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Abstract

By constructing a dynamic general equilibrium model this paper studies the optimal pollution emission and abatement policies for a small economy. We show that in autarky government issues an optimal level of emission permits but its optimal abatement level is zero at the unique steady-state equilibrium. On the other hand, government employs an optimal two-dimensional policy with free trade if the country is an exporter of a pollution intensive good. That is, government issues pollution permits (thus controls the emission level) while it undertakes a positive level of abatement activity. In other words, an optimal combination of emission cap and pollution abatement is required for a small trading economy with comparative advantage in dirty goods.

1 Introduction

Academic community as well as policy makers have been engaged in contentious debates regarding the effects of international trade on environmental degradation for the past few decades. Whether or not trade plays a negative role on environmental quality has remain a subject of intense study. Aside from any role trade plays in this regard, numerous policy recommendations have been put forth including various forms of Pigovian tax, emission permits, and pollution abatement activities. Each of these issues has been studied individually and mostly within static frameworks. The purpose of our paper is to offer a holistic dynamic theory of emission permits and pollution abatement in a general equilibrium set up both under autarky and free trade for a small open economy.

The literature on trade and environment is extensive and can be traced back to the pioneering work of Baumol (1971). In one strand of literature the core issue is the effect of trade liberalization in the presence of environmental problems, for example, see Baumol (1971) and Pethig (1976), and Siebert (1977). In these studies, a country faces two distortions: trade barriers and inefficient environmental policies. Then, a question that researchers ask is whether the standard theorems of trade regarding gains from trade hold since a reduction in trade barriers can alleviate or worsen the problems raised due to inefficient environmental policies.

Another major theme of this literature has been the effect of international trade on environmental quality, see for example Blackhurst (1977), Copeland and Taylor (1995), and Beladi and Oladi (2011), among others. Accordingly, the effect of trade liberalization on environment in a country depends on pollution intensity of its exports/imports as well as the magnitude of income elasticity of marginal environmental damage. As this issue ties with the country's pattern of trade, comparative advantage also plays an important role. This also has led to the development of yet another strand of literature known as pollution haven hypothesis which predicts that countries with weak environmental standards will specialize in production of dirty goods. Pioneering work of Pething (1976) indicated that, given the exogenous pollution intensities, countries with laxer environmental standards export dirty goods.

On the policy front, various policy regimes (environmental and commercial policies) and their effectiveness in protecting the environment have been studied in Markusen (1975), Long (1992), Dockner and Long (1993), Benchekroun and Long (1998), Chao and Yu (1999, 2000) and Chander and Khan (2001).[1] While some studies such as Pethig (1976) considered the effects of environmental policies on international competitiveness of a country, others investigated environmentally related tariffs on pollution intensive industries (see, for example, Markusen (1975)). Regarding the domestic environmental policy instruments, in addition to the Pigovian tax and its extension as in Lee and Batabyal (2002), some researchers have suggested and studied the use of pollution permits, see for example Crocker (1966), among others. Copeland and Taylor (1995) extended the concept of permits by allowing the permits to be traded internationally and showed that global trade in permits can reduce global pollution even if the government's supply of permits is not restricted. Yet another policy instrument suggested by some authors is pollution abatement activities, for example, see Khan (1996) and Chao and Yu (1999), among others.

In this paper we construct a dynamic general equilibrium model of pollution permits and abatement for a small economy. Therefore, our paper contributes to the last strand of literature. We assume that our economy produces and consumes two commodities. Both production sectors use labor and pollution (as a by-product). The government interferes by using two instruments. On the one hand, it imposes a cap on emission and sells permits to the production sectors. On the other hand, it cleans up the stock of pollution, financed by a lump sum tax. To our knowledge, no

[1] See also Mohtadi (1992), Batabyal (1998) and Eglin (2001), among others.

attempt has been made in the literature to investigate optimal pollution permits and abatement in a holistic dynamic general equilibrium setup for a small open economy. We characterize the steady-state equilibrium both under autarky and free trade equilibrium. Interestingly, we show that government does not undertake any abatement activity in autarky, while the optimal policy under free trade requires a positive level of pollution abatement given that the country is an exporter of pollution intensive good. That is, while only emission cap would be sufficient for a closed economy, this one dimensional policy is not enough for a trading economy if it exports dirty goods. Under this scenario government will have to employ a combination of optimal emission cap (and issue permits accordingly) and abatement.

The organization of the rest of the paper is as follows. Section 2 presents the autarky model and draws conclusions with regard to optimum levels of permits and pollution treatment. We then modify our setup for a small trading economy and characterize our steady state equilibrium in Section 3. Finally, Section 4 concludes the paper.

2 The model and autarky equilibrium

Assume an economy that produces two goods, 1 and 2, using labor and pollution with Cobb-Douglas production technology given by:

$$Y_i = E_i^{\alpha_i} L_i^{\beta_i}, \quad i = 1, 2 \tag{1}$$

where Y_i, E_i, and L_i are the production level, emission, and labor usage of sector i. We assume that both sectors exhibit constant returns to scale, i.e., $\alpha_i + \beta_i = 1$, and that $\alpha_1 < \alpha_2$ and $\beta_1 > \beta_2$ implying that sector 1 is labor intensive and sector 2 is emission intensive. Furthermore, we assume that both good markets are competitive. The competitive market assumption implies that:

$$E_i = \frac{p_i \alpha_i Y_i}{\tau} \quad i = 1, 2 \tag{2}$$

$$L_i = \frac{p_i \beta_i Y_i}{w} \quad i = 1, 2 \tag{3}$$

where p_i, τ, and w are the real price of good i, emission permit real price, and the real wage rate, respectively, all expressed in terms of good 1. We normalize the initial good prices to unity throughout the paper. The labor and emission permit market clearing conditions are:

$$L_1 + L_2 = L \tag{4}$$

$$E_1 + E_2 = E \tag{5}$$

where L is the constant stock of labor and E is the total amount of pollution permits to be determined by the government.

Turning now to the household problem, assume that the household preference is represented by:

$$U = C_1^{\theta_1} C_2^{\theta_2} - \gamma X^2 \tag{6}$$

where, $C_i, i = 1, 2$, is the consumption of good i and X is the accumulated stock of pollution, $\gamma > 0$, and $\theta_i \in (0, 1), i = 1, 2$. Thus, γX^2 is the pollution damage to the consumers. We further postulate that $\theta_1 + \theta_2 < 1$ to ensure concavity of the indirect utility function that we will derive shortly. The household budget constraint is given by:

$$C_1 + p_2 C_2 = wL + \tau E - T \tag{7}$$

where T is the real lump-sum tax that the government imposes to finance pollution abatement. However, the government return the permit revenues back to the household as a transfer payment. Maximizing (6) subject to (7) leads to the following demand functions for goods 1 and 2, respectively.[2]

$$C_1 = \frac{(wL + \tau E - T)\theta_1}{\theta_1 + \theta_2} \tag{8}$$

$$C_2 = \frac{(wL + \tau E - T)\theta_2}{p_2(\theta_1 + \theta_2)} \tag{9}$$

The following lemma identifies the household's indirect utility function, which will be used as a measure of welfare in the rest of this section.

Lemma 1. *The indirect utility function is given by:* $W = \Omega(L)E^{\sigma} - \gamma X^2$ *where* $0 < \sigma = \alpha_1\theta_1 +$

[2]Note that pollution stock does not influence the marginal rate of substitution for goods and thus the household's choice. Nevertheless, this stock affects the household's welfare. This is consistent with the literature.

$\alpha_2\theta_2 < 1$ *and* $d\Omega/dL > 0$.

Proof. The good market equilibrium conditions $C_i = Y_i, i = 1, 2$, and equations (8) and (9) imply:

$$\frac{(wL + \tau E - T)\theta_1}{Y_1(\theta_1 + \theta_2)} = 1 \tag{10}$$

$$p_2 = \frac{(wL + \tau E - T)\theta_2}{Y_2(\theta_1 + \theta_2)} \tag{11}$$

Next use equations (2) and (3) to obtain:

$$\frac{E_1}{E_2} = \frac{\alpha_1 Y_1}{p_2 \alpha_2 Y_2} \tag{12}$$

$$\frac{L_1}{L_2} = \frac{\beta_1 Y_1}{p_2 \beta_2 Y_2} \tag{13}$$

By substituting equations (10) and (11) in equations (12) and (13), we get:

$$\frac{E_1}{E_2} = \frac{\theta_1\alpha_1}{\theta_2\alpha_2} \tag{14}$$

$$\frac{L_1}{L_2} = \frac{\theta_1\beta_1}{\theta_2\beta_2} \tag{15}$$

We use equations (14) and (15) as well as the equilibrium conditions (4) and (5) to obtain:

$$\frac{E_1}{E} = \frac{\theta_1\alpha_1}{\theta_1\alpha_1 + \theta_2\alpha_2} \tag{16}$$

$$\frac{L_1}{L} = \frac{\theta_1\beta_1}{\theta_1\beta_1 + \theta_2\beta_2} \tag{17}$$

Therefore, the equilibrium factor usage (2) and (3) can be rewritten as:

$$E_i = \frac{\theta_i\alpha_i}{\theta_1\alpha_1 + \theta_2\alpha_2} E \quad i = 1, 2 \tag{18}$$

$$L_i = \frac{\theta_i\beta_i}{\theta_1\beta_1 + \theta_2\beta_2} L \quad i = 1, 2 \tag{19}$$

Finally, substitute equations (18) and (19) in equation (1) to obtain the equilibrium production levels. Then, using the commodity market clearing conditions as well as equation (6), we get

$W = \Omega(L)E^{\sigma} - \gamma X^2$ where σ is as defined by the lemma and Ω is defined as:

$$\Omega(L) = \theta_1\theta_2 \left(\frac{\theta_1}{\theta_1\alpha_1 + \theta_2\alpha_2}\right)^{\theta_1+\theta_2} \left(\alpha_1^{\alpha_1}\beta_1^{\beta_1}\right)\left(\alpha_2^{\alpha_2}\beta_2^{\beta_2}\right) L^{\theta_1\beta_1+\theta_2\beta_2} \tag{20}$$

Clearly, $\Omega > 0$ and $d\Omega/dL > 0$. □

Turning now to the government's problem, we assume a benevolent government that maximizes the household's welfare by issuing pollution permits and cleaning up the environment. Given that the government issues a pollution level of E and spends T on abatement, the law of motion for pollution stock is given by:

$$\dot{X} = E - \eta X - T^{v} \tag{21}$$

where $\eta \in (0,1)$ is a constant natural rate of pollution absorption, as in Long (1992). Moreover, T^{v} is the pollution treatment technology with $v \in (0,1)$, according to which pollution abatement technology exhibits diminishing marginal productivity.[3] Thus, the government's problem is given by:

$$\max_{E,T} \int_0^{\infty} [\Omega(L)E^{\sigma} - \gamma X^2] \exp(-\rho t) dt \tag{22}$$

$$s.t. \quad \dot{X} = E - \eta X - T^{v} \tag{23}$$

with initial conditions $E(0) = E_0$, $T(0) = T_0$ and $X(0) = X_0$. Moreover, ρ denotes the discount rate. Then, the Hamiltonian function for the government's problem is given by:

$$H = [\Omega(L)E^{\sigma} - \gamma X^2] \exp(-\rho t) + \psi(E - \eta X - T^{v}) \tag{24}$$

The corresponding first order conditions, in addition to equation (23), are:

$$\frac{\partial H}{\partial E} = (\sigma \Omega E^{\sigma-1}) \exp(-\rho t) + \psi = 0 \tag{25}$$

$$\frac{\partial H}{\partial T} = -\psi v T^{v-1} = 0 \tag{26}$$

$$\dot{\psi} = -\frac{\partial H}{\partial X} = (2\gamma X) \exp(-\rho t) + \psi\eta \tag{27}$$

[3]One can see a parallel between our view of abatement formulated here in our model and capital formation in growth theory literature. That is, in our model real lump-sum tax (in terms of the numeriare good)is the resource used in abatement of pollution.

We next study the steady-state properties of our closed economy. The following proposition characterizes the steady-state equilibrium.

Proposition 1. *There exists a unique steady-state equilibrium at which $T = 0$, $E > 0$, and $X > 0$. Moreover, the steady-state values of X and E increase if the endowment of labor increases.*

Proof. By substituting ψ, obtained from equation (25), in (27), we get:

$$\dot{\psi} = [2\gamma X - \eta\sigma\Omega(L)E^{\sigma-1}]\exp(-\rho t) \tag{28}$$

Moreover, equation (20) implies that Ω is positive. Given any non-trivial positive choice of E and a finite value of t, equation (25) shows that ψ is non-zero. Thus, we conclude from equation (26) that at any equilibrium (including the its time path) $T = 0$. Therefore, at steady state we have:

$$E - \eta X = 0 \tag{29}$$

$$2\gamma X - \sigma\Omega(\rho + \eta)E^{\sigma-1} = 0 \tag{30}$$

where we have transformed the problem to a current value problem. By solving these two equations we get:

$$E^s = [\frac{\sigma\Omega\eta(\rho+\eta)}{2\gamma}]^{1/(2-\sigma)} \tag{31}$$

$$X^s = [\frac{\sigma\Omega\eta^{\sigma-1}(\rho+\eta)}{2\gamma}]^{1/(2-\sigma)} \tag{32}$$

where the superscript s denote the steady-state value. Clearly, equations (31) and (32) indicate that $E^s > 0$ and $X^s > 0$. It is also evident from these equations that $E^s > 0$ and $X^s > 0$. Then, it also follows from these these equations and Lemma 1 that both E^s and X^s are increasing in L.

Finally, it is evident from equation (29) that a function that traces E and X loci at which $\dot{X} = 0$ is linear and strictly increasing. On the other hand, the E and X loci along which $\dot{\psi} = 0$ is strictly decreasing since by differentiating equation (30) we get $dE/dX = 2\gamma/[\sigma(1-\sigma)(\rho+\eta)E^{\sigma-2}] < 0$. Recall that $\sigma \in (0, 1)$. These guarantee that our steady state equilibrium is unique. □

Figure 1 shows the steady-state equilibrium. The intersection of these curves gives us the steady-state values of X and E. An increase in labor endowment shifts up the curve along which $\dot{\psi} = 0$, leading to an increase in steady-state values of E and X. As the size of an economy is identified by the size of the labor endowment is our model, the above proposition states that a larger economy will emit more and have a greater pollution stock.

As we are addressing a general theory of production, pollution, abatement efforts, and international trade in the context of a small open economy, it is of paramount importance to address the possible differences that arise from differing degrees of natural absorption rates for various pollutants. Nuclear wastes are clearly different from carbon emission as it could take decades for nuclear waste to be absorbed by nature. Our framework allows for such a difference in pollutants. Therefore, our results may be different when we apply it to various type of pollutants with different natural rate of absorption. The following proposition formally addresses this issue.

Proposition 2. *The steady state level of emission permit is increasing in* η*, while the steady state level of pollution is decreasing in* η *for all* $0 < \eta < \bar{\eta}$ *and increasing for all* $\eta > \bar{\eta}$*, where* $\bar{\eta} = (1/\sigma - 1)\rho$*. Moreover,* $\bar{\eta} < 1$ *for sufficiently large* $\alpha_i, i = 1, 2$*.*

Proof. Equation (31) directly implies that steady-state permit level is increasing in η. To see the effect of a change in η on steady-state level of pollution, we differentiate equation (32) to get $\partial X^s/\partial\eta = (\sigma\Omega/2\gamma\eta^{2-\sigma})[(\sigma-1)\rho+\sigma\eta]$, implying that $\partial X^s/\partial\eta = 0$ if and only if $(\sigma-1)\rho+\sigma\eta = 0$. This, in turn, implies that $\bar{\eta} = (1/\sigma - 1)\rho$. Clearly, $\bar{\eta} > 0$. Note that $0 < \sigma < 1$. It is also easy to see $\bar{\eta} < 1$ if and only if $\sigma \equiv \theta_1\alpha_1 + \theta_2\alpha_2 > \rho/(1+\rho)$, which is held if $\alpha_i, i = 1, 2$ is sufficiently large. □

The above results deserve more attention. The first part of this proposition is quite intuitive. At the first glance, it seems intuitive that, ceteris paribus, an increase in η should decrease the steady-state pollution level. This is not generally true given that $\alpha_i, i = 1, 2$, is large. To see this, suppose this condition is not met. That is, the emission marginal productivity is low in both sectors for any given level of E and L. Then, the steady-state pollution level is decreasing in η for all η as one may expect. That is, emission increases by less than the amount of higher natural absorption. Thus, the overall stock of pollution falls. However, assume that $\alpha_i, i = 1, 2$ is sufficiently large such that $\theta_1\alpha_1 + \theta_2\alpha_2 > \rho/(1+\rho)$. Note that $\rho/(1+\rho) < 1/2$ for all $\rho \in (0, 1)$. Then, as emission marginal productivity is high in both sectors, when natural pollution absorption is already high, the emission usage increases by a greater amount than the increase in natural absorption due to an increase in η, resulting in an increase in steady-state pollution level. The relationship between steady-state quantities of permit and pollution and natural absorption rate (η) is depicted in Figure 2.

Finally, the last comparative analysis that we consider is a change in discount factor. The following result formally highlights the role that the rate of time preference plays in our model.

Proposition 3. *An increase in the rate of time preference ρ will increase both the steady-state levels of emission and pollution stock.*

Proof. It is immediate from equations (31) and (32) since $1/(2-\sigma) > 0$. □

3 International trade

In this section we assume that our small economy is open to international trade. We assume that the international relative price of good 2 is greater than its autarky equilibrium price so that good 2 (1) is exportable (importable). Also, for mathematical simplicity, assume that the autarky relative price is unity. Rybczynski theorem implies that at equilibrium:

$$\frac{Y_2}{Y_1} = \delta(.)E \tag{33}$$

where $\delta > 0$ if and good 1 (2) is labor (emission) intensive and good prices are constant. We also assume in the remainder of the paper that L is constant. Let also $S_1(.;E)$ be the equilibrium supply of good 1. It again follows from Rybczynski theorem that $\partial S_1/\partial E < 0$. The following lemma characterizes the indirect utility function for the open economy case.

Lemma 2. *The indirect utility function for our small open economy is given by: $W^o = \Delta[(1 + p_2\delta E)S_1(.;E) - T]^\epsilon - \gamma X^2$ where $\Delta = [\theta_1/(\theta_1+\theta_2)]^{\theta_1}[\theta_2/p_2(\theta_1+\theta_2)]_2^\theta > 0$ and $0 < \epsilon = \theta_1 + \theta_2 < 1$.*

Proof. Using equations (2)-(5) and (33) we derive:

$$\tau E = [\alpha_1 + p_2\alpha_2\delta E]S_1(.;E) \tag{34}$$

$$wL = [(1-\alpha_1) + p_2(1-\alpha_2)\delta E)]S_1(.;E) \tag{35}$$

By substituting the above expressions in equations (8) and (9), we get:

$$C_1 = \frac{\theta_1[(1+p_2\delta E)S_1(.;E) - T]}{\theta_1+\theta_2} \tag{36}$$

$$C_2 = \frac{\theta_2[(1+p_2\delta E)S_1(.;E) - T]}{p_2(\theta_1+\theta_2)} \tag{37}$$

Finally, substitute equations (36) and (37) in equation (6) to conclude the lemma. □

We now shall turn to government problem. The government faces the following Maximization problem:

$$\max_{E,T} \int_0^{\infty} \left(\Delta[(1+p_2\delta E)S_1(.;E) - T]^{\epsilon} - \gamma X^2\right)\exp(-\rho t)dt \tag{38}$$

$$s.t. \quad \dot{X} = E - \eta X - T^{v} \tag{39}$$

$$E(0) = E_0, T(0) = T_0, X(0) = X_0 \tag{40}$$

The Hamiltonian for this problem is given by:

$$H = \left(\Delta[(1+p_2\delta E)S_1(.;E) - T]^{\epsilon} - \gamma X^2\right)\exp(-\rho t) + \phi(E - \eta X - T^{v}) \tag{41}$$

Therefore, the first order conditions are given by:

$$\epsilon\Delta[p_2\delta S_1 + (1+P_2\delta E)\frac{\partial S_1}{\partial E}][(1+p_2\delta E)S_1 - T]^{\epsilon-1}\exp(-\rho t) + \phi = 0 \tag{42}$$

$$\epsilon\Delta[(1+p_2\delta E)S_1 - T]^{\epsilon-1}\exp(-\rho t) + \phi\nu T^{\nu-1} = 0 \tag{43}$$

$$\dot{\phi} = 2\gamma X\exp(-\rho t) + \phi\eta \tag{44}$$

As in the preceding section, we characterize the steady-state equilibrium of our economy. Specifically, we showed in the previous section that the steady-state equilibrium level of pollution abatement is zero. It is interesting to see if this is also true for a small trading economy. To answer this question, let $E = \xi(X)$ be the loci along which $\dot{\phi} = 0$, i.e., equation (44) is equal to zero. Similarly, let $E = \zeta(X)$ be the loci along which $\dot{X} = 0$. In what follows we characterize these functions which, in turn, we will use in characterizing the steady state equilibrium.

Lemma 3. $\Lambda \equiv p_2\delta S_1 + (1+p_2\delta E)\partial S_1/\partial E > 0$ *at equilibrium.*

Proof. Differentiating equation (33) with respect to E and using the resulting equation as well as equation (33), we get:

$$p_2\delta S_1 + (1 + p_2\delta E)\partial S_1/\partial E = \frac{p_2 Y_2}{E} + \frac{\partial Y_2/\partial E - \delta Y_2}{(\delta E)^2} + \frac{p_2(\partial Y_2/\partial E - \delta Y_2)}{\delta E} \tag{45}$$

Thus, $Sign(\Lambda) = Sign\left(p_2 Y_2 \delta^2 E + (\partial Y_2/\partial E - \delta Y_2) + p_2(\partial Y_2/\partial E - \delta Y_2)\delta E\right)$. After simplifying the right-hand-side of this expression, we get $Sign(\Lambda) = Sign(\partial Y_2/\partial E + p_2\delta E \partial Y_2/\partial E - \delta Y_2)$. Thus, $Sign(\Lambda) = Sign\left((\partial Y_2/\partial E)/Y_2 + \delta[\kappa_{Y_2E} - 1]\right) > 0$, where $\kappa_{Y_2E} \equiv (\partial Y_2/\partial E)E/Y_2$ is the elasticity of output of sector 2 with respect to E. Recall also that we have normalized the initial good prices to unity. The inequality follows from the fact that $\partial Y_2/\partial E > 0$ due to Rybczynski theorem and from the fact that $\kappa_{Y_2E} > 1$ due to Jones' magnification effect of a factor endowment increase (see Jones, 1967). Recall also that sector 2 is emission intensive. This concludes the lemma. □

Lemma 4. *ξ (ζ)is strictly decreasing (increasing) in X.*

Proof. Use equations (42) and (43) to obtain expressions for ϕ and T in terms of E and X. We substitute these expression in equation (44), and set the resulting equation equal to zero, to get:

$$2\gamma X - \epsilon\eta\Delta\Lambda\left[(1 + p_2\delta E)S_1 - (\nu\Lambda)^{\frac{1}{1-\nu}}\right]^{\epsilon-1} = 0 \tag{46}$$

where Λ is defined as in Lemma 3. Totally differentiating equation (46) and simplifying the result, we get:

$$\frac{dE}{dX}|_{\dot{\phi}=0} = \frac{2\gamma}{2\epsilon\eta\delta\Delta p_2 \frac{\partial S_1}{\partial E}\Pi^{\epsilon-1} + \epsilon\eta\Delta(\epsilon-1)[\Lambda^2 - \frac{2\delta p_2 \nu^{\frac{1}{1-v}}}{1-\nu}\frac{\partial S_1}{\partial E}\Lambda^{\frac{1}{1-v}}]\Pi^{\epsilon-2}} \tag{47}$$

where $\Pi \equiv (1 + p_2\delta E)S_1 - T > 0$ as the equilibrium consumption levels of both goods are strictly positive. Recall also that $\partial S_1/\partial E < 0$ due to Rybczynski theorem and that $\epsilon - 1 < 0$. It then follows from Lemma 3 that $dE/dX|_{\dot{\phi}=0} < 0$.

Similarly, by using equations (42) and (43) to rewrite equation (39) in terms of E and X and then setting it equal to zero, we get:

$$E - \eta X - (\nu[p_2\delta S_1 + (1 + p_2\delta E)\partial S_1/\partial E])^{\frac{\nu}{1-\nu}} = 0 \tag{48}$$

Totally differentiating equation (48) with respect to E and X, we get:

$$\frac{dE}{dX}|_{\dot{X}=0} = \frac{\eta}{1 - 2\delta p_2 \nu^{\frac{\nu}{1-\nu}} \frac{\partial S_1}{\partial E} \Lambda^{\frac{2\nu-1}{\nu}}} \tag{49}$$

Again, as $\partial S_1/\partial E < 0$, it follows from Lemma 3 that ζ is increasing in E. □

We now turn to the main question of this section. Do the results of Proposition 1 hold under free trade? Specifically, does the government implement zero-abatement policy at the steady-state equilibrium, as we concluded under autarky? The short answer is "no." The following proposition addresses this question formally.

Proposition 4. *We assume that under free trade the pollution intensive good is the exportable good. Then, at the unique free trade steady-state equilibrium $E^s > 0$, $X^s > 0$, and $T^s > 0$.*

Proof. It follows from Lemma 3 that at a steady state equilibrium $S_1 > 0$, implying that $E^s > 0$. To show that $X^s > 0$, assume the negation, i.e., $X^s = 0$. Then, it follows from equation (48) that $E = (\nu\Lambda)^{\nu/(1-\nu)}$. By substituting this in equation (46) and maintaining our negation assumption, we can get:

$$S_1 = \frac{E^{\frac{1}{\nu}}}{1 + \delta E} \tag{50}$$

Now, differentiate equation (50) with respect to S_1 and E to obtain:

$$\frac{\partial S_1}{\partial E} = \frac{\frac{1}{\nu} E^{\frac{1-\nu}{\nu}} + (\frac{1-\nu}{\nu}) E^{\frac{1}{\nu}}}{(1 + \delta E)^2} \tag{51}$$

implying that $\partial S_1/\partial E > 0$ as $0 < \nu < 1$, which contradicts Rybczynski theorem. Therefore, we must have $X^s > 0$.

Next, we have to show that $T^s > 0$. To do this we use equations (43) and (44) to get:

$$T = \left(\nu [p_2 \delta S_1 + (1 + p_2 \delta E) \frac{\partial S_1}{\partial E}] \right)^{\frac{1}{1-\nu}} \tag{52}$$

It then follows from Lemma (3) that $T^s > 0$ since $\nu > 0$. Finally, this equilibrium is unique since ξ (ζ) is strictly decreasing (increasing) by Lemma (4). □

We show the steady state equilibrium in Figure 3, where we depict both ξ and ζ. In contrast with the autarky case, it is no longer sufficient to leave the pollution reduction to the natural level. As the economy becomes more specialize in pollution intensive good and therefore emits more, government has to interfere yet in another dimension of the economy to achieve optimality. Recall that even under autarky government interferes as it chooses the emission level by selling permits. Here in free trade case, in addition to choosing the level of emission and selling permits, government taxes the consumers which is used in financing abatement.

4 Conclusion

We developed a dynamic general equilibrium model of pollution permit, abatement, and international trade for a small open economy. The economy produces and consumes two goods, a labor intensive and a pollution intensive good. Government uses two policy tools. On the one hand, it can control the quantity of emission by selling permits. On the other hand, it can conduct pollution abatement activities financed by a lump-sum tax. Apart from the interesting results we presented in this paper, our model offers a holistic view of an economy that produces and exports a pollution intensive good and its government has in its disposal both abatement and pollution permits as policy tools. This aspect of our paper in itself contributes to the literature.

We characterized the steady-state equilibrium under both autarky and free trade. As our main result, we showed that the autarky steady-state equilibrium level of abatement is zero. Although it is sufficient for government to choose a positive emission level to achieve social optimum under autarky, we indicated that such a one-dimensional policy is not enough with free trade assuming that the country specializes and exports the emission intensive good. Under such a scenario, government needs to employ a combination of optimal emission cap (and issue permits accordingly) and pollution abatement.

Our model has various features that can be extended and used in future research. One can use our framework to study issues such as global pollution and international environmental agreements. As another extension, it would be interesting to expand our analysis by incorporating capital formation. Yet another interesting extension is to look at the dynamics of comparative advantage using our setup. In particular, one can address questions such as whether the dynamics of pollution permits and abatement would alter a country's comparative advantage and pattern of trade.

Appendix

It is easier to work with current value problem when analyzing stability of our control problems. First consider the autarky case. Given that $T(t) = 0$ for all t, our system of differential equations can be re-written as:

$$\dot{X} = E - \eta X \tag{53}$$

$$\dot{\lambda} = 2\gamma X + (\rho + \eta)\lambda \tag{54}$$

where $E = (-\lambda/\delta\Omega)^{1/(\sigma-1)}$ and $\lambda \equiv \psi \exp[\rho t]$. Jacobian of the linearized system around the steady-state is given by:

$$J = \begin{pmatrix} -\eta & \frac{1}{1-\sigma}(\frac{\rho+\eta}{2\gamma}) \\ 2\gamma & \rho + \eta \end{pmatrix} \tag{55}$$

where $tr(J) = \rho > 0$. Moreover, as $|J| = -(\rho + \eta)[\eta + 1/(1 - \sigma)] < 0$, our equilibrium is stable in saddle point sense. Recall that $\sigma \in (0, 1)$.

Turn now to the open economy case. The first order conditions of the current value problem are given by:

$$\epsilon\Delta[P_2\delta S_1 + (P_1 + P_2\delta E)\frac{\partial S_1}{\partial E}][(P_1 + P_2\delta E)S_1 - T]^{\epsilon-1} + \lambda = 0 \tag{56}$$

$$\epsilon\Delta[(P_1 + P_2\delta E)S_1 - T]^{\epsilon-1} + \lambda\nu T^{\nu-1} = 0 \tag{57}$$

$$\dot{\lambda} = 2\gamma X + \lambda(\rho + \eta) \tag{58}$$

as well as equation (39). Let $E = \chi_1(\lambda)$ and $T = \chi_2(\lambda)$ be the solution to the system of equations (54) and (55). Recall that we already established that such solution exists at steady-state. We next characterize these functions. By substituting equation (54) in equation (55), simplifying and differentiating with respect to E and T, we obtain:

$$\frac{dT}{dE} = \frac{2\nu P_2\delta}{1-\nu}\Lambda^{\frac{\nu}{1-\nu}}\frac{\partial S_1}{\partial E} < 0 \tag{59}$$

where Λ is defined as in Lemma 3. The inequality follows from Lemma 3 and Rybczynski theorem.

Next for simplicity re-write equation (54) as λ in terms of E and T, then differentiate the resulting equation to obtain:

$$\frac{d\lambda}{dE} = \Delta\epsilon\left[-2P_2\delta\Pi^{\epsilon-1}\frac{\partial S_1}{\partial E} + (1-\epsilon)\Lambda[\Lambda - \frac{dT}{dE}][(P_1 + P_2\delta E)S_1 - T]^{\epsilon-2}\right] \tag{60}$$

where Π is defined as in the proof of Lemma 4 and is positive. As we have already established that $dT/dE < 0$ (due to equation 57), it follows from Lemma 3 and Rybczynski theorem that $d\lambda/dE > 0$. Then, it follows from implicit function theorem that $\chi_1'(E) \equiv dE/d\lambda > 0$. Moreover, this as well as equation (56) imply that $\chi_2'(\lambda) = (dT/dE)(dE/d\lambda) < 0$.

Re-write the system of differential equations (39) and (56) as:

$$\dot{X} = \chi_1(\lambda) - \eta X - [\chi_2(\lambda)]^\nu \tag{61}$$

$$\dot{\lambda} = 2\gamma X + \lambda(\rho + \eta) \tag{62}$$

The Jacobian matrix of linearized system (around the steady-state equilibrium) is given by:

$$J_F = \begin{pmatrix} -\eta & \chi_1'(\lambda^s) - \nu[\chi_2(\lambda^s)]^{\nu-1}\chi_2'(\lambda^s) \\ 2\gamma & \rho + \eta \end{pmatrix} \tag{63}$$

As $|J_F| = -\eta(\rho+\eta) - 2\gamma(\chi_1'(\lambda^s) - \nu[\chi_2(\lambda^s)]^{\nu-1}\chi_2'(\lambda^s)) < 0$, since $\chi_1'(\lambda) > 0$ and $\chi_2'(\lambda) < 0$, the steady state equilibrium is stable in saddle point sense. Note that $tr(J_A) = \rho > 0$.

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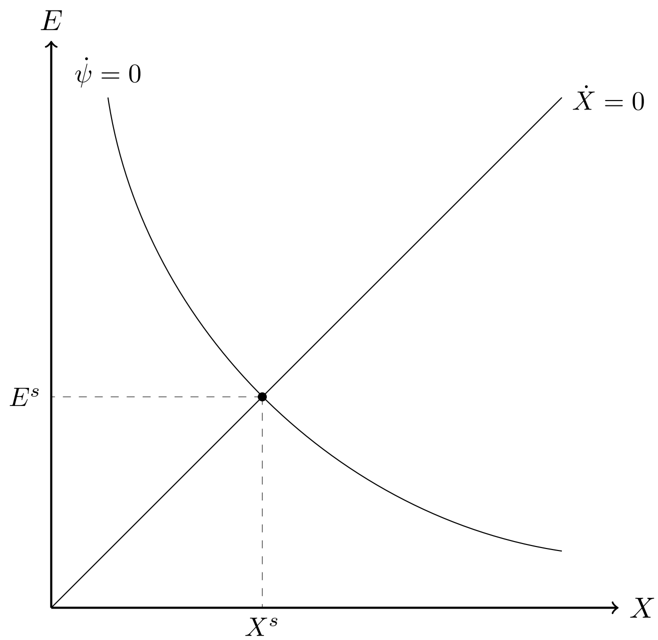

Figure 1: Autarky steady-state equilibrium

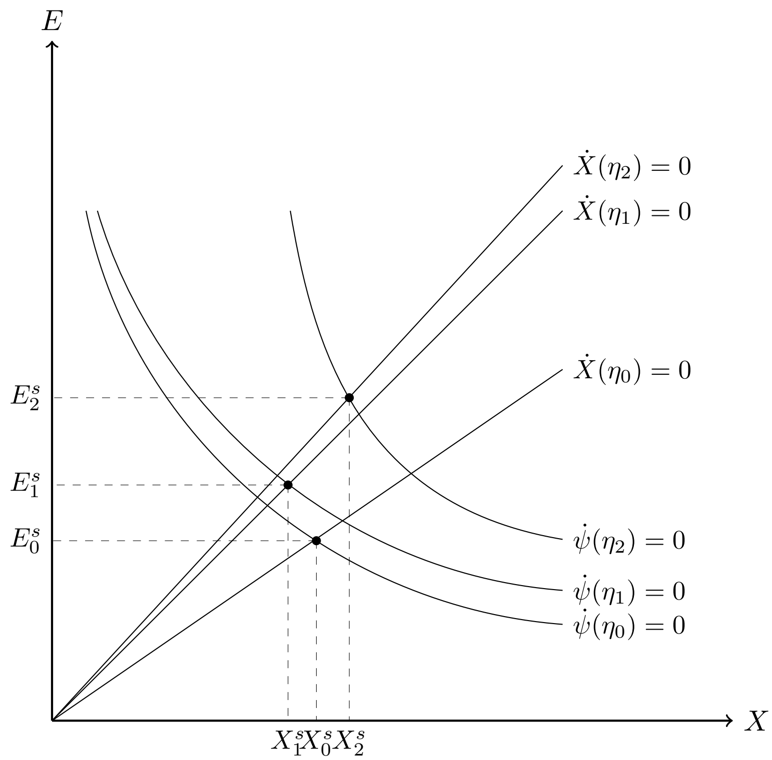

Figure 2: The effect of a change in η on equilibrium: $\eta_0 < \eta_1 < \eta_2$

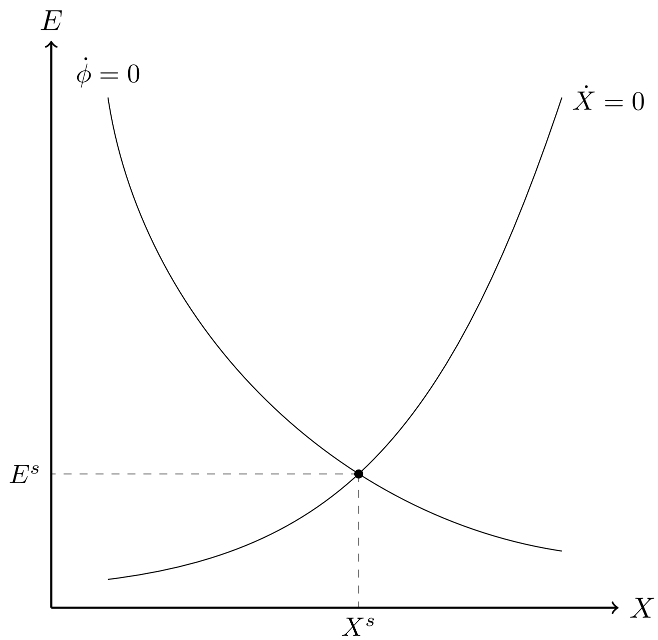

Figure 3: Open economy steady-state equilibrium