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A Mixed GOLE Model of Cross-Border Mergers and Trade

We construct a tractable open economy general equilibrium model of a mixed oligopoly. Our model is then applied to capture the incentives for and implications of cross-border horizontal mergers and trade in the presence of a public firm. Absent any possibility of cross-border mergers, an increase in the degree of privatization will result in a shrinking of the extensive margins of trade. Cross-border mergers will mitigate, by aligning specialization toward the direction of comparative advantage, the effect of privatization on the extensive margins of trade. Allowing firms to move sequentially will magnify the effect that cross-border mergers have on the extensive margins of trade: the magnification effect will be larger when the private firms lead than it will be if the private firms follow.

JEL Classification Code:	F10, F12, L13			
Keywords:	General Equilibrium, International Trade, Multinational			
	Corporations,	Cross-border	Merger,	Diversification,
	Comparative Advantage, Privatization, M-GOLE.			

1. Introduction

We construct a tractable Mixed General Oligopolistic Equilibrium (M-GOLE) model to reflect upon the role of privatization in the incentives for cross-border mergers and implications for the extensive margins of international trade. From an analytical perspective, to the best of our knowledge, this is the first general equilibrium model of a mixed oligopoly.

Cross-border mergers have been intensifying significantly as capital reallocation between firms is increasingly facilitated through interactions between financial liberalization policies and regional agreements. This millennium's wave of cross-border mergers was preceded by five successive waves in close proximity — one appearing at the dawn of the 20th century which was followed by four dominant waves at the ends of the 1920s, 1960s, 1980s and 1990s.¹ During the widely celebrated merger boom witnessed during this millennium, the value of worldwide mergers reached an all-time record nearly half of which was due to cross-border mergers². The steady growth of cross-border mergers pulled up the value of cross-border mergers to a record \$1,637 billion in 2007 spanning a total number of 10,145 such transactions.³ There is a growing

consensus⁴ that, notwithstanding the low tides due to the global crises, "a new wave" of cross-border mergers is imminent. As such, it is not at all surprising that mergers across borders continue to draw increasing attention since they not only yield cost-reducing efficiencies but, at the same time, present new opportunities for firms to intensify market power with international reach.⁵ At the same time, recent dramatic financial events have brought about the creation of new sectors where private and public organizations vie to supply the same customers. This has renewed the interest in the interaction among state-owned agents and private suppliers.

A cross-border horizontal merger involves firms producing substitutes in two distinct countries with the consequence that such a merger will remove direct competitive pressures absent other constraining factors or offsetting efficiencies.⁶ Cross-border horizontal mergers present a greater challenge for competition authorities in the absence of complete privatization since the strategic interactions between public and private firms will inevitably affect the intensity of competition as the structure of ownership of a firm inevitably affects its actions and alters the strategic environment.⁷ In today's economy, several industries experience the interplay of private and public agents. As pointed out by De Fraja (2009), on the one hand, the markets for cars, ships or steel manufacturers, or traditional insurers, started off as fully private markets, and some firms became public at a later stage. Unlike many of the public utilities, which were nationalized with a view to prevent monopoly suppliers of essential services from exploiting their monopoly power, and where, typically, the entire industry was taken over by the state sector, firms in these industries were nationalized to stop them from going bankrupt, which could have labor market, and other economic social and political negative consequences, and therefore, following nationalization, operated in the same market as the firms which remained private. On the other hand, recent dramatic financial events have brought about the creation of a totally new and utterly unexpected new sector where private and public organizations vie to supply the same customers: several banks in several OECD countries have been effectively nationalized. As history repeats itself, in view of the significance of such ownership structure, it becomes imperative to explore the effects of the interaction among state-owned agents and private suppliers on the emerging waves of cross-border mergers.

Our key innovation, through a merger of Neary (2007) and Beladi et al. (2010), stems from constructing a tractable general equilibrium model of mixed oligopoly that distinguishes a domestic firm from a foreign firm even in the absence of any friction allowing us to link cross-border mergers, international trade, and privatization. While the literature on cross-border mergers, a survey of which remains outside the scope of this paper, is still at its infancy, Neary (2007) constructed the first analytically tractable general equilibrium model of cross-border mergers where he showed how trade liberalization can trigger international merger waves through bilateral mergers in which it is profitable for low-cost firms to buy out higher-cost foreign rivals. As such, international differences in access to technology can generate incentives for bilateral mergers in which low-cost firms located in one country acquire high-cost firms located in another. In consequence, cross-border mergers facilitate specialization in the direction of a nation's comparative advantage.⁸ Beladi et al. (2010) presented a partial equilibrium model of cross-border horizontal mergers where an increase in the degree of privatization (which, absent any provision for mergers, raises the incentives for diversification of international production) at home magnifies the potential gains from a take-over of a home firm by a foreign firm but dampen the potential gains from a take-over of a foreign firm by a home firm.

The rest of the paper is organized as follows. In the next section, we present our model and propositions. The sensitivity of our construct is discussed in section 3. We draw our conclusions in section 4.

2. Model and Propositions

Consider a stylized world containing two countries each with a continuum of atomistic industries, indexed by $z \in [0, 1]$. Each industry supports a homogeneous good produced by n^* foreign firms competing, `a la Cournot, with n privately owned home firms and one public firm (i.e. owned partially by the home government). Let $\alpha \in (0,1)$, the proportion of privately held shares in the public firm, measure the degree of privatization. All firms in a given location have identical unit cost of production: c for home firms and c^* for foreign firms. We assume away any fixed cost which, otherwise, would provide a trivial rationale for mergers.⁹ The output of the industry is given

by
$$\tilde{y} = \left(\sum_{i=1}^{n} y_i + \sum_{j=1}^{n^*} y_j^* + y_p\right)$$
 where y_i $(i = 1, 2, ..., n)$ is the output of a privately owned

home firm, y_j^* ($j = 1, 2, ..., n^*$) is the output of a foreign firm, and y_p is the output of the public firm.

Following the Dornbusch-Fischer-Samuelson (DFS) exposition of the Ricardian theory, let countries differ in their access to technology reflected in unit labor requirements denoted by $\beta(z)$ and $\beta^*(z)$ with wages w and w^{*} at home and abroad

respectively. For expositional convenience, we assume that $\beta(z)$ is increasing and $\beta^*(z)$ is decreasing in z which can then be interpreted as an index of foreign comparative advantage with home's relative productivity $\left(\frac{\beta^*(z)}{\beta(z)}\right)$ decreasing as z increases.

Let home demand for variety z be x(z) and foreign demand for the same variety be $x^*(z)$. Let preferences be characterized by an additive utility function of the form

(1)
$$U\left[\left\{x^{(-)}(z)\right\}\right] = \int_{0}^{1} \left[ax^{(-)}(z) - \frac{1}{2}bx^{(-)}(z)^{2}\right] dz$$

There is a single representative consumer, in each country, who maximizes (30) subject to the budget constraint

(2)
$$\int_{0}^{1} p(z)x(z)dz \le I$$

Where p(z) is the price of variety z and I is aggregate income. This yields, for each country, an inverse demand function for each good which is linear in its own price conditional on the marginal utility of income (λ)

(3)
$$p(z) = \frac{1}{\lambda} [a - bx(z)]$$

where $\lambda = \frac{a\mu_1^p - bI}{\mu_2^p}$.

The effects of prices on λ are summarized by the first and second moments of the distribution of prices

(4)
$$\mu_1^P \equiv \int_0^1 p(z) dz$$

(5)
$$\mu_2^P \equiv \int_0^1 p(z)^2 dz$$

It follows that, under free trade, the world demand $(\bar{x}(z))$ for each variety z is

$$(6) p(z) = a' - b' \overline{x}(z)$$

where $a' \equiv \frac{\overline{a}}{\overline{\lambda}} = \frac{a+a^*}{\lambda+\lambda^*}$ and $b' \equiv \frac{b}{\overline{\lambda}}$ with a and a^* being the intercepts and b the

common slope for home demand and foreign demand respectively. $\overline{\lambda}$ is the world marginal utility of income which we choose as the *numeraire*. We will, hereinafter, normalize the wages to $W = \overline{\lambda}w$ and $W^* = \overline{\lambda}w^*$. ¹⁰ We will also assume $\min(n, n^*) > \max\left[2, \frac{1}{2}\left(\frac{(a'-c^*) + (a'-c)}{(c^*-c)}\right)\right]$ which, as will be apparent in subsequent

analyses, imposes a *sufficient* condition for removing any incentive for bilateral mergers within a country. This generalizes analogous conditions derived by Salant *et al.* (1983) to the extent that we allow the existence of a public firm.

Wages are determined by the full employment conditions

(7)
$$L = \int_{0}^{\tilde{z}^{*}} \beta(z)n\tilde{y}(W,z,n)\Big|_{C}dz + \int_{\tilde{z}^{*}}^{\tilde{z}} \beta(z)n\tilde{y}(W,W^{*},z,n,n^{*})\Big|_{C}dz$$

(8)
$$L^{*} = \int_{\tilde{z}}^{1} \beta^{*}(z) n^{*} \tilde{y}^{*}(W^{*}, z, n^{*}) \Big|_{C} dz + \int_{\tilde{z}^{*}}^{\tilde{z}} \beta(z) n^{*} \tilde{y}^{*}(W, W^{*}, z, n, n^{*}) \Big|_{C} dz$$

where L and L^* denote the supply of labor and \tilde{z} and \tilde{z}^* are the threshold sectors for the extensive margins of trade, at home and abroad respectively.

Each privately owned home firm will

Maximize:
$$\Pi_i = (a'-b'\,\tilde{y}-c)y_i$$
 $i = 1,2,...,n$

Each foreign firm will

$$Maximize: \quad \Pi_{j}^{*} = (a'-b'\,\tilde{y}-c^{*})y_{j}^{*} \qquad j = 1,2,...,n^{*}$$

The public firm will

$$Maximize: \quad \Pi_{P} = (a'-b'\,\widetilde{y}-c)y_{P} + (1-\alpha)\left(\sum_{i=1}^{n}\Pi_{i} + \frac{b'\,\widetilde{y}^{2}}{2}\right)$$

i.e. the publicly-owned firm's objective function is a weighted average of its own profits and social welfare, where the weight is the degree of privatization. Social welfare, in turn equals the profts of all home frms and the surplus which accrues to all consumers. This reflects an underlying model of bargaining between the public and the private shareholders. The board of this firm consists of the home government's representatives who advocate domestic welfare (consumer and producer surplus) and the representatives of the private shareholders who advocate domestic profit. Since α is the proportion of privately held shares in the public firm and the home government owns the rest: bargaining will involve α percent representatives with a goal of maximizing domestic profits and $(1 - \alpha)$ percent representatives with a goal of maximizing domestic welfare.¹¹ The best-response functions of the $(n + n^* + 1)$ firms can be written as

(9)
$$y_{i} = \frac{1}{2b'} \left(a' - b' \left[\sum_{\substack{k=1\\k \neq i}}^{n} y_{k} + \sum_{j=1}^{n^{*}} y_{j}^{*} + y_{p} \right] - c \right) \qquad \forall \quad i = 1, 2, ..., n$$

(10)
$$y_{j}^{*} = \frac{1}{2b'} \left(a' - b' \left[\sum_{i=1}^{n} y_{i} + \sum_{\substack{k=1\\k \neq j}}^{n^{*}} y_{k}^{*} + y_{p} \right] - c^{*} \right) \qquad \forall \qquad j = 1, 2, ..., n^{*}$$

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(11)
$$y_{p} = \frac{1}{b'(1+\alpha)} \left(a' - b' \left[\sum_{i=1}^{n} y_{i} + \alpha \sum_{j=1}^{n^{*}} y_{j}^{*} \right] - c \right)$$

The firms produce

(12)
$$y_i(n,n^*)\Big|_C = \left(\frac{\alpha a' - (n^* + \alpha)c + n^*c^*}{b'[\alpha(n+1) + n^* + 1]}\right)$$
 $\forall \quad i = 1, 2, ..., n$

(13)
$$y_j^*(n,n^*)\Big|_C = \left(\frac{\alpha a' + [\alpha(n-1)+1]c - [\alpha n+1]c^*}{b'[\alpha(n+1)+n^*+1]}\right) \quad \forall \quad j = 1,2,...,n^*$$

(14)
$$y_P(n,n^*)\Big|_C = \left(\frac{a'((1-\alpha)n^*+1) - [1+n^*-nn^*(1-\alpha)]c + [\alpha(n+1)-n]n^*c^*}{b'[\alpha(n+1)+n^*+1]}\right)$$

It may be noted that as competition stiffens (i.e. if there is an increase in n and/or n^*), the output of a fully owned public firm will rise if domestic firms are more efficient (i.e. $c > c^*$) and decline if foreign firms are more efficient (i.e. $c^* > c$), with $(1+n^*)(a'-c)+nn^*(c-c^*)>0$ imposing a condition sufficient for the public firm to survive competition from the private sector.

The industry output and price are

(15)
$$\widetilde{y}(n,n^*)\Big|_{C} = \left(\frac{(a'-c)(1+\alpha n) + n^*(a'-c^*)}{b'[\alpha(n+1) + n^* + 1]}\right)$$

(16)
$$p(n,n^*)\Big|_{C} = \left(\frac{\alpha a' + (\alpha n + 1)c + n^* c^*}{\alpha (n+1) + n^* + 1}\right)$$

In the pre-merger equilibrium, the profits of the firms are

(17)
$$\Pi_{i}(n,n^{*})|_{C} = b' (y_{i}(n,n^{*})|_{C})^{2} \qquad \forall \quad i = 1,2,...n$$

(18)
$$\Pi_{j}^{*}(n,n^{*})\Big|_{C} = b'\Big(y_{j}^{*}(n,n^{*})\Big|_{C}\Big)^{2} \qquad \forall \qquad j = 1,2,...,n^{*}$$

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(19)
$$\Pi_{P}(n,n^{*})\Big|_{C} = B_{C}\Big(y_{P}(n,n^{*})\Big|_{C}\Big)^{2}$$

where
$$B_{c} = \frac{\left[\alpha(a'-c) + n^{*}(c^{*}-c)\right]}{a'\left[n^{*}(1-\alpha) + 1\right] + c\left[nn^{*}(1-\alpha) - n^{*} - 1\right] + c^{*}\left[n^{*}\left(\alpha(n+1) - n\right)\right]}b'$$
.

It will be profitable for a home firm to produce *iff* its unit cost does not exceed a weighted average of the demand intercept and the unit cost of foreign firms, where the weight attached to the former is decreasing in the number of foreign firms and increasing the degree of privatization of the publicly owned home firm:

(20)
$$c \leq \xi_{0C} a' + (1 - \xi_{0C}) c^*$$

where
$$\xi_{0C} = \left(\frac{\alpha}{n^* + \alpha}\right) \in (0,1)$$
.

Analogously, it will be profitable for a foreign firm to produce *iff* its unit cost does not exceed to a weighted average of the demand intercept and the unit cost of domestic firms, where the weight attached to the former is decreasing in the number of home firms and increasing the degree of privatization of the publicly owned home firm:

(21)
$$c^* \leq \xi_{0C}^* a' + (1 - \xi_{0C}^*)c$$

where
$$\xi_{0C}^* = \left(\frac{\alpha}{\alpha(n+1)+1}\right) \in (0,1)$$
.

Figure 1 below captures the effect of a higher degree of privatization, before mergers are allowed, on the extent of specialization of international production and extensive margins of trade. When the cost of every firm exceeds a', in region O, then the good is not produced at all. In region H only the home can compete and in region F only the foreign firms can compete. Region HF is a cone of diversification (in terms of the goods' origin) where both home and foreign firms can co-exist.

The threshold sectors pinning down the extensive margins of trade, denoted by \tilde{z} and \tilde{z}^* at home and abroad respectively, are determined (conditional on wages) by

(22)
$$W\beta(\tilde{z}) - \xi_{0C}a' - (1 - \xi_{0C})W\beta^*(\tilde{z}) = 0$$

(23)
$$W^*\beta^*(\tilde{z}^*) - \xi_{0C}^*a' - (1 - \xi_{0C}^*)W^*\beta(\tilde{z}^*) = 0$$

This is depicted by ZZ where, given wages, the home country specializes in $z \in [0, \tilde{z}^*)$, the foreign country specializes in $z \in (\tilde{z}, 1]$, and production is diversified in $z \in [\tilde{z}, \tilde{z}^*]$.

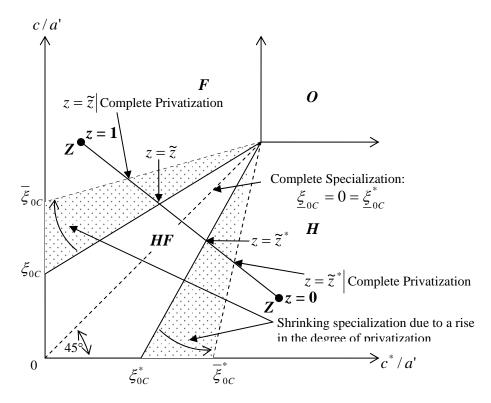


Figure 1: Privatization and Pre-Merger Trading Equilibrium in a Simultaneous Game

[Note:
$$\underline{\xi}_{0C}^{(\)} = \underset{\alpha \to 0}{Limit} \, \xi_{0C}^{(\)}$$
 and $\overline{\xi}_{0C}^{(\)} = \underset{\alpha \to 1}{Limit} \, \xi_{0C}^{(\)}$]

When the degree of privatization of the public firm rises, the regions of specialization (*H* and *F*) shrink and the cone of diversification (*HF*) expands. Consequently, in the premerger trading equilibrium, the extensive margin of trade shrinks on the face of privatization. If the public firm were to be completely privatized (i.e. if $\alpha = 1$),

international production would attain the highest degree of diversification causing the extensive margins to shrink to a minimum. At the other extreme, if the public firm were to be wholly owned by the home government (i.e. if $\alpha = 0$), the pattern of international production would mimic the outcome of a perfectly competitive limit (which would, otherwise, require $n \to \infty$ and $n^* \to \infty$) with *HF* collapsing to a 45° line as each country specializes completely in line with her comparative advantage replicating the extensive margins of trade that would prevail in a Ricardian world. Our first proposition follows.

Proposition **I**. The extensive margins of trade shrink with a rise in the degree of privatization when public and private firms move simultaneously.

In other words, a fall in α moves the international equilibrium closer to that implied by competitive behavior. Even more strongly, a single fully-publicly-owned firm (the case when $\alpha = 0$) is sufficient to restore the efficient competitive pattern of production. Intuitively, the greater the degree of privatization the further is the division of labor from the competitive benchmark. Intuitively, the lower is α , the more the public firm uses its choice of output to offset the negative effects on home welfare of decisions by other firms. If the firm is fully publicly owned, it tries to completely offset these negative effects.

Let us now turn to the possibility of mergers. A merger, under conditions of free and frictionless trade (i.e. absent any tariff or transportation cost), effectively implies that one of the participating firms is closed down since there is no incentive for a firm to operate more than one plant. Closing down $(n - \tilde{n})$ private firms at home raises the output of the remaining private firms (at home and abroad) by

$$y_{i}(\tilde{n}, n^{*})|_{C} - y_{i}(n, n^{*})|_{C} = y_{j}^{*}(\tilde{n}, n^{*})|_{C} - y_{j}^{*}(n, n^{*})|_{C} = \theta_{0C}(n - \tilde{n})y_{i}(n, n^{*})|_{C}$$

$$\forall i = 1, 2, ..., \tilde{n} \text{ and } j = 1, 2, ..., n^*$$

where
$$\theta_{0C} = \frac{\alpha}{\left[1 + n^* + \alpha(1 + \tilde{n})\right]}$$
.

Analogously, closing down $(n^* - \tilde{n}^*)$ foreign firms raises the output of the remaining private firms (at home and abroad) by

$$y_{i}(n,\tilde{n}^{*})|_{C} - y_{i}(n,n^{*})|_{C} = y_{j}^{*}(n,\tilde{n}^{*})|_{C} - y_{j}^{*}(n,n^{*})|_{C} = \theta_{1C}(n^{*}-\tilde{n}^{*})y_{j}^{*}(n,n^{*})|_{C}$$
$$\forall i = 1,2,...,n \text{ and } j = 1,2,...,\tilde{n}^{*}$$

where $\theta_{1C} = \frac{1}{\left[1 + \alpha(1+n) + \tilde{n}^*\right]}$.

The net gain from a merger between two privately owned home firms is

(24)
$$G_{HH}|_{C} = b' \Big(y_{i}(n,n)^{*} \Big|_{C} \Big)^{2} \Big[\Big(1 + \theta_{0C} \Big|_{\tilde{n}=n-1} \Big)^{2} - 2 \Big]$$
 $(i = 1,2,...n)$

It follows from (24), that there exists a threshold degree of privatization beyond which there is no incentive for a merger between two home firms

(25)
$$\underline{\alpha}_{0C} = \left(\frac{1}{n}\right) \left(\frac{1}{\sqrt{2}-1} - (n^*+1)\right)$$

The net gain from a merger between two foreign firms is

(26)
$$G_{FF}|_{C} = b' \left(y_{j}^{*}(n, n^{*}) \Big|_{C} \right)^{2} \left[\left(1 + \theta_{1C} \Big|_{\tilde{n}^{*} = n^{*} - 1} \right)^{2} - 2 \right]$$
 $\left(j = 1, 2, ..., n^{*} \right)$

It follows from (26), that there exists a threshold degree of privatization beyond which there is no incentive for a merger between two foreign firms

(27)
$$\underline{\alpha}_{1C} = \left(\frac{1}{n+1}\right) \left(\frac{1}{\sqrt{2}-1} - n^*\right)$$

It follows directly from (25) and (27) that, notwithstanding the degree of privatization, $n^* > 2$ imposes a condition *sufficient* to remove any incentive for a merger between two firms within the same country. Let us now focus on the incentives for mergers across borders.

The net gain from a take-over of a private home firm by a foreign firm is

(28)
$$G_{FH}|_{C} = b' (y_{i}(n, n^{*})|_{C})^{2} N_{0} [c - \xi_{1C}a' - (1 - \xi_{1C})c^{*}]$$
 $(i = 1, 2, ..., n)$

where
$$\xi_{1C} = \frac{N_1}{N_0}$$
.
 $N_1 = \alpha \left(2\alpha n (1 - \alpha) + 2n^* (1 - \alpha) + \alpha^2 (n^2 - 1) + 2\alpha (n - 1) + n^{*2} + 1 + 2\alpha nn^* \right)$ and
 $N_0 = \left(\alpha n^* (1 - \alpha) + \alpha^3 (n^2 - 1) + \alpha n^{*2} + 2\alpha^3 n^2 + 2\alpha nn^* + \alpha^2 n^2 n^* + 4\alpha^2 nn^* + n^* + 3\alpha n^* + 3\alpha + 6\alpha^2 n - \alpha^3 + 2n^{*2} + n^{*3} + 2\alpha nn^{*2} \right)$

Incentives for a takeover of a private home firm by a foreign firm exists iff

$$c > \xi_{1C}a' + (1 - \xi_{1C})c^*$$

where $0 < \xi_{1C} < \xi_{0C} < 1$.

The net gain from a take-over of a foreign firm by a privately owned home firm is

(29)
$$G_{HF}|_{C} = b' (y_{j}^{*}(n,n^{*})|_{C})^{2} N_{2} [c^{*} - \xi_{1C}^{*}a' - (1 - \xi_{1C}^{*})c]$$
 $(j = 1,2,...n^{*})$
where $\xi_{1C}^{*} = \frac{N_{3}}{N_{2}}$.
 $N_{3} = \alpha (2\alpha (n+1)(n^{*}-1) + [n^{*}(n^{*}-2)-1] + 2\alpha^{2}n + \alpha^{2}n^{2} + \alpha^{2})$ and
 $N_{2} = ([\alpha (n+1) + (n^{*}+1)](n^{*}-1) + 2\alpha^{2}(n+n^{*}) + 3\alpha (n^{*} + \alpha^{2}n)(n+1) + \alpha^{2}n^{2}(n+\alpha) + \alpha n^{*2} + \alpha nn^{*}(4\alpha^{2} + n^{*} + 2\alpha n^{2}) + 2n^{*2} + \alpha^{2}(1+\alpha))$
Incentives for a takeover of a foreign firm by a private home firm exists *iff*

$$c^* > \xi_{1C}^* a' + (1 - \xi_{1C}^*)c$$

where $0 < \xi_{1C}^* < \xi_{0C}^* < 1$.

Intuitively, a lower degree of private ownership of the public firm implies that it gives greater weight to social welfare and, consequently, the incentive of a foreign firm to take over a private domestic firm but raises the incentive of a domestic firm to take over a foreign firm. As such, the existence of the public firm (or reduction of privatization) dampens the potential gains from a take-over of a home firm by the foreign firm but magnifies the potential gains from a take-over of a foreign firm by the home firm. The lower the degree of privatization, the more the public firm uses its choice of output to offset the negative effects on home welfare of decisions by other firms.

Our next proposition follows.

Proposition **II**. Cross-border mergers will mitigate the effect of privatization on the extensive margins of trade.

Cross-border mergers will induce expansion and contraction of sectors as high-cost firms in one country are bought out by low-cost foreign rivals in another. At any given wages, expanding firms will a) increase their output by only a fraction of the output of the firms which are taken over and b) have lower labor requirements per unit output than the contracting ones. Consequently, the total demand for labor will fall pressing wages down to restore equilibrium in the labor market which, in turn, encourages hiring of labor at the intensive margin. The lower wages raise the profitability of high-cost firms, at the margin, placing them outside the reach of takeovers thereby dampening the initial wave of mergers.

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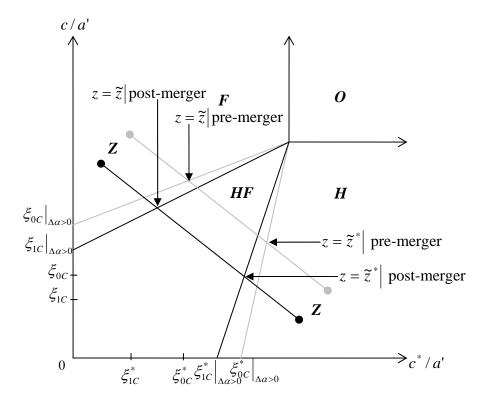


Figure 2: Privatization and Post-Merger Trading Equilibrium in a Simultaneous Game

This is captured in figure 2 where a merger-induced fall in wages causes the ZZ locus to shifts toward the origin which will mitigate the effect of privatization on the extensive margins of trade by aligning production and trade patterns toward the direction of comparative advantage (i.e. what would prevail in a competitive Ricardian world).

3. Sensitivity

In this section, we look at the sensitivity of our results to the order of the move by allowing the possibility that the public and private firms can move sequentially (`a la Stackelberg). ¹² Consider first the case where the public firm moves before the private firms (home and foreign) get to move. Each privately owned home firm will

$$Maximiz_{\{y_i\}} i = \left(a' - b' \left[\sum_{i=1}^{n} y_i + \sum_{j=1}^{n^*} y_j^* + y_p\right] - c\right) y_i \qquad i = 1, 2, ..., n$$

Each foreign firm will

$$Max_{\{y_i^*\}}^{i} ize: \quad \Pi_{jF}^* = \left(a' - b' \left[\sum_{i=1}^n y_i + \sum_{i=1}^n y_j^* + y_p\right] - c^*\right) y_j^* \qquad \qquad j = 1, 2, ..., n^*$$

The best-response functions of the $(n + n^*)$ privately owned home and foreign firms are

(30)
$$y_{iF} = \frac{1}{2b'} \left(a' - b' \left[\sum_{\substack{k=1\\k \neq i}}^{n} y_k + \sum_{j=1}^{n^*} y_j^* + y_p \right] - c \right) \qquad \forall \quad i = 1, 2, ..., n$$

(31)
$$y_{jF}^{*} = \frac{1}{2b'} \left(a' - b' \left[\sum_{i=1}^{n} y_{i} + \sum_{\substack{k=1\\k\neq j}}^{n^{*}} y_{k}^{*} + y_{p} \right] - c^{*} \right) \qquad \forall \qquad j = 1, 2, ..., n^{*}$$

Using backward induction, the public firm's objective will be to

$$Maximize: \quad \Pi_{PL} = \left(a' - b' \left[\sum_{i=1}^{n} y_{iF} + \sum_{i=1}^{n^*} y_{jF}^* + y_P\right] - c\right) y_P + (1 - \alpha) \left(\sum_{i=1}^{n} \Pi_i + \frac{b' \tilde{y}^2}{2}\right)$$

In equilibrium, the public firm will produce

$$(32) \quad y_{p}(n, n^{*})\Big|_{L} = \left(\frac{a' [\alpha n + (2 - \alpha)n^{*} + 1] - [1 + n^{*} (n^{*} + 2(\alpha n + 1) - n) + \alpha n]c + [n^{*} + \alpha(n + 1) - n(1 - \alpha)]n^{*}c^{*}}{b' [2(n^{*} + \alpha n) + (1 + \alpha)]}\right)$$

The equilibrium output of the $(n + n^*)$ privately owned home and foreign firms will be

(33)
$$y_i(n,n^*)\Big|_F = \left(\frac{\alpha a' - (n^* + \alpha)c + n^*c^*}{b' [2(n^* + \alpha n) + (1 + \alpha)]}\right)$$
 $\forall \quad i = 1, 2, ..., n$

(34)
$$y_{j}^{*}(n,n^{*})\Big|_{F} = \left(\frac{\alpha a' + [n(1+2\alpha)+1]c - [n^{*}+\alpha(1+2n)+1]c^{*}}{b'[2(n^{*}+\alpha n)+(1+\alpha)]}\right) \quad \forall \qquad j = 1,2,...,n^{*}$$

It will be profitable for a home firm to produce if its unit cost does not exceed a weighted average of the demand intercept and the unit cost of foreign firms, where the weight attached to the former is decreasing in the number of foreign firms and increasing the degree of privatization of the publicly owned home firm:

$$c \leq \xi_{0F} a' + (1 - \xi_{0F}) c^*$$

where
$$\xi_{0F} = \xi_{0C} = \left(\frac{\alpha}{n^* + \alpha}\right) \in (0,1)$$
.

Analogously, it will be profitable for a foreign firm to produce if its unit cost does not exceed to a weighted average of the demand intercept and the unit cost of domestic firms, where the weight attached to the former is decreasing in the number of home firms and increasing the degree of privatization of the publicly owned home firm:

$$c^* \leq \xi_{0F}^* a' + (1 - \xi_{0F}^*)c$$

where $\xi_{0C}^* > \xi_{0F}^* = \left(\frac{\alpha}{n^* + \alpha(2n+1)}\right) \in (0,1).$

The industry output and price are

(35)
$$\widetilde{y}(n,n^*)\Big|_L = \left(\frac{(2\alpha n+1)(a'-c)+n^*\left[(a'-c)+(a'-c^*)\right]}{b'\left[\alpha(2n+1)+2n^*+1\right]}\right)$$

(36)
$$p(n,n^*)\Big|_L = \left(\frac{\alpha a' + (2\alpha n + n^* + 1)c + n^*c^*}{\alpha(2n+1) + 2n^* + 1}\right)$$

In the pre-merger equilibrium, the profits of the firms are

(37)
$$\Pi_{i}(n,n^{*})|_{F} = b' (y_{i}(n,n^{*})|_{F})^{2} \qquad \forall \quad i = 1,2,...n$$

(38)
$$\Pi_{j}^{*}(n,n^{*})\big|_{F} = b'\big(y_{j}^{*}(n,n^{*})\big|_{F}\big)^{2} \qquad \forall \qquad j = 1,2,...,n^{*}$$

(39)
$$\Pi_{P}(n,n^{*})\Big|_{L} = B_{L}\Big(y_{P}(n,n^{*})\Big|_{L}\Big)^{2}$$

where
$$B_L = \frac{\left[\alpha(a'-c) + n^*(c^*-c)\right]}{a'\left[n^*(2-\alpha) + \alpha n + 1\right] - c\left[(n^*+1)^2 + n\left(n^*(2\alpha-1) + \alpha\right)\right] + c^*\left[n^*\left(\alpha(2n+1) - n + n^*\right)\right]}b'$$

Figure 3 captures the changes in the pre-merger and post-merger trading equilibria when the public firm moves first as the cone of diversification (*HF*) shrinks.

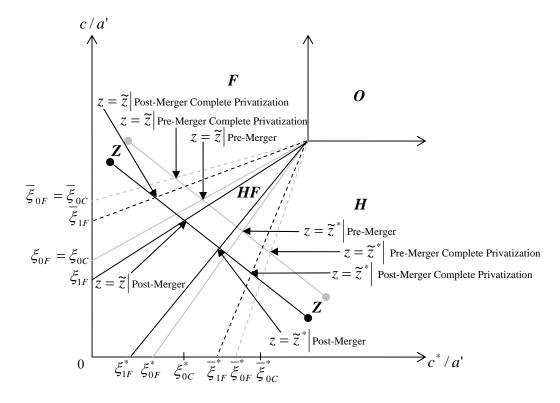


Figure 3: Pre-Merger and Post-Merger Trading Equilibria when the Private Firms Follow

[Note:
$$\overline{\xi}_{()}^{()} = \underset{\alpha \to 1}{Limit} \xi_{()}^{()}$$
]

Consider next the case where the public firm moves only after the private firms (home and foreign) have moved. The public firm will

$$Maximize: \quad \Pi_{PF} = \left(a' - b' \left[\sum_{i=1}^{n} y_i + \sum_{i=1}^{n^*} y_j^* + y_p\right] - c\right) y_p + (1 - \alpha) \left(\sum_{i=1}^{n} \Pi_i + \frac{b' \tilde{y}^2}{2}\right)$$

The best-response function of the public firm can be written as

(40)
$$y_{PF} = \frac{1}{b'(1+\alpha)} \left(a' - b' \left[\sum_{i=1}^{n} y_i + \alpha \sum_{j=1}^{n^*} y_i^* \right] - c \right)$$

Using backward induction, each home firm's objective will be to

$$Maximize: \quad \Pi_{iL} = \left(a' - b' \left[\sum_{i=1}^{n} y_i + \sum_{j=1}^{n^*} y_j^* + y_{PF}\right] - c\right) y_i \qquad i = 1, 2, ..., n$$

and each foreign firm will

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$$Maximize: \quad \Pi_{jL}^{*} = \left(a' - b' \left[\sum_{i=1}^{n} y_{i} + \sum_{i=1}^{n^{*}} y_{j}^{*} + y_{PF}\right] - c^{*}\right) y_{j}^{*} \qquad \qquad j = 1, 2, ..., n^{*}$$

The equilibrium output of the $(n + n^*)$ privately owned home and foreign firms will be

(41)
$$y_{iL} = \frac{\alpha a' - (n^* (1 + \alpha) + \alpha)c + n^* (1 + \alpha)c^*}{\alpha b' (n + n^* + 1)} \quad \forall \quad i = 1, 2, ..., n$$

(42)
$$y_{jL}^* = \frac{\alpha a' + (n(1+\alpha)+1)c - (1+\alpha)(1+n)c^*}{\alpha b'(n+n^*+1)}$$
 $\forall \quad j = 1, 2, ..., n^*$

In equilibrium, the public firm will produce

$$(43) y_{P}(n,n^{*})\Big|_{L} = \left(\frac{a'\left[1+n^{*}(1-\alpha^{2})\right] - \left[\alpha+\alpha n^{*}(1+\alpha)+nn^{*}(\alpha^{2}+\alpha^{3}-1-\alpha)\right]c - \left[(1+\alpha)\left(n(1-\alpha^{2})-\alpha^{2}\right)\right]n^{*}c^{*}}{b'\left[(n+n^{*}+1)\alpha(1+\alpha)\right]}\right)$$

It will be profitable for a home firm to produce if its unit cost does not exceed a weighted average of the demand intercept and the unit cost of foreign firms, where the weight attached to the former is decreasing in the number of foreign firms and increasing the degree of privatization of the publicly owned home firm:

$$c \leq \xi_{0L}a' + (1 - \xi_{0L})c^*$$

where $\xi_{0C} > \xi_{0L} = \left(\frac{\alpha}{n^*(1+\alpha) + \alpha}\right) \in (0,1)$.

Analogously, it will be profitable for a foreign firm to produce if its unit cost does not exceed to a weighted average of the demand intercept and the unit cost of domestic firms, where the weight attached to the former is decreasing in the number of home firms and increasing the degree of privatization of the publicly owned home firm:

$$c^* \leq \xi_{0L}^* a' + (1 - \xi_{0L}^*)c$$

where $\xi_{0C}^* > \xi_{0L}^* = \left(\frac{\alpha}{(1+\alpha)(1+n)}\right) \in (0,1)$.

The industry output and price are

(44)
$$\widetilde{y}(n,n^*)\Big|_F = \left(\frac{(n(1+\alpha)+1)(a'-c)+n^*(1+\alpha)(a'-c^*)}{b'(1+\alpha)(n+n^*+1)}\right)$$

(45)
$$p(n,n^*)\Big|_F = \left(\frac{\alpha a' + (n(1+\alpha)+1)c + n^*(1+\alpha)c^*}{[(1+\alpha)(n+n^*+1)]}\right)$$

In the pre-merger equilibrium, the profits of the firms are

(46)
$$\Pi_{i}(n,n^{*})\Big|_{L} = \frac{\alpha}{1+\alpha} b'\Big(y_{i}(n,n^{*})\Big|_{L}\Big)^{2} \qquad \forall \quad i = 1,2,...n$$

(47)
$$\Pi_{j}^{*}(n,n^{*})\Big|_{L} = \frac{1}{1+\alpha} b' \Big(y_{j}^{*}(n,n^{*})\Big|_{L} \Big)^{2} \qquad \forall \qquad j = 1,2,...,n^{*}$$

(48)
$$\Pi_{P}(n,n^{*})\Big|_{F} = B_{F}\Big(y_{P}(n,n^{*})\Big|_{F}\Big)^{2}$$

where
$$B_F = \frac{\left[\alpha(a'-c) + n^*(1+\alpha)(c^*-c)\right]}{a'\left[1 + n^*(1-\alpha^2)\right] - c\left[n^*\left(\alpha(1+\alpha) + n(1+\alpha-\alpha^2+\alpha^3)\right)\right] - c^*\left[n^*(1+\alpha)\left(n(1-\alpha^2) + \alpha^2\right)\right]}b'$$

Figure 4 captures the changes in the pre-merger and post-merger trading equilibria when the public firm follows as the cone of diversification (HF) shrinks even more than it does when the public firm leads.

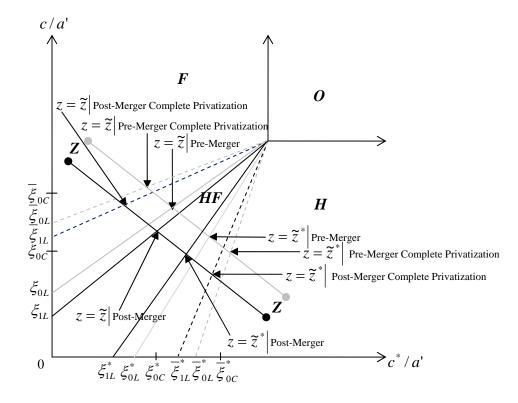


Figure 4: Pre-Merger and Post-Merger Trading Equilibria when the Private Firms Lead

[Note:
$$\overline{\xi}_{0(\cdot)}^{(\cdot)} = \underset{\alpha \to 1}{Limit} \xi_{0(\cdot)}^{(\cdot)}$$
]

In sum, the larger the cost differential, the greater is the gain from a low cost firm taking over a high cost firm. While cross-border mergers mitigate (by facilitating specialization toward the direction of comparative advantage) the effect of privatization on the extensive margins of trade, the mitigating effect is magnified when the public and private firms move sequentially with the magnification effect enhanced when private firms lead.

4. Conclusion

Cross-border mergers have increasingly evolved into an effective strategy used by a large number of companies with global presence. Notwithstanding the fact that a third of worldwide mergers involve firms from different countries, the vast majority of the academic literature on mergers has been primarily limited to intra-national mergers. We hope to have taken a step forward, along the path of continued efforts to capture the

incentives for and implications of cross-border mergers, by constructing an analytically tractable general equilibrium model of a mixed oligopoly that can capture the role of mergers and trade across borders. Our model offers a lens to look through the general equilibrium implications of the interactions between cross-border mergers and privatization for the extensive margins of trade. The practical relevance of our M-GOLE model follows directly from the recent dramatic financial events that have brought about the creation of new modes of competition, where private and public organizations vie to supply the same customers, renewing interests in the interaction among state-owned agents and private suppliers. We believe that the implications of our model are of critical importance on the face of the growing consensus that "a new wave" of cross-border mergers is likely to be triggered by the imminent exit of public funds from ailing industries in the immediate aftermath of the current global economic crises. We have shown that, absent cross-border mergers, the extensive margins of trade will shrink with a rise in the degree of privatization. Cross-border mergers will mitigate the effect of privatization on the extensive margins of trade by aligning specialization toward the direction of comparative advantage. This mitigating effect will be magnified if the firms move sequentially and the magnification effect would be larger when the private firms lead than it would be if the private firms follow. Some interesting extensions, we are working on, involves incorporating costs of complexity á la Gale and Sabourian (2005) and urban unemployment á la Oladi and Gilbert (2011), in our otherwise M-GOLE model.

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Endnotes

¹ While much of the earlier merger activity was confined to North America and Great Britain, the most recent wave has engulfed all of the major industrial countries of the world.

² Cross-border mergers and acquisitions account for a significant and growing share of global FDI flows. Between 1996 and 2005, the annual average value of cross-border mergers and acquisitions worldwide was \$533 billion, or about 70% of annual world FDI flows (*source*: UNCTAD, 2009).

³ The volume of cross-border mergers has been growing worldwide, from 30 percent of the total merger volume in 1998 to 45 percent in 2007. *Source*: World Investment Report, UNCTAD, 2009.

⁴ Source: World Investment Prospects Survey (WIPS) 2009–2011, UNCTAD, 2009.

⁵ There are at least three reasons why it is interesting to examine mergers and acquisitions from an international perspective. First, cross-border mergers and acquisitions have fueled the growth in international production for more than a decade. Specifically, most foreign direct investment is carried out through the acquisition of foreign firms' assets rather than the creation of new firms, also known as greenfield investment. Second, there is evidence that economic integration affects mergers and acquisitions, and by forcing industries to restructure. This restructuring is often accomplished through mergers and acquisitions. Third, both cross-border mergers and mergers between domestic firms engaged in international trade pose challenges for competition policy.

⁶ See Perry and Porter (1985) and Farrell and Shapiro (1990) on cost synergies in horizontal mergers.

⁷ Even mergers of companies with headquarters in the same country, though do not fit into the strict definition of cross-border mergers, are often transnational in nature. For instance, when Boeing acquired McDonnell Douglas, the two American companies had to integrate operations in dozens of countries around the world. This was just as true for other supposedly single-country mergers, such as the \$27 billion dollar merger of Swiss drug makers Sandoz and Ciba-Geigy (now Novartis).

⁸ Brakman *et al.* (2012) have investigated the role of comparative advantage in explaining the direction of cross-border mergers and found strong evidence that acquiring firms operate in sectors with a comparative advantage. See also Batra's (2002) critique of the basic comparative advantage gains-from-trade model.

⁹ It may be noted that sunk costs have no effect on merger decisions as they cannot be recouped.

¹⁰ W and W^* can be interpreted as marginal real wages since they equal nominal wages deflated by the marginal cost of utility. For homothetic preferences, W and W^* would measure the real wages.

¹¹ Following Bös (1991), such a bargaining will yield a mixed objective between profits and welfare in which each carries the respective weight of the representatives. For an analogous argument, see Chao and Yu (2006).

¹² See Hamilton and Slutsky (1990).