concentrated.

These results shed some light on current policies in Europe. Specifically, it appears that members of the Eurozone are moving towards fiscal integration in order to reduce their exposure to financial crises and create more harmony in the effects of monetary policy. This paper also calls for more integration in the banking sector. As long as significant differences in banking structure prevail, the effects of monetary policy on economic activity and stability will be highly asymmetric. More importantly, liquidity injections in economies where the banking sector is highly concentrated and government debt to total output is significant, such as Greece, might yield unfavorable results.

The paper is organized as follows. In Section 2, we describe the model and study the behavior of depositors and factors markets. Section 3 studies an economy with a perfectly competitive banking sector. An economy where the banking sector is fully concentrated is analyzed in Section 4. We conclude in Section 5. Most of the technical details are presented in the Appendix.

## 2 Environment

Consider a discrete-time economy with two geographically separated, yet symmetric locations or islands. Each location is populated by an infinite sequence of two-period lived overlapping generations. Let  $t = 1, 2, \dots, \infty$ , index time. Within each generation, there are two types of agents: workers (or depositors) and

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<sup>7</sup>A number of studies examine the effects of open market operations under a perfectly competitive environment. Among these studies we cite, Wallace (1984), Bhattacharya et al. (1997), and Schreft and Smith (1998).

bankers. At the beginning of each time period, a continuum of depositors is born. The population of depositors is equal to one. By comparison, there are  $N \geq 1$  bankers.

Bankers and depositors are assumed to derive utility from consuming the economy's single consumption good only when old,  $c_t$ . Each depositor is endowed with one unit of labor effort when young and is retired when old. Because there is no disutility from labor effort, labor is supplied inelastically in factor markets. The preferences of a typical worker are,  $u(c_t) = \ln c_t$ .<sup>8</sup> Furthermore, bankers receive no endowments and are assumed to be risk neutral.

The consumption good is produced by a representative firm using capital and labor as inputs. The production function is of the Cobb-Douglas form, with  $Y_t = AK_t^\alpha L_t^{1-\alpha}$ , where  $Y_t$ ,  $K_t$ , and  $L_t$  are period  $t$  aggregate output, capital stock, and labor, respectively. In addition,  $A$  is a technology parameter and  $\alpha \in (0, 1)$  reflects capital's share of total output. Equivalently, output per worker is expressed by  $y_t = Ak_t^\alpha$ , with  $k_t = \frac{K_t}{L_t}$  is the capital labor ratio. Further, we assume that the capital stock depreciates completely in the production process. As in standard one sector models with complete depreciation, one unit of foregone consumption in period  $t$  generates one unit of new capital in  $t + 1$ .

There are three types of assets in this economy: Fiat money, government debt, and physical capital. Denote the total amount of real money balances and government debt by  $m_t$  and  $b_t$ , respectively. Furthermore, at the initial date 0, the generation of old depositors at each location is endowed with the aggregate nominal money stock,  $M_0$ .

Government bonds are assumed to mature in one period. Additionally, one dollar invested in bonds pays a gross nominal interest rate,  $I_t$  when held between periods  $t$  and  $t + 1$ . Define the price level,  $P_t$ , to be the dollar value of a unit of goods in period  $t$ , which is assumed to be common across locations. In this manner, the real return on government debt that matures in  $t+1$  is:  $R_t^b = I_t \frac{P_t}{P_{t+1}}$ , where  $\frac{P_t}{P_{t+1}}$  is the gross real return to money.

Following Schreft and Smith (1997), depositors are subject to relocation shocks. After portfolios are made, a fraction of young depositors is randomly chosen to relocate. The probability of relocation,  $\pi$ , is public information and the same in each location.<sup>9</sup>

If an agent moves to another location, limited communication and spatial separation prevent her from trading claims on assets in her home location. As in standard random relocation models, fiat money alleviates these trade frictions. Further, it is the only asset that can be carried across islands. Therefore, depositors who learn they will be relocated will liquidate all their asset holdings into currency. Random relocation thus plays the same role that liquidity preference shocks perform in Diamond and Dybvig (1983). In addition, it provides a fundamental role for financial intermediation.

<sup>8</sup>The primary insights of the paper hold under general CRRA preferences with a coefficient of risk aversion less than unity.

<sup>9</sup>Because there is a unit mass of depositors, the probability of relocation also reflects the number of relocated agents.

In this setting, bankers serve two primary purposes in the economy. First, they insure depositors against random relocation shocks by diversifying idiosyncratic risk. Additionally, they invest in money and capital markets on behalf of their depositors. Unlike depositors, bankers are not subject to relocation shocks.

The final agent in the economy is a government (monetary authority) that issues one period bonds and fiat money in order to finance interest payments on previously issued debt. The government's budget constraint is:

$$R_{t-1}^b b_{t-1} = \frac{M_t - M_{t-1}}{P_t} + b_t \quad (1)$$

where  $\frac{M_t - M_{t-1}}{P_t}$  is real seigniorage revenue in period  $t$ .

Following Wallace (1984), we assume that the monetary authority targets the composition of government liabilities in the economy. In particular, the central bank chooses a fixed bonds to money ratio,  $\mu$ , in period zero, where

$$\mu \equiv \frac{b_t}{m_t} > 0 \quad (2)$$

As in previous studies such as Wallace (1984), a change in  $\mu$  can be thought as permanent open market operations. Specifically, a tight monetary policy is reflected by a higher  $\mu$  (open market sale).

## 2.1 Trade

### 2.1.1 Factors Markets

In period  $t$ , a representative firm rents capital and hires workers in perfectly competitive factor markets at rates  $r_t$  and  $w_t$ , respectively. The inverse demands for labor and capital by a typical firm are expressed by

$$w_t = (1 - \alpha) A k_t^\alpha \equiv w(k_t) \quad (3)$$

and

$$r_t = f'(k_t) = \alpha A k_t^{\alpha-1} \quad (4)$$

### 2.1.2 Depositors

In period  $t$ , a depositor receives her income,  $w_t$ , from working, which is entirely saved. Given that financial intermediaries are costless to access, it is easy to verify that financial intermediation always dominates direct investment in equilibrium. Therefore, all savings are intermediated. In order to make the analysis more tractable, we assume that depositors face an exogenous reservation expected utility received in absence of banks, which we denote by  $\underline{u}$ .<sup>10</sup> Thus,

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<sup>10</sup>As we discuss in the following section, the primary insights of the paper continue to hold when this assumption is relaxed.



depositors are willing to participate in the banking sector if they receive at least  $\underline{u}$ . That is, we assume that the following participation constraint holds at every point in time:

$$\pi \ln c_t^m + (1 - \pi) \ln c_t^n \geq \underline{u} \quad (5)$$

where  $c_t^m$  and  $c_t^n$  are the levels of consumption if an agent relocates (a mover) and does not relocate (non-mover), respectively. Under this condition, all savings are intermediated.

### 3 Perfectly Competitive Banking System

As a benchmark, we assume that the banking sector is perfectly competitive. Because bankers are Bertrand-Nash competitors, perfect competition is realized when the number of banks exceeds unity. At the beginning of period  $t$ , each banker announces deposit rates taking the announced rates of return of other banks as given. Specifically, a bank promises a gross real return on deposits,  $r_t^m$  if a young individual is relocated and a gross real return  $r_t^n$  if not. Rates of return are chosen such that depositors participate in the banking sector. That is, (5) must hold.

In period  $t$ , each bank invests all deposits in the economy's assets. In particular, a bank's portfolio choice involves determining the amount of capital investment,  $k_{t+1}$ , and the amount of government liabilities,  $m_t$  and  $b_t$  to acquire.<sup>11</sup> A typical bank's balance sheet is expressed by:

$$k_{t+1} + m_t + b_t = w_t \quad (6)$$

Because depositors' types are publicly observable, banks are able to offer deposits contracts that are contingent on the realization of the shock. As relocated agents need cash to transact, payments made to movers, satisfy:

$$\pi c_t^m = \pi r_t^m w_t = m_t \frac{P_t}{P_{t+1}} \quad (7)$$

Furthermore, we choose to study equilibria in which money is dominated in rate of return. Therefore, banks will not hold excess reserves. A bank's total payments to non-movers are therefore paid out of its revenue from capital and government debt. The constraint on payments to non-movers is such that:

$$(1 - \pi) c_t^n = (1 - \pi) r_t^n w_t = r_{t+1} k_{t+1} + R_t^b b_t \quad (8)$$

Finally, agents must not have an incentive to misrepresent their types ex-post. Therefore, the following incentive compatibility constraint must also hold:

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<sup>11</sup>One may slightly change the environment to incorporate credit markets. For example, consumer goods firms may be thought as a separate group of agents that own the capital stock but receive no endowments when young. Therefore, they must borrow from banks to finance their investment in capital goods and produce when old. Such changes will not affect the primary insights of the paper.

$$\frac{r_t^n}{r_t^m} \geq 1 \quad (9)$$

Perfect competition eliminates any equilibrium profits and renders banks choose their portfolio to maximize the expected utility of their depositors. A typical bank's problem is summarized by:

$$\underset{r_t^m, r_t^n, m_t, k_{t+1}, b_t}{Max} \quad \pi \ln r_t^m w_t + (1 - \pi) \ln r_t^n w_t \quad (10)$$

subject to (5)-(9).

In equilibrium, a bank will invest in capital and hold government bonds up to the point where both assets yield the same rate of return. Therefore, the following no-arbitrage condition between private capital and government bonds must hold:

$$r_{t+1} = R_t^b = R_t \quad (11)$$

Furthermore, the demand for real money balances is such that:

$$m_t = \pi w_t \quad (12)$$

Because depositors have logarithmic preferences, the demand for cash reserves is price insensitive under a perfectly competitive banking system. This happens because the income and substitution effects from a change in the return to different assets exactly offset each other.<sup>12</sup>

Using the expression for money demand, (12) into the bank's balance sheet, (6), the demand for government debt is:

$$b_t^d = w_t - m_t - k_{t+1} \quad (13)$$

Finally, using (6), (7), (8), and (12), the relative return to depositors is:

$$\frac{r_t^n}{r_t^m} = I_t \quad (14)$$

which indicates that depositors receive a lower amount of insurance under a higher interest rate. Moreover, the contract between the bank and its depositors is incentive compatible when money is dominated in rate of return. That is,  $I_t \geq 1$ .

### 3.1 General Equilibrium

We proceed to characterize the equilibrium for the economy with a perfectly competitive banking sector. Equilibrium is characterized by a set of non-

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<sup>12</sup>As we demonstrate in the following section, the bank's portfolio responds to the return to different assets when the banking sector is not competitive. This takes place because the return to different assets directly affects the profitability of the bank and therefore, its portfolio composition.

negative quantities,  $(k_{t+1}, b_t, m_t, L_t)$  and prices,  $(r_{t+1}, R_t^b, \frac{P_t}{P_{t+1}}, w_t)$  that clear capital, bonds, money, and labor markets.

In equilibrium, the labor market clears, with  $L_t = 1$ , and labor receives its marginal product, (3). Furthermore, from the expression for money demand, (12), and the ratio of government liabilities, (2), the supply of bonds is  $b_t^s = \mu m_t$ . Therefore, the bond market clears when:

$$w_t - m_t - k_{t+1} = \mu m_t$$

which yields the general equilibrium amount of capital supplied by banks or the law of motion for capital:

$$k_{t+1} = [1 - (1 + \mu) \pi] w(k_t) \equiv \psi(k_t) \quad (15)$$

where  $(1 + \mu) \pi$  is the fraction of deposits allocated towards government liabilities.

Furthermore, imposing equilibrium on the money market and combining the government budget constraint, (1), with (2) and (12), nominal interest rates are such that:

$$I_t = \frac{1}{\frac{1+\mu}{\alpha A} \frac{k_{t+1}}{k_t^\alpha} - \mu} \quad (16)$$

Equivalently, prices evolve according to:

$$\frac{P_{t+1}}{P_t} = \frac{I_t}{f'(k_{t+1})} = \frac{k_{t+1}^{1-\alpha} k_t^\alpha}{(1 + \mu) k_{t+1} - \mu \alpha A k_t^\alpha} \quad (17)$$

Finally, in equilibrium, the incentive compatibility constraint, (9), must be satisfied, which requires money to be dominated in rate of return:

$$I_t \geq 1 \quad (18)$$

In sum, equations (4) – (5) and (15) – (18) characterize the behavior of the economy at a particular point in time.

We proceed to examine the behavior of the economy in the steady-state. Imposing steady-state on the evolution of capital, (15), the stationary level of capital formation is:

$$k^{PC} = ((1 - \alpha) A [1 - (1 + \mu) \pi])^{\frac{1}{1-\alpha}} \quad (19)$$

where the superscript,  $PC$ , reflects the outcome under a perfectly competitive banking sector. Using (19) into (12), the amount of money balances in the economy is:

$$m^{PC} = \pi [(1 - \alpha) A]^{\frac{1}{1-\alpha}} ([1 - (1 + \mu) \pi])^{\frac{\alpha}{1-\alpha}} \quad (20)$$

Additionally, imposing steady-state on (16) and using (19) into (11), the real

and the nominal return to capital are respectively,

$$R^{PC} = \frac{\alpha}{(1 - \alpha) [1 - (1 + \mu) \pi]} \quad (21)$$

and

$$I^{PC} = \frac{\alpha}{(1 + \mu) (1 - \alpha) [1 - (1 + \mu) \pi] - \alpha \mu} \quad (22)$$

**Proposition 1.** *Suppose  $\underline{u}$  is sufficiently small. Under this condition, a steady-state exists and is unique if  $\mu \in (\mu_0, \mu_1)$ , where  $\mu_0 = \frac{1-2\alpha}{(1-\alpha)\pi} - 1$  and  $\mu_1 = \frac{1-\pi}{\pi} > \mu_0$ . Moreover, the steady-state is globally stable.*

The first condition in Proposition 1 is necessary for depositors to participate in the banking sector. Moreover, the second condition guarantees that all assets are held in positive amounts in equilibrium. In particular, from the expression for capital, (19), capital investment is positive if the fraction of deposits allocated towards government liabilities is below unity. This takes place if  $\mu < \mu_1$ . In addition, money is dominated in rate of return if the ratio of government liabilities is above some level,  $\mu_0$ . Intuitively, under a higher debt to reserves ratio, the monetary authority raises the inflation tax to cover additional debt obligations incurred by the government. Therefore, money is dominated in rate of return when  $\mu > \mu_0$ .

Interestingly, monetary tightening (a higher  $\mu$ ) reduces capital formation and raises nominal interest rates when the banking sector is perfectly competitive. Specifically, a tight monetary policy raises the amount of public debt in banks' portfolios, which reduces their ability to invest in capital. The lower amount of capital raises its return.

We proceed to study the stability of the steady-state. The dynamical properties of the economy can be derived from the law of motion of capital, (15). It is easily verified that  $\psi'(k_t) > 0$  and  $\psi''(k_t) < 0$ , which implies that  $\psi(k_t)$  is concave in  $k_t$  as illustrated in Figure 1 below. Consequently, the steady-state equilibrium is stable. Furthermore, using (15) into (16) and (17), there is a unique nominal interest rate and a unique inflation rate for any initial capital stock,  $k_0$ . Therefore, the steady-state is determinate as there is a unique trajectory taking the economy to its stationary level.

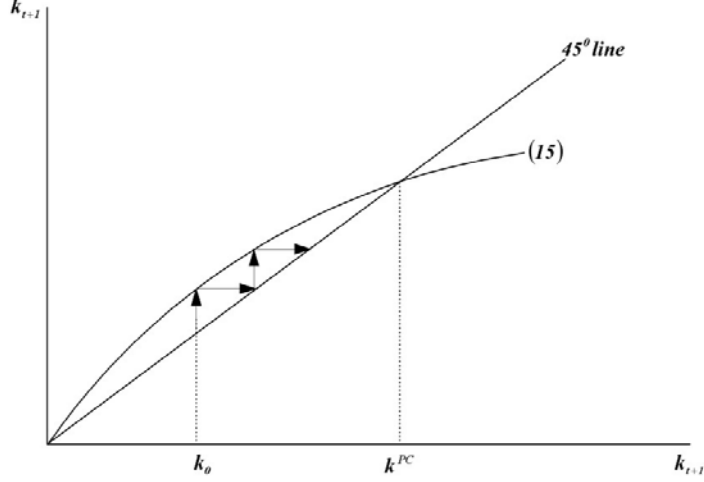


Figure 1: Equilibrium Under Perfect Competition

## 4 A Monopoly Banking System

In contrast to the previous section, we now examine an economy where there is one banker,  $N = 1$ . At the beginning of period  $t$ , the banker announces deposit rates,  $r_t^m$  and  $r_t^n$ . Given that the banking sector is fully concentrated, the bank exerts its market power by extracting all surplus from deposit markets. Hence, the participation constraint, (5) holds with equality, with:

$$\pi \ln r_t^m w_t + (1 - \pi) \ln r_t^n w_t = \underline{u} \quad (23)$$

As the banker values old age consumption, she invests all deposits in asset markets and makes her portfolio and pricing decisions,  $(m_t, k_{t+1}, b_t, r_t^m, r_t^n, r_{t+1})$  to maximize profits in  $t + 1$ ,  $\Pi_{t+1}$ , where

$$\Pi_{t+1} = \underset{m_t, k_{t+1}, b_t, r_t^m, r_t^n}{Max} r_{t+1} (k_{t+1}) k_{t+1} + R_t^b b_t + m_t \frac{P_t}{P_{t+1}} - \pi r_t^m w_t - (1 - \pi) r_t^n w_t \quad (24)$$

subject to (6), (7), and (9). Further, payments made to non-relocated agents are made out of the return from private capital and government bonds. The banker is willing to provide financial services only if she makes positive profits. Thus, the constraint on payments to non-movers is such that:

$$(1 - \pi) r_t^n w_t < r_{t+1} k_{t+1} + R_t^b b_t \quad (25)$$

Because the bank is the sole supplier of capital, it faces a downward sloping

demand for capital, (4). Therefore, the bank takes into account that it must charge firms a lower rental rate to stimulate capital consumption. Given that both the banker and the government have market power in the bonds market, we simplify the analysis by assuming that both parties take the market interest rate as given.

In sum, the bank maximizes (24) subject to (4), (6), (7), (9), (23), and (25). Substituting the binding constraints into the objective function, the problem is reduced into:

$$\Pi(k_{t+1}, b_t) = \underset{k_{t+1}, b_t}{Max} \alpha A k_{t+1}^\alpha + R_t^b b_t - \frac{(1-\pi) \pi^{\frac{\pi}{1-\pi}} e^{\frac{u}{1-\pi}} \left(\frac{P_{t+1}}{P_t}\right)^{\frac{\pi}{1-\pi}}}{[w(k_t) - k_{t+1} - b_t]^{\frac{\pi}{1-\pi}}} \quad (26)$$

The profit maximizing choice of capital is such that:

$$\frac{\partial \Pi(k_{t+1}, b_t)}{\partial k_{t+1}} \equiv \Pi_1(k_{t+1}, b_t) = \alpha^2 A k_{t+1}^{\alpha-1} - \frac{\pi}{1-\pi} \frac{(1-\pi) \pi^{\frac{\pi}{1-\pi}} e^{\frac{u}{1-\pi}} \left(\frac{P_{t+1}}{P_t}\right)^{\frac{\pi}{1-\pi}}}{[w(k_t) - k_{t+1} - b_t]^{\frac{\pi}{1-\pi}+1}} = 0 \quad (27)$$

where the term  $\alpha^2 A k_{t+1}^{\alpha-1}$ , is the marginal revenue from investing in capital.

Moreover,  $\frac{\pi}{1-\pi} \frac{(1-\pi) \pi^{\frac{\pi}{1-\pi}} e^{\frac{u}{1-\pi}} \left(\frac{P_{t+1}}{P_t}\right)^{\frac{\pi}{1-\pi}}}{[w(k_t) - k_{t+1} - b_t]^{\frac{\pi}{1-\pi}+1}}$  is the marginal cost of capital, which reflects the additional payments the monopoly has to make to non-movers when it increases its capital investment. Specifically, for a given stock of government debt, the bank will hold less cash balances when private capital increases. Therefore, the return to relocated agents declines, which requires the bank to increase its payments in the good state (if depositors do not relocate) to prevent deposit withdrawals.

Analogously, the profit maximizing choice of government bonds is such that:

$$\frac{\partial \Pi(k_{t+1}, b_t)}{\partial b_t} \equiv \Pi_2(k_{t+1}, b_t) = R_t^b - \frac{\pi}{1-\pi} \frac{(1-\pi) \pi^{\frac{\pi}{1-\pi}} e^{\frac{u}{1-\pi}} \left(\frac{P_{t+1}}{P_t}\right)^{\frac{\pi}{1-\pi}}}{[w(k_t) - k_{t+1} - b_t]^{\frac{\pi}{1-\pi}+1}} = 0 \quad (28)$$

which has similar interpretation as (27).

Using (27) and (28), the bank holds capital and issues credit to the public sector up to the point where both assets yield the same rate of return, at the margin. That is, the following no-arbitrage condition must hold:

$$\alpha^2 A k_{t+1}^{\alpha-1} = R_t^b \quad (29)$$

In comparison to a perfectly competitive banking system, market power in capital markets, generates a wedge between the average return on capital and that on government debt. That is, capital earns a premium over government bonds.

Moreover, the marginal gains from capital investment are lower compared to a perfectly competitive banking sector.

Using (6), (27), (29), and the definition of the nominal interest rate, the equilibrium demand for money by the bank is:

$$m_t \equiv m\left(I_t, \frac{P_t}{P_{t+1}}\right) = m_t = \pi e^{\frac{u}{I_t^{1-\pi}}} \frac{P_{t+1}}{P_t} \quad (30)$$

In contrast to a competitive banking system, the demand for cash reserves is strictly decreasing in the nominal return on government bonds due to a higher cost of holding money. However, the demand for cash reserves is strictly increasing with the inflation rate for a given nominal interest rate. Intuitively, for a given portfolio choice, relocated agents receive lower consumption under a higher inflation tax. Therefore, the bank must hold more cash reserves in order to induce its depositors to retain their deposits in the bank.

In addition, using (5), (7), and (30), the return to each type of depositor and the relative return on deposits are respectively:

$$r_t^m w_t = \frac{e^{\frac{u}{I_t^{1-\pi}}}}{I_t^{1-\pi}} \quad (31)$$

$$r_t^n w_t = e^{\frac{u}{I_t^\pi}} I_t^\pi \quad (32)$$

and

$$\frac{r_t^n}{r_t^m} = I_t \quad (33)$$

#### 4.1 General Equilibrium

In equilibrium, all markets clear. From the work under a perfectly competitive banking system, the supply of government debt is expressed by  $b_t^s = \mu m\left(I_t, \frac{P_t}{P_{t+1}}\right)$ , where  $m\left(I_t, \frac{P_t}{P_{t+1}}\right)$  is obtained from (30). In addition, using the expression for money demand, (30) and the definition of the policy tool,  $\mu$ , into the bank's balance sheet, (15), to obtain the general equilibrium supply of capital by the monopolist:

$$k_{t+1} = w(k_t) - (1 + \mu) m(I_t, \rho_t) \quad (34)$$

where  $\rho_t = \frac{P_t}{P_{t+1}}$  is the gross real return to money balances between  $t$  and  $t + 1$ .

Furthermore, using the expression for money demand, (30), the no-arbitrage condition, (29), and the definition of the policy tool,  $\mu$ , into the government budget constraint, equilibrium in the money market generates the evolution of the real return to money:

$$\rho_{t+1} = \frac{(1 + \mu) I_t^{1-\pi}}{(1 + I_t \mu) I_{t+1}^{1-\pi}} \quad (35)$$

and from the definition of  $I_t$  and the no-arbitrage condition, (29), the equilibrium inflation rate is:

$$\frac{1}{\rho_t} = \frac{I_t}{\alpha^2 A k_{t+1}^{\alpha-1}} = \frac{(1 + I_{t-1} \mu) I_t^{1-\pi}}{(1 + \mu) I_{t-1}^{1-\pi}} \quad (36)$$

Equations (34) - (36) characterize the behavior of the economy at a particular point in time. We proceed our analysis by studying the steady-state behavior of the economy under a monopoly banking system.

#### 4.1.1 Steady-State Analysis

Imposing steady-state on (36), the steady-state inflation rate is  $\frac{P_{t+1}}{P_t} = \frac{1+\mu I}{1+\mu}$ . Substituting into (30), the steady-state demand for money by the bank is:

$$m(I) = \frac{\pi e^{\frac{u}{\mu}}}{I^{1-\pi}} \frac{1 + \mu I}{1 + \mu} \quad (37)$$

We begin with the following observation:

**Lemma 1.**  $\frac{dm}{dI} \leq (>) 0$  for all  $I \leq (>) \tilde{I}$ , where  $\tilde{I} = \frac{1}{\mu} \frac{1-\pi}{\pi}$ .

In contrast to the competitive case, Lemma 1 indicates that the effect of interest rates on the demand for money balances is non monotonic when the banking sector is not competitive. Intuitively, a change in the nominal interest rate has two effects on the bank's decision to hold cash balances. First, a higher nominal return on long term investments lowers the incentive to hold cash reserves (the dominated asset) as profits are generated from bonds and physical capital. However, a higher nominal interest rate also raises the government's debt obligations, which requires the monetary authority to increase the inflation tax. This in turn adversely affects the return to relocated agents. The bank responds by holding more cash balances to prevent deposit withdrawals.

At low levels of interest, the gains from holding more cash reserves under a higher interest rate are much lower than the additional costs in terms of lower profits. This occurs because the bank is already providing a lot of insurance against liquidity risk when the cost of holding money is low. However, as interest rates increase beyond some level, depositors are receiving a low amount of insurance against relocation shocks. Therefore, it becomes too costly for the bank to cut its money holding under a higher interest rate as depositors will pull their money out of the bank. Consequently, the bank responds to a higher interest rate by holding more cash reserves at the cost of making less profits when interest rates are high enough.

Imposing steady-state on (34) and using the money demand equation, (37), the supply of capital in the steady-state is:



$$k = w(k) - (1 + \mu) \frac{\pi e^{\frac{u}{I}}}{I^{1-\pi}} \frac{1 + \mu I}{1 + \mu} \quad (38)$$

and the condition equating the marginal return to capital and the return to government bonds is:

$$\frac{(1 + \mu)I}{1 + \mu I} = \alpha^2 A k^{\alpha-1} \quad (39)$$

The stationary behavior of an economy with a fully concentrated banking sector is summarized by (38) and (39). Using the no-arbitrage condition, (39) into (38), the system of equations is reduced into:

$$\Gamma(k) = \Omega(k) + \lambda(k, \mu) = 1 \quad (40)$$

which reflects the total assets to deposits ratio. Specifically,  $\Omega(k) = \frac{k}{w(k)} = \frac{k^{1-\alpha}}{(1-\alpha)A}$  is the capital to deposits ratio and  $\lambda(k, \mu) = \frac{m+b}{w} = \frac{\pi e^{\frac{u}{I}}}{\alpha^{2(1-\pi)} A^{2-\pi(1-\alpha)}} \frac{(1+\mu)^{1-\pi} k^{1-2\alpha}}{(k^{1-\alpha} - \frac{\mu}{1+\mu} \alpha^2 A)^\pi}$ , is the fraction of deposits allocated towards government liabilities. To begin, the behavior of  $\lambda$  is summarized in the following Lemma.

**Lemma 2.** *The locus defined by  $\lambda(k, \mu)$  satisfies:*

i. *If  $\alpha \geq \frac{1}{2}$ ,  $\frac{d\lambda}{dk} < 0$ ,  $\lim_{k \rightarrow \check{k}} \lambda \rightarrow +\infty$ , and  $\lim_{k \rightarrow \infty} \lambda \rightarrow 0$ , with  $\check{k} = \left( \frac{\alpha^2 A \mu}{(1+\mu)} \right)^{\frac{1}{1-\alpha}}$ .*

ii. *If  $\alpha < \frac{1}{2}$ ,  $\frac{d\lambda}{dk} \leq (>) 0$  if  $k \leq (>) \check{k}$ , where  $\check{k} = \left( \frac{\mu}{1+\mu} \frac{\alpha^2 A}{1 - \frac{(1-\alpha)\pi}{(1-2\alpha)}} \right)^{\frac{1}{1-\alpha}} > \tilde{k}$ .  
Moreover,  $\lim_{k \rightarrow \check{k}} \lambda \rightarrow +\infty$  and  $\lim_{k \rightarrow \infty} \lambda \rightarrow \infty$ .*

The intuition behind Lemma 3 is as follows. A change in the level of capital formation affects the fraction of deposits the bank allocates toward government liabilities in two ways. First, for a given nominal interest rate, and therefore, stock of government liabilities, total deposits increase under a higher level of capital formation through higher wages, which lowers  $\lambda$ . However, the capital stock and the nominal interest rate are inversely related by the no-arbitrage condition, (29). Therefore a change in the capital stock, has a non-monotonic effect on the total amount of government liabilities held by the bank as discussed in Lemma 2.

More specifically, when wages (or the level of output) are highly sensitive to changes in the capital stock,  $\alpha > \frac{1}{2}$ , the bank unambiguously allocates a smaller fraction of its deposits into government liabilities under a higher level of capital formation (lower  $I$ ).

Next, suppose  $\alpha \leq \frac{1}{2}$ . Under this condition, the impact of a change in the level of investment on  $\lambda$  depends on how the level of government liabilities changes with  $k$ . When the capital stock is low (high nominal interest rates), the bank holds less cash balances and government debt under a higher level of

capital formation (lower  $I$ ).<sup>13</sup> Combined with a higher level of deposits, the bank allocates a smaller fraction of its deposits towards government liabilities when the level of capital formation is low. By comparison, the bank holds more cash reserves under a higher  $k$ , when the nominal interest rate (capital stock) is below (above) some threshold level. Therefore,  $\lambda$  is increasing with  $k$  beyond  $\tilde{k}$ .

We proceed by summarizing the properties of (40) in the following Lemma.

**Lemma 3.** *The locus defined by  $\Gamma(k)$ , behaves as follows:  $\frac{d\Gamma}{dk} \leq (>) 0$  for all  $k \leq (>) \hat{k}$ , where  $\hat{k} > \tilde{k}$  is defined in the appendix. Moreover,  $\lim_{k \rightarrow \hat{k}} \Gamma(k) \rightarrow +\infty$  and  $\lim_{k \rightarrow \infty} \Gamma(k) \rightarrow +\infty$ .*

The behavior of total assets to deposits ratio, follows directly from our analysis of the ratio of government liabilities to deposits, in Lemma 2. The primary difference underlies in the fact that the non-monotonicity of  $\Gamma$  is independent of the capital share of total output. A graphical representation of  $\Gamma(k)$  is provided in Figure 2 below. We discuss existence and uniqueness of equilibria in the following Proposition:

**Proposition 2.** *Suppose  $\underline{u} < \underline{u}_0$  and  $\pi > \pi_0$ . Under these conditions:*

- a. A steady-state with a low level of capital formation exists and is unique if  $\mu < \tilde{\mu}$  and  $A < A_0$ .*
- b. A steady-state with a high level of capital formation exists and is unique if  $\mu > \tilde{\mu}$  and  $A > A_0$ .*
- c. Two steady-state equilibria exist if  $\mu > \tilde{\mu}$ ,  $A < \min(A_0, A_1)$ , and  $\underline{u} > \underline{u}_1$ .*

The existence of steady-state equilibria with active money, capital, and bond markets, requires a number of conditions to be satisfied in equilibrium. First, the system defined by (40) should have at least one solution,  $k^*$ . Second, the incentive compatibility constraint must be satisfied, where  $r^n \geq r^m$ . Third, money must be dominated in rate of return by bonds and private capital. Finally, the bank must make non-negative profits.

From the expression for the relative return to depositors, (33), the contract between the bank and its depositors is incentive compatible when money is dominated in rate of return that is:

$$I \geq 1 \tag{41}$$

Upon using the no-arbitrage condition, (39), money is dominated in rate of return if:  $k^* < (\alpha^2 A)^{\frac{1}{1-\alpha}} = \bar{k}$ .

Furthermore, using the equilibrium conditions derived above into the bank's profit function, (26), the bank's profits are:

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<sup>13</sup>By definition of the policy tool,  $b$  and  $m$  move in the same direction for a given  $\mu$ .

$$\Pi = \left\{ [\pi(1-\alpha)(1+\mu) + \alpha] k^{1-\alpha} - (1-\pi(1+\mu)) \alpha A(1-\alpha) \right\} \frac{\alpha A k^{2\alpha-1}}{\pi(1+\mu)} \quad (42)$$

It is clear that the term in curly brackets is increasing in  $k$ . Therefore, profits

are positive if  $k^* > \underline{k} = \left( \frac{A(1-\alpha)}{\frac{1+\mu}{\left(\frac{1-\pi(1+\mu)}{\pi}\right)^\alpha + 1}} \right)^{\frac{1}{1-\alpha}}$ .

From the characterization of  $\Gamma$ , the locus defined by (40), has at least two solutions if the inflection point lies below the one line. That is,  $\Gamma(\hat{k}) < 1$ . Since  $\Gamma(k)$  shifts upwards under a higher  $\underline{u}$ , there exists a  $\underline{u}_0$ , where  $\Gamma(\hat{k}) < 1$  for all  $\underline{u} < \underline{u}_0$ . Intuitively, as the bank is extracting all surplus from the deposit market, the bank makes lower payments to its depositors under a lower reservation utility, for a given level of deposits (capital stock). Specifically, profit maximizing requires the bank to hold less cash balances, which reduces the amount of government liabilities and therefore the total assets to deposits ratio by the bank. Therefore,  $\Gamma$  intersects the one line twice if the first condition in Proposition 2 holds as illustrated in Figure 2 below. Denote the two solutions of (40) by  $k_A$  and  $k_B$ , reflecting the capital stock for economies  $A$  and  $B$ , respectively.

The second condition in the Proposition is necessary for the upper bound on capital formation to exceed the lower bound necessary for profits to be positive. That is,  $\underline{k} < \bar{k}$ . Clearly, the capital stock in economy  $A$  is much lower compared to that in economy  $B$ . From (39), this also implies that the nominal interest rate and inflation are much higher in the economy with low levels of capital formation. When the first two conditions in Proposition 2 hold, three possible cases emerge, given that a trade-off between the bank's profitability and the amount of insurance provided exists. We illustrate each case in in Figures 2-4 below.

First, under Case a, we have  $\underline{k} < k^A < \bar{k} < k^B$ , and economy  $A$  only exists and is unique. When the level of capital stock is low, the amount of profits the bank is generating is decreasing with the level of total factor productivity. As we demonstrate in the appendix, the nominal interest rate and inflation are increasing with the level of technology in the low-capital economy. By (32), this implies that the bank is also making higher payments to non-movers, and therefore earning lower profits.<sup>14</sup> Given that inflation is significant in the low capital economy, this effect dominates the gains the bank generates from a higher return to capital that comes about under a higher level of productivity.

<sup>14</sup>Equivalently, by Lemma 2, the bank must hold more cash reserves under higher nominal interest rates to insure its depositors against liquidity risk, when the initial interest rate (capital stock) is high (low). By the definition of the policy tool, the level of cash and bonds holdings move in the same direction for a given stance of monetary policy. Therefore, the total amount of government liabilities is higher in the low capital economy under a higher level of technological change. This in turn translates into a lower level of investment and profits. This necessarily holds for all  $\alpha \leq .5$ .

In this manner, when the level of total factor productivity is sufficiently low, the bank is making positive profits in both economies  $A$  and  $B$ . However, the bank is providing too much insurance in the high capital economy when the ratio of government liabilities is below some threshold level. That is,  $I_B < 1$  when  $\mu < \tilde{\mu}$ . Consequently, economy  $B$  is not feasible.

By comparison, money is dominated in rate of return in both economies when the ratio of government liabilities is above  $\tilde{\mu}$ . However, the bank is making negative profits in economy  $A$  given that the level of productivity is high enough. In this manner, under the conditions in Case  $b$ , we have  $k^A < \underline{k} < k^B < \bar{k}$ , and economy  $B$  only exists and is unique.

Finally, when the ratio of government liabilities is significant and the level of productivity is low enough, as under Case  $c$ , both solutions are feasible. Under the conditions provided in Case  $c$ , we have  $\underline{k} < k^A < k^B < \bar{k}$ , and therefore, both economies  $A$  and  $B$  exist.

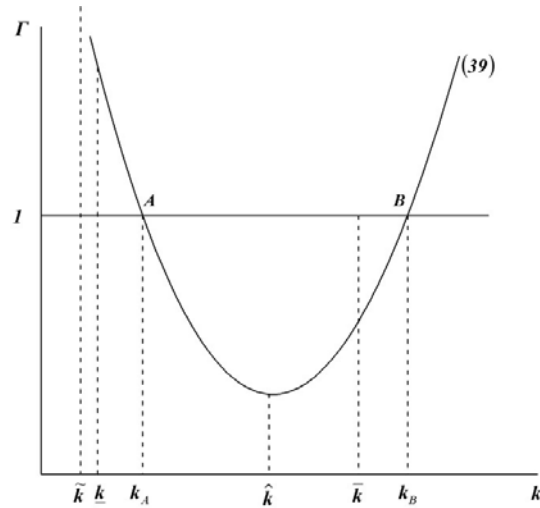


Figure 2. Unique Steady-State Under Case a

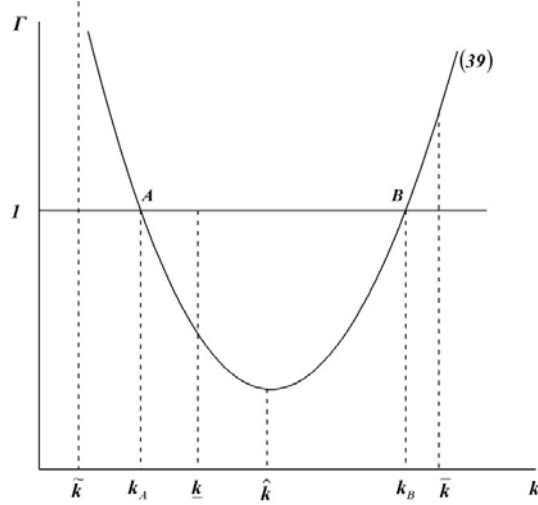


Figure 3. Unique Steady-State Under Case b

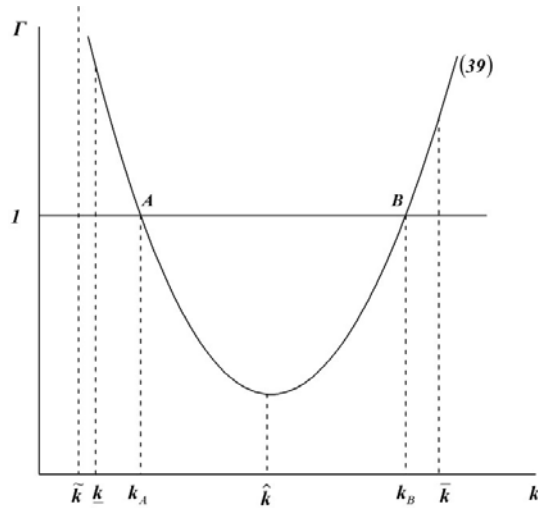


Figure 4. Multiple Steady-States Under Case c

We proceed to study the implications of banking competition for capital formation, risk sharing, and inflation. In order to make the analysis more tractable, we primarily focus on the case where the steady-state under imperfect competition is unique. More specifically, we compare the outcome under perfect competition to that where the monopolist generates the maximum amount of investment (economy  $B$ ), as under Case  $b$  in Proposition 2. Let the variables under perfect competition and imperfect competition be indexed by the superscripts

$PC$  and  $IC$ , respectively. Moreover, suppose the conditions in Proposition 1 and Proposition 2, Case  $b$ , hold.

**Proposition 3.**

*i. Suppose  $A > A_2$ . Under this condition,  $k^{PC} < k^{IC}$ ,  $I^{PC} > I^{IC}$ , and  $\left(\frac{P_{t+1}}{P_t}\right)^{PC} > \left(\frac{P_{t+1}}{P_t}\right)^{IC}$ .*

*ii. Suppose  $A < A_2$  and  $\underline{u} < \underline{u}_2$ . Under this condition,  $k^{PC} > k^{IC}$ . Moreover,  $I^{PC} > I^{IC}$  and  $\left(\frac{P_{t+1}}{P_t}\right)^{PC} > \left(\frac{P_{t+1}}{P_t}\right)^{IC}$ .*

Interestingly, Proposition 3 indicates that nominal interest rates and inflation are lower under a monopolistic banking system. However, the impact of banking structure on capital formation depends on the level of productivity in the economy.

Intuitively, a change in the level of total factor productivity has two effects on portfolio allocation when the banking sector is fully concentrated. First, a higher level of productivity raises the marginal return from capital relative to other assets, which promotes capital formation. Additionally, the level of deposits (wages) is higher, which increases the bank's ability to invest in different assets. By comparison, the second effect is only operative under a perfectly competitive banking sector. This takes place because the level of productivity has no impact on the real return to capital when the banking sector is competitive. In this manner, banking competition promotes capital formation when the level of productivity is below some threshold level.

Furthermore, for a given stock of capital, the marginal return to capital is lower under a monopoly banking system. No-arbitrage between physical capital and government bonds also implies that the real cost of government debt is lower under a fully concentrated banking system. More importantly, the bank is holding a larger amount of government liabilities for a given stance of monetary policy. The higher monetary base and a lower real cost of debt reduce the need for the inflation tax and therefore put a downward pressure on inflation and nominal interest rates.<sup>15</sup> Moreover, because the bank has market power in the market for deposits, it has an incentive to make lower payments to its depositors in the event they do not relocate. This can only be achieved by compensating depositors in the bad state (if they relocate). Therefore, although depositors receive a lower welfare under a monopoly bank, they are better insured against liquidity risk in comparison to a competitive banking sector.

We proceed to examine how the effects of open market operations depend on the competitive structure of the banking system in the following Proposition.

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<sup>15</sup>As we demonstrate in the appendix, the real cost of government debt is much lower in a fully concentrated banking sector compared to a competitive banking economy. This result holds for any level of capital formation where an equilibrium in a fully concentrated banking sector exists. This result is driven by the fact that the bank takes the actions of the government in the bonds market as given. In upcoming work, we examine the consequences of relaxing this assumption.

**Proposition 4.** *Suppose two steady-states exist in an economy with a fully concentrated banking sector. In the economy with a high level of capital formation, a tight monetary policy adversely affects investment and output. By comparison, a tight monetary policy promotes capital investment and output in the economy with a low level of capital.*

The result in Proposition 4 indicates that the competitive structure of the banking system bears significant consequences for monetary policy. However, the effects of monetary policy also depend on other distortions in financial markets such as the amount of government debt and the level of inflation in the economy, which in turn affect the incentive of the bank to respond in a particular manner to a change in the degree of liquidity in the economy.

Unlike a competitive banking system, monetary policy affects the economy through *three* primary channels. First, a tight monetary policy (higher  $\mu$ ) raises the amount of debt in the bank's portfolio, which reduces its ability to invest in capital goods. Secondly, a higher debt to reserves ratio raises the government's obligations and the need to inflate. The bank responds to the higher inflation tax by holding more cash reserves in order to compensate relocated agents for the loss in purchasing power. Thus, fewer resources are allocated towards capital investment. Finally, a tight monetary policy raises the nominal return from capital for a given nominal interest rate on bonds due to higher inflation. The higher relative return to capital stimulates investment until both government bonds and private capital yield the same rate of return at the margin.

When the level of capital formation is high, as in an economy like *B* in Figure 4 above, the bank is holding a small amount of government liabilities in its portfolio. Moreover, the nominal interest rate and inflation are low. Therefore, the bank is providing a lot of insurance against relocation shocks. More importantly, the marginal effect of inflation on the real return to government bonds is small. Consequently, the first two effects dominate, and a tight monetary policy reduces capital investment and raises nominal interest rates.

By comparison, the bank is holding a lot of cash balances and government bonds in the low-capital economy. Furthermore, the inflation tax is significant when the nominal interest rate is high. Therefore, a change in the ratio of government liabilities has a substantial impact on the return to capital. Hence, the amount of capital must also adjust significantly to satisfy the no-arbitrage condition, (39). In this manner, the indirect impact of a higher  $\mu$  through prices dominates the direct effect on the bank's balance sheet in the low-capital economy. As a result, capital investment increases with the ratio of government liabilities and nominal interest rates fall.<sup>16</sup> In sum, in response to a tight monetary, the bank conserves significantly on cash holding, which frees up resources in its portfolio to increase its capital investment.<sup>17</sup>

<sup>16</sup>In a two-sector production economy, Ghossoub (2012) demonstrates that a change in the rate of money creation can also have asymmetric effects under an imperfectly competitive banking system.

<sup>17</sup>It is important to note that the non-monotonicity in the effects of monetary policy can still hold when the reservation utility varies with the policy tool. For example, suppose agents

## 4.2 Dynamical Equilibria

In section 4.1, we demonstrate that two steady states may arise under a fully concentrated banking sector. We proceed to study the implications of banking competition for the stability of steady-state equilibria and the number of dynamical equilibria. Due to the high degree of non-linearity under a monopolistic banking system, we focus our attention on the local stability properties of the system in the neighborhood of the steady-states.

### 4.2.1 Local Dynamics

The dynamic behavior of the economy is summarized by the system of equations, (34) - (36). Using the no-arbitrage condition, (36), and the evolution of capital, (34), nominal interest rates evolve according to:

$$I_{t+1} = \left( \frac{\alpha^{2\frac{1}{1-\alpha}} A^{\frac{1}{1-\alpha}}}{(1+\mu)\pi e^{\frac{u}{\alpha}}} \right)^{\frac{1-\alpha}{\alpha+\pi(1-\alpha)}} \frac{1}{\rho_{t+1}^{\frac{\alpha}{\alpha+\pi(1-\alpha)}}} \left( \frac{(1-\alpha)\rho_{t+1}^{\frac{1}{1-\alpha}} I_{t+1}^{\frac{1}{1-\alpha}}}{\rho_t^{\frac{1}{1-\alpha}} I_t^{\frac{1}{1-\alpha}}} - 1 \right)^{\frac{1-\alpha}{\alpha+\pi(1-\alpha)}} \quad (43)$$

Moreover, using the implicit function theorem, the evolution of the real return to money, (35) can be expressed as:

$$\rho_{t+1}(I_t, \rho_t) = \frac{(1+\mu)}{(1+I_t\mu)} \frac{I_t^{1-\pi}}{(I_{t+1}(I_t, \rho_t))^{1-\pi}}$$

The stability properties of a steady state are generated from the eigenvalues of the Jacobian matrix:

$$J = \begin{bmatrix} \frac{\partial I_{t+1}}{\partial I_t} & \frac{\partial I_{t+1}}{\partial \rho_t} \\ \frac{\partial \rho_{t+1}}{\partial I_t} & \frac{\partial \rho_{t+1}}{\partial \rho_t} \end{bmatrix}_{ss}$$

Denote the determinant and trace of  $J$  by  $D$  and  $T$  respectively. The discriminant,  $\Delta$ , is  $\Delta = T^2 - 4D$ . The elements of the Jacobian are given by:

$$\frac{\partial I_{t+1}}{\partial I_t} \Big|_{ss} = \frac{\left[ \rho \frac{\mu I}{(1+\mu)} - (1-\pi) \right] \left( \frac{1}{1-\alpha} \frac{(1+\mu)\pi e^{\frac{u}{\alpha}}}{\alpha^{\frac{3\alpha-1}{1-\alpha}} A^{\frac{1}{1-\alpha}}} I^{\frac{\alpha+\pi(1-\alpha)}{1-\alpha}} \rho^{\frac{\alpha}{1-\alpha}} - \rho \right) - \alpha \rho}{\left[ \frac{[\alpha+\pi(1-\alpha)]}{I} \right] - \alpha(1-\pi)} \frac{1}{1-\alpha} \frac{(1+\mu)\pi e^{\frac{u}{\alpha}}}{\alpha^{\frac{3\alpha-1}{1-\alpha}} A^{\frac{1}{1-\alpha}}} I^{\frac{\alpha+\pi(1-\alpha)}{1-\alpha}} \rho^{\frac{\alpha}{1-\alpha}} + \pi \rho \quad (44)$$

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receive a lower utility in absence of banks under a higher ratio of government liabilities. This causes the locus defined by (40), to shift downwards, which can partially offset the direct effects of a tight monetary policy on the bank's portfolio choice. Whether the direct of indirect effects dominates, the effects of monetary policy can still vary across steady-states.



$$\frac{\partial I_{t+1}}{\partial \rho_t} \Big|_{SS} = \frac{-\alpha I}{\pi \left\{ \frac{(1+\mu)\pi e^{\underline{u}}}{(1-\alpha)\alpha^2 I^{\frac{1-\alpha}{\alpha}} A^{\frac{1}{1-\alpha}}} \rho^{\frac{\alpha}{1-\alpha}} I^{\frac{\alpha-(1-\pi)(1-\alpha)}{1-\alpha}} - \rho \right\}} \quad (45)$$

$$\frac{\partial \rho_{t+1}}{\partial I_t} \Big|_{SS} = \rho \left( -\rho \frac{\mu}{(1+\mu)} + \frac{(1-\pi)}{I} \left( 1 - \frac{\partial I_{t+1}}{\partial I_t} \Big|_{SS} \right) \right) \quad (46)$$

$$\frac{\partial \rho_{t+1}}{\partial \rho_t} \Big|_{SS} = -\frac{(1-\pi)(1+\mu)}{(1+I\mu)} \frac{1}{I} \frac{\partial I_{t+1}}{\partial \rho_t} \Big|_{SS} \quad (47)$$

Moreover, the eigenvalues of  $J$  may be obtained by solving the following equation:

$$p(\lambda) = |J - \lambda I| = 0$$

$$|J - \lambda I| = \begin{vmatrix} \frac{\partial I_{t+1}}{\partial I_t} \Big|_{SS} - \lambda & \frac{\partial I_{t+1}}{\partial \rho_t} \Big|_{SS} \\ \frac{\partial \rho_{t+1}}{\partial I_t} \Big|_{SS} & \frac{\partial \rho_{t+1}}{\partial \rho_t} \Big|_{SS} - \lambda \end{vmatrix}$$

Numerical results indicate that when two steady-states exist, they are both approachable. More specifically, the indeterminacy of equilibria present under a perfectly competitive banking system may prevail under a monopoly bank. That is, when two steady-states exist, they can both be saddle-path stable. Therefore, lack of competition in the banking sector does not necessarily imply more instability when multiple steady-state equilibria exist.

The following example illustrates this point. Suppose  $A = 3.5$ ,  $\alpha = .35$ ,  $\pi = .482$ ,  $\underline{u} = .5505$ , and  $\mu = .4$ . Under these parameters, the capital stock in the low-capital economy,  $k_A$ , and the corresponding eigenvalues of the Jacobian matrix, are respectively,  $k_A = .245$ ,  $\lambda_1 = .324$ , and  $\lambda_2 = -2.696$ . Moreover,  $k_B = .271$ ,  $\lambda_1 = .349$ , and  $\lambda_2 = -2.558$ .<sup>18</sup>

Interestingly, when the economy with a low-level of investment exists and is unique, the economy displays Hopf bifurcation. More importantly, monetary policy bears significant consequences for the number of dynamical equilibria (determinacy) and the stability of this stationary equilibrium. For instance, when the ratio of government liabilities is sufficiently low, the steady-state is a saddle. As the ratio exceeds some threshold level,  $\mu_3$ , the steady-state becomes indeterminate (sink).

Additionally, when  $\mu \in (\mu_3, \mu_4)$ , the eigenvalues of the Jacobian matrix become complex conjugates. Therefore the economy exhibits endogenous volatility as cyclical trajectories emerge. However, since the determinant of the Jacobian is less than one, the steady-state displays damped oscillatory behavior (sink). Finally, Hopf bifurcation occurs for  $\mu > \mu_4$ , as the determinant exceeds unity and the steady-state becomes a source (determinate). The following set of examples in Table 1 below illustrates this result. Suppose the parameters of the economy are such that:  $A = 6$ ,  $\alpha = .335$ ,  $\pi = .9$ ,  $\underline{u} = .52$ . Under these

<sup>18</sup>Although we are able to provide examples under which the high capital steady-state is a sink, the parameter values for  $\alpha$ , exceed .5, which may not be empirically plausible. Therefore, we drop that observation.

parameters, an economy like  $A$  in Figure 2 exists and is unique.

$\mu$	0.020	0.030	0.040	0.092
$k_A$	0.051	0.058	0.066	0.106
$\lambda_1$	0.031	-0.404	<i>complex</i>	<i>complex</i>
$\lambda_2$	-1.259	-0.595	<i>complex</i>	<i>complex</i>
$D$	-0.039	0.240	0.400	1.000
$T$	-1.228	-0.999	-0.869	-0.846
$\Delta$	1.665	0.036	-0.846	-3.285

Table 1: Bifurcation in the Low-Capital Economy

The examples above indicates that imperfect competition can be a source of endogenous fluctuations and bifurcation.<sup>19</sup> Furthermore, the equilibrium indeterminacy that could arise in an economy with a concentrated banking sector can be avoided if the central bank injects significant amounts of liquidity into the banking sector.

## 5 Conclusion

Fiscal problems in a number of countries that belong to the European monetary union, such as Greece, triggered significant liquidity injections by the European central bank into the banking sector throughout Europe. Moreover, members of the Euro area are considering fiscal integration to create more stability and harmony in the effects of monetary policy. While it may be a step in the right direction, financial markets in Europe are still not well integrated. For example, there are significant differences in the degree of banking concentration across members of the Euro area, which could affect the degree of banking competition and have implications for monetary policy.

This manuscript studies how the effects of liquidity injections or contractions by the central bank depend on the industrial organization of the banking sector. In a setting where fiat money and financial intermediaries play important economic functions, we demonstrate that the degree of banking competition bears significant consequences for the number of dynamical and steady-state equilibria. More importantly, we show that the effects of monetary policy become non-monotonic when the banking sector is not competitive. That is, output increases under higher degrees of liquidity when the banking sector is perfectly competitive. However, liquidity injections through an open market purchase can

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<sup>19</sup>It is important to note that the number of equilibria in an economy with a perfectly competitive banking sector can become indeterminate when the coefficient of risk aversion is different from unity. However, endogenous volatility and bifurcation cannot occur when the production function is of the Cobb-Douglas as it is in this manuscript. Schreft and Smith (1998) highlight this point in an economy where banks are perfectly competitive and the coefficient of risk aversion is above unity. A formal proof for the case where  $\theta < 1$  can be furnished upon request.

be contractionary when the banking sector is fully concentrated. As we demonstrate in the text, such unfavorable outcome occurs in economies where government liabilities constitute a large fraction of deposits in the banking system and inflation is high enough. Finally, we demonstrate that sufficient liquidity injections by the monetary authority can shield the economy from deterministic cycles and endogenous volatility that could arise under an imperfectly competitive banking sector. The results in this manuscript call for more financial integration among members of the European monetary union to promote more symmetry in the effects of monetary policy.

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## 6 Technical Appendix

**1. Proof of Lemma 2.** By definition of  $\lambda$ , we have:

$$\lambda(k, \mu) = \frac{\pi e^{\frac{\mu}{k}}}{\alpha^{2(1-\pi)} A^{2-\pi} (1-\alpha)} \frac{(1+\mu)^{1-\pi} k^{1-2\alpha}}{\left(k^{1-\alpha} - \frac{\mu}{1+\mu} \alpha^2 A\right)^\pi} \quad (48)$$

Next, differentiate (48) with respect to  $k$  to obtain:

$$\frac{d\lambda}{dk} = \frac{\pi e^{\frac{\mu}{k}} (1+\mu)^{1-\pi}}{\alpha^{2(1-\pi)} A^{2-\pi} (1-\alpha)} \left[ \frac{(1-2\alpha) k^{-2\alpha}}{\left(k^{1-\alpha} - \frac{\mu}{1+\mu} \alpha^2 A\right)^\pi} - \frac{\pi (1-\alpha) k^{1-3\alpha}}{\left(k^{1-\alpha} - \frac{\mu}{1+\mu} \alpha^2 A\right)^{1+\pi}} \right]$$

It is easy to verify that  $\frac{d\lambda}{dk} \geq 0$  if:

$$k \geq \left( \frac{\mu}{1+\mu} \frac{\alpha^2 A}{1 - \frac{(1-\alpha)\pi}{(1-2\alpha)}} \right)^{\frac{1}{1-\alpha}} \equiv \check{k}$$

Positive money holding requires that  $\lambda > 0$ . This condition is satisfied if  $k > \tilde{k} = \left( \frac{\alpha^2 A \mu}{(1+\mu)} \right)^{\frac{1}{1-\alpha}}$ .

With some simple algebra,  $\check{k} > \tilde{k}$  if  $-\frac{(1-\alpha)\pi}{(1-2\alpha)} < 0$ . Therefore, if  $\alpha \geq \frac{1}{2}$ ,  $\check{k} < \tilde{k}$ , and  $\frac{d\lambda}{dk} < 0$  for all  $k > \tilde{k}$ . By comparison,  $\check{k} > \tilde{k}$ , if  $\alpha < \frac{1}{2}$ . Therefore,  $\frac{d\lambda}{dk} < 0$  for all  $k \in (\tilde{k}, \check{k})$  and  $\frac{d\lambda}{dk} \geq 0$  for all  $k \geq \check{k}$ . Using l'Hôpital's rule, it can be verified that  $\lim_{k \rightarrow \infty} \lambda \rightarrow \infty$ . This completes the proof of Lemma 2.

**2. Proof of Lemma 3.** Differentiating  $\Gamma(k)$  from (40) with respect to  $k$  yields:

$$\Gamma'(k) = \frac{k^{-\alpha}}{A} + \frac{\pi e^{\frac{\mu}{k}} (1+\mu)^{1-\pi}}{\alpha^{2(1-\pi)} A^{2-\pi} (1-\alpha)} \left[ \frac{(1-2\alpha) k^{-2\alpha}}{\left(k^{1-\alpha} - \frac{\mu}{1+\mu} \alpha^2 A\right)^\pi} - \frac{\pi (1-\alpha) k^{1-3\alpha}}{\left(k^{1-\alpha} - \frac{\mu}{1+\mu} \alpha^2 A\right)^{1+\pi}} \right]$$

With some simplifying algebra,  $\Gamma'(k) \geq 0$  if:

$$\frac{\alpha^{2(1-\pi)} A^{1-\pi} \left(k^{1-\alpha} - \frac{\mu}{1+\mu} \alpha^2 A\right)^\pi k^\alpha}{\pi e^{\frac{\mu}{k}} (1+\mu)^{1-\pi}} + \frac{(1-2\alpha)}{(1-\alpha)} - \frac{\pi}{\left(1 - \frac{\mu}{1+\mu} \frac{\alpha^2 A}{k^{1-\alpha}}\right)} \geq 0 \quad (49)$$

Denote the term on the left-hand-side of (49) by  $\chi(k)$ . It is clear that  $\frac{d\chi}{dk} > 0$ ,  $\lim_{k \rightarrow \tilde{k}} \chi(k) \rightarrow -\infty$  and  $\lim_{k \rightarrow \infty} \chi(k) \rightarrow +\infty$ . In this manner, there exists a value

of the capital stock, denoted by  $\hat{k}$ , with  $\hat{k} > \tilde{k}$ , beyond which,  $\chi(k) \geq 0$  or equivalently,  $\Gamma'(k) \geq 0$ . For all  $k \leq (>) \hat{k}$ ,  $\chi(k) \leq (>) 0$  and therefore,  $\Gamma'(k) \leq (>) 0$ . This result holds for all  $\alpha \in (0, 1)$ . The remaining characteristics of  $\Gamma$  are trivial. This completes the proof of Lemma 3.

**3. Proof of Proposition 2.** As discussed in the text, several conditions must be satisfied in equilibrium. To begin,  $\Gamma$  intersects the one line twice if  $\Gamma(\hat{k}) < 1$ . Using the definition of  $\hat{k}$ , where (49) holds with equality, we have:

$$\left( \hat{k}^{1-\alpha} - \frac{\mu}{1+\mu} \alpha^2 A \right)^\pi = \frac{\pi e^{\underline{u}} (1+\mu)^{1-\pi}}{\alpha^{2(1-\pi)} A^{1-\pi}} \left( \frac{\pi}{\left(1 - \frac{\mu}{1+\mu} \frac{\alpha^2 A}{\hat{k}^{1-\alpha}}\right)} - \frac{(1-2\alpha)}{(1-\alpha)} \right) \frac{1}{\hat{k}^\alpha}$$

substituting into the expression for  $\Gamma$  and simplifying, we get:

$$\Gamma(\hat{k}) = \left[ \frac{\pi(1-\alpha) + \left(1 - \frac{\mu}{1+\mu} \frac{\alpha^2 A}{\hat{k}^{1-\alpha}}\right) \alpha}{\pi(1-\alpha) - (1-2\alpha) \left(1 - \frac{\mu}{1+\mu} \frac{\alpha^2 A}{\hat{k}^{1-\alpha}}\right)} \right] \frac{\hat{k}^{1-\alpha}}{(1-\alpha)A}$$

Therefore,  $\Gamma(\hat{k}) \leq 1$  if:

$$\left[ \frac{\pi(1-\alpha) + \left(1 - \frac{\mu}{1+\mu} \frac{\alpha^2 A}{\hat{k}^{1-\alpha}}\right) \alpha}{\pi(1-\alpha) - (1-2\alpha) \left(1 - \frac{\mu}{1+\mu} \frac{\alpha^2 A}{\hat{k}^{1-\alpha}}\right)} \right] \frac{\hat{k}^{1-\alpha}}{(1-\alpha)A} \leq 1 \quad (50)$$

Clearly, the term on the left-hand-side of (50) is increasing in  $\hat{k}$ . Moreover, by definition of  $\hat{k}$ , we have  $\frac{d\hat{k}}{d\underline{u}} > 0$ . Therefore, there exists a value of  $\underline{u}$ , denoted by  $\underline{u}_0$ , below which  $\Gamma(\hat{k}) < 1$ . In this manner, (40) has two solutions when  $\underline{u} < \underline{u}_0$ . The second condition in the proposition is derived in the text, guarantees that  $\underline{k} < \bar{k}$ .

We proceed to prove Case *a* in the Proposition. As indicated in the text, an economy like *A*, in Figure 2 exists and is unique if  $\underline{k} < k^A < \bar{k} < k^B$ . It is clear that  $k^A < \bar{k} < k^B$  if  $\Gamma(\bar{k}) < 1$ . Using the expression for  $\bar{k}$  derived in the text into  $\Gamma$ , and simplifying,  $\Gamma(\bar{k}) < 1$  if  $\mu < \frac{(1-\alpha-\alpha^2)A^{\frac{1}{1-\alpha}}}{\pi e^{\underline{u}} \alpha^{\frac{2\alpha}{1-\alpha}}} - 1 = \tilde{\mu}$ . Furthermore,  $\underline{k} < k^A$  if  $\Gamma(\underline{k}) > 1$  and  $\underline{k} < \hat{k}$ . However, since  $\underline{k} < \bar{k}$ ,  $\Gamma(\underline{k}) > 1$  and  $\Gamma(\bar{k}) < 1$  imply that  $\underline{k} < \hat{k}$ . Using the expression for  $\underline{k}$  into  $\Gamma$ ,  $\Gamma(\underline{k}) > 1$

$$\text{if } A < \left[ \frac{\pi e^{\underline{u}} \left(\frac{1-\pi}{\pi} - \mu\right)}{\alpha^{1-2\pi} \left(\frac{(1-\alpha)(1+\mu)}{\frac{1+\mu}{\left(\frac{1-\pi}{\pi} - \mu\right)^\alpha} + 1} - \mu\alpha^2\right)^\pi} \right]^{(1-\alpha)} \left( \frac{\left(\frac{1+\mu}{\left(\frac{1-\pi}{\pi} - \mu\right)^\alpha} + 1\right)}{(1-\alpha)} \right)^\alpha = A_0. \text{ Under these}$$

condition, money is dominated in rate of return and profits are positive in economy *A*. While profits are positive in economy *B*, money is not dominated

in rate of return. Therefore, economy  $A$  exists and is unique under the conditions in Case  $a$ .

Subsequently, we examine Case  $b$  in the Proposition. An economy like  $B$ , in Figure 2 exists and is unique if  $k^A < \underline{k} < k^B < \bar{k}$ . Following our proof for Case  $a$ ,  $k^A < \underline{k} < k^B$  if  $\Gamma(\underline{k}) < 1$ . This takes place if  $A > A_0$ . Moreover,  $k^B < \bar{k}$  if  $\Gamma(\bar{k}) > 1$ , and therefore when  $\mu > \tilde{\mu}$ . Under these conditions, economy  $B$  exists and is unique as profits are negative in economy  $A$ .

Finally, two steady-states exist as under Case  $c$ , in the Proposition, if both  $k^A$  and  $k^B$  fall in the feasible range. That is, we need  $\underline{k} < k^A < k^B < \bar{k}$ . This requires that both  $\Gamma(\bar{k})$  and  $\Gamma(\underline{k})$  to exceed unity. From the work above,  $\underline{k} < k^A$  if  $A < A_0$  and  $k^B < \bar{k}$  if  $\mu > \tilde{\mu}$ . Since existence of two equilibria under Case  $c$  requires that both  $\Gamma(\bar{k}) > 1$  and  $\Gamma(\underline{k}) > 1$ , we need to make sure that  $\underline{k} < k^A$  and  $k^B < \bar{k}$ . Equivalently, we need to find conditions under which  $\Gamma'(\bar{k}) > 0$  and  $\Gamma'(\underline{k}) < 0$ .

Using the expression for  $\bar{k}$ , where  $\bar{k} = (\alpha^2 A)^{\frac{1}{1-\alpha}}$  and from our proof of Lemma 3,  $\Gamma'(\bar{k}) > 0$  if:

$$A > \left( \frac{\pi e^{\underline{u}} (1 + \mu)}{\alpha^{\frac{2}{1-\alpha}}} \left[ \pi (1 + \mu) - \frac{(1 - 2\alpha)}{(1 - \alpha)} \right] \right)^{1-\alpha} = A_1$$

Analogously,  $\Gamma'(\underline{k}) < 0$ , where  $\underline{k} = \left( \frac{A(1-\alpha)}{\left(\frac{1+\mu}{\left(\frac{1-\pi}{\pi}-\mu\right)\alpha}+1\right)} \right)^{\frac{1}{1-\alpha}}$ , if:

$$\underline{u} > \ln \frac{\alpha^{2-\pi} A^{\frac{1}{1-\alpha}} \left( \frac{(1-\alpha)[1-\pi(1+\mu)]}{\alpha+\pi(1-\alpha)(1+\mu)} - \frac{\mu\alpha}{1+\mu} \right)^\pi \left( \frac{(1-\alpha)[1-\pi(1+\mu)]\alpha}{(1-\alpha)(1+\mu)\pi+\alpha} \right)^{\frac{\alpha}{1-\alpha}}}{\pi (1 + \mu)^{1-\pi} \left[ \frac{\pi}{1 - \frac{\mu}{1+\mu} \frac{\alpha[(1-\alpha)(1+\mu)\pi+\alpha]}{(1-\alpha)[1-\pi(1+\mu)]}} - \frac{1-2\alpha}{1-\alpha} \right]} = \underline{u}_1$$

As a final step, we show the effects of a change in the level of technological change on nominal interest rates. Using the no-arbitrage condition, (39), into the equilibrium condition, (40), the equilibrium condition is reduced into a one in  $I$  .:

$$\Gamma(I) = \frac{\alpha^2}{(1-\alpha)(1+\mu)} \frac{1+\mu I}{I} + \frac{(1+\mu)^{\frac{1-\alpha-1+2\alpha}{1-\alpha}} \pi e^{\underline{u}}}{(1-\alpha) A^{\frac{1}{1-\alpha}} \alpha^{\frac{2\alpha}{1-\alpha}}} \left[ \frac{1}{I} + \mu \right]^{\frac{1-2\alpha}{1-\alpha}} I^\pi = 1 \quad (51)$$

Differentiating with respect to  $I$  and some simplifying algebra to obtain:

$$\Gamma'(I) = -\frac{\alpha^2}{(1-\alpha)(1+\mu)} \frac{1}{I^2} + \frac{(1+\mu)^{\frac{1-\alpha-1+2\alpha}{1-\alpha}} \pi e^{\underline{u}}}{(1-\alpha) A^{\frac{1}{1-\alpha}} \alpha^{\frac{2\alpha}{1-\alpha}}} \left\{ -\frac{1-2\alpha}{1-\alpha} \frac{1}{1+\mu I} + \pi \right\} \frac{1}{I^{1-\pi}} \left[ \frac{1}{I} + \mu \right]^{\frac{1-2\alpha}{1-\alpha}}$$

It is easy to verify that  $\Gamma'(I) \leq 0$  if:



$$\left\{ \pi - \frac{1-2\alpha}{1-\alpha} \frac{1}{1+\mu I} \right\} [1 + \mu I]^{\frac{1-2\alpha}{1-\alpha}} I^{\pi + \frac{\alpha}{1-\alpha}} \leq \frac{A^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}}}{(1+\mu)^{\frac{1}{1-\alpha}} \pi e^{\frac{\mu}{\alpha}}} \quad (52)$$

Given that the term on the left-hand-side of (52) is strictly increasing in  $I$  for all  $\alpha \leq .5$ , there exists an  $\hat{I} : \Gamma'(I) \leq (>) 0$  for all  $I \leq (>) \hat{I}$ . In this manner,  $\Gamma(I)$  is  $U$  shaped and could intersect the one line twice. Moreover, for a given  $I$ ,  $\Gamma(I)$  shifts downwards under a higher level of total factor productivity, which implies that nominal interest rates decrease (increase) in the low (high) interest economy. This completes the proof of Proposition 2.

**4. Proof of Proposition 3.** We begin by showing that the nominal interest rate and therefore inflation is lower under a monopoly banking sector. From (16) and (36), the nominal return on government bonds under monopoly and perfect competition are respectively:

$$I^{IC} = \frac{1}{\frac{(1+\mu)(k^{IC})^{1-\alpha}}{\alpha^2 A} - \mu}$$

and

$$I^{PC} = \frac{1}{\frac{(1+\mu)(k^{PC})^{1-\alpha}}{\alpha A} - \mu}$$

Some simplifying algebra imply that:  $I^{IC} < I^{PC}$  if  $\alpha^{\frac{1}{1-\alpha}} k^{PC} < k^{IC}$ . Using the expression for  $k^{PC}$  from (19), this condition can be written as:

$$k^{IC} > (\alpha(1-\alpha)A[1 - (1+\mu)\pi])^{\frac{1}{1-\alpha}} \equiv k_1$$

It is sufficient to show that  $k_1 < \underline{k}$ . Using the expression for  $\underline{k}$ ,  $k_1 < \underline{k}$  if  $\alpha < 1$ , which always holds. Therefore, if an equilibrium under a monopoly economy exists, the nominal interest rate is lower compared to that under a perfectly competitive banking sector. Note that this also implies that the real return on government bonds is always lower under a monopoly bank. That is:  $R^{IC} < R^{PC}$ . Using the definition of  $R^{IC}$  and  $R^{PC}$ ,  $R^{IC} < R^{PC}$  if  $\alpha^{\frac{1}{1-\alpha}} k^{PC} < k^{IC}$ , which is identical to the condition derived above.

It remains to compare the level of capital stock corresponding to an economy like  $B$  in Figure 2, to that under perfect competition. Specifically,  $k^{PC} < k_B$  if  $\Gamma(k^{PC}) < 1$ . Upon using the definition of  $k^{PC}$ , (19) into (40),  $\Gamma(k^{PC}) < 1$  if:

$$A > \frac{e^{\frac{\mu}{\alpha}(1-\alpha)} [1 - (1+\mu)\pi]^{1-2\alpha}}{\{(1-\alpha)(1+\mu)[1 - (1+\mu)\pi] - \mu\alpha^2\}^{\pi(1-\alpha)} \alpha^{2(1-\alpha)(1-\pi)} (1-\alpha)^\alpha} \equiv A_2$$

By comparison,  $k^{PC} > k_B$  if  $\Gamma(k^{PC}) > 1$  and  $k^{PC} > \hat{k}$ . From our proof of Lemma 3,  $k^{PC} > \hat{k}$  when  $k^{PC} > k_B$  if  $\Gamma'(k^{PC}) > 0$ . Using the expression for  $\Gamma'$  from Lemma 3 and the definition of  $k^{PC}$ ,  $\Gamma'(k^{PC}) > 0$  if:

$$\underline{u} < \ln \frac{\alpha^{2(1-\pi)} A^{\frac{1}{1-\alpha}} \left( (1-\alpha) [1 - (1+\mu)\pi] - \frac{\mu}{1+\mu} \alpha^2 \right)^\pi ((1-\alpha) [1 - (1+\mu)\pi])^{\frac{\alpha}{1-\alpha}}}{\pi (1+\mu)^{1-\pi} \left( \frac{\pi}{\left(1 - \frac{\mu}{1+\mu} \frac{\alpha^2}{((1-\alpha)[1 - (1+\mu)\pi]}\right)} - \frac{(1-2\alpha)}{(1-\alpha)} \right)} \equiv \underline{u}_2$$

This completes the proof of Proposition 3.